THE WAGE RATE AND THE PROFIT RATE IN THE PRICE OF PRODUCTION EQUATION: A NEW SOLUTION TO AN OLD PROBLEM

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Résumé

Le but de cet article est de démontrer que, même si la solution de Marx au problème de la transformation peut être modifiée, ses conclusions restent valables. La nouvelle solution qui est proposée est fondée sur la contrainte d'un taux de profit moyen commun aux deux espaces de valeur et un taux de salaire nominal qui est déterminé simultanément avec les prix. Notre solution diverge de celle de Duménil-Foley-Lipietz quant à l'hypothèse d'un taux de salaire nominal supposé ici endogène et déterminé par la concurrence sur le marché du travail. On ne suppose plus que le salaire est fixé par la valeur d'un panier de subsistance, même si tel est le cas dans l'espace de valeur-travail. Comme on peut le constater, cette solution diffère aussi nettement de celle de Morishima et des néo-ricardiens qui ont accepté le postulat de Samuelson quant à l'indépendance des deux espaces. Notre solution est une alternative à celle de Marx, car elle transforme tous les coûts et maintient les deux contraintes macro et un taux général de profit commun aux deux espaces de valeur.

Mots clés: Transformation, valeur, prix, plus value, profit, salaire, capital, travail, équilibre

Classification JEL: B-14, B-24, D-33, D-46, D-57, E-11, P-16.

Abstract

The aim of this paper is to demonstrate that, even if Marx's solution to the transformation problem can be modified, his basic conclusions remain valid. The proposed alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices. This solution diverges from the Duménil-Foley-Lipietz solution on the assumption of the money wage rate which is assumed here as an endogeneous variable. The money wage level is determined by competition on the labor market. It is no more assumed to be fixed by the value of a subsistence basket, although it is still the case in the labor space. This is also quite different from the Morishima or the neo-ricardian solution, who have accepted the samuelsonian postulate of independence between the two spaces. This solution is an alternative to the Marx solution because it fully transforms the cost of production and maintains the two macro constraints and a general profit rate between the monetary and the social spaces.

Key words: Transformation, value, price, surplus value, profit, wage, capital, labor, equilibrium

JEL classification: B-14, B-24, D-33, D-46, D-57, E-11, P-16.

Introduction[1]

The debate around the transformation problem is more than a century old and it continues to fuel passionate discussions among marxian and neo-ricardian economists and, to a lesser extent, mainstream economists, namely since Samuelson (1971) outlined the independence between the two value systems [2]. The latter views it as a good illustration of the redundancy of the old ricardian and marxian labor value theory while radical economists, who still believe in the validity of an alternative approach to marginal utility theory, periodically attempt to give an adequate answer to valid objections formulated against Marx's way of establishing the correspondence between labor values and prices. Moreover, the « new » solutions proposed by Duménil-Foley-Lipietz (1982) or by the neo-ricardians like Morishima (1973) do not offer an adequate answer to Samuelson's critique. Some radical economists pursue the debate either by looking for a monetary solution in a static framework (Wolff-Roberts-Callari, 1982), either by proposing a dynamic approach (Shaikh 1977, Naples 1989, Freeman 1995, Freeman and Carchedi 1995). As was pointed out accurately by L. Gill (1996, p.532) in quoting M. Desai ... "the debate does not seem to end and is viewed as a rare example of a problem which continues to produce new solutions or reformulation of old solutions in a new mathematical language.... [perhaps] there is more than a simple technical question." Indeed, since the turn of the century, neo-classical economists have attempted to demonstrate the irrelevance of Marx's theory of value.

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[1] This article is a major revision of a first paper written on the new solution to the transformation problem which was published in the working paper 9625, Dept. of Economics, Uninersity of Montreal, under the title « The transformation problem: an alternative solution with an identical aggregate profit rate in the labor value space and the monetary space ». The revised solution presented here is more formal and general.

[2] For a review of this debate, see in particular Dostaler (1978), Beaud and Dostaler (1993), Eatwell, Milgate and Newman (1987-90), Gill (1996), Laibman (1973), Mandel and Freeman (1984), Samuelson (1971), Steedman (1981), Sweezy (1949). The authors who have contributed with new solutions to the debate are Dumenil (1980,1983), Freeman and Carchadi (1995), Foley (1982,1986), Lipietz (1982,1983), Morishima and Seton (1961), Morishima (1973), Naples (1989), Seton (1957), Shaikh (1977), Sraffa (1960), Wolff,Roberts and Callari (1982).

Marx's solution to the transformation problem[3] is based on the hypothesis of the non- transformation of the cost of production (constant and variable costs). It is a simplifying assumption which allows him to concentrate on the distribution of the surplus value and the profit rate in the labor value space and the monetary space. His main interest was to show that, although the profit rate can be different from one sector to another in the value space, the monetary profit rate in a competitive world must be the same in all sectors. The reason why the profit rate varies from one sector to another in the value space is caused by the differences in organic composition of capital due to unequal development between industries. It is also because some sectors do not produce any surplus value, such as the circulation sphere, in particular, the financial sector. Marx was well aware that the non transformation of the production costs was a simplifying assumption and did not consider that his conclusion would be substantially changed if that assumption were modified[4].

His main conclusion is twofold: on a micro basis, prices differ from values -and this is why a labor value theory is relevant as an explanation- but on a macro basis, there are two macro constraints which ought to be satisfied if the principle of value conservation is to be preserved. The first constraint is that the sum of values must be equal to the sum of prices, i.e. the aggregate gross production must be equal in both monetary and abstract labor spaces. The second constraint is that the sum of profits must be equal to the sum of surplus values. Since the transformation problem is a static analysis similar to a general calculable equilibrium approach[5], the same amount of value must be preserved in both spaces when one chooses a general equivalent form for measuring prices. There is more than one way of choosing that price standard:

(i) Choose a level of expenditure common to both spaces and one macro constraint. This is the Marx solution with the hypothesis of no-cost transformation.

(ii) Choose a level of wage common to both spaces and one macro constraint. This is the Duménil-Foley-Lipietz solution (1980, 1982, 1982) of the money wage determined exogeneously and fixed to the nominal wage in the value space

(iii) Choose a level of **real wage** common to both spaces and one macro constraint. This is the Morishima solution (1973) of money wages determined simultaneously with prices. It is also the neo-ricardian solution based on Sraffa's composite commodity (1960) as a standard of value[6].

Only the Marx solution preserves the same general profit rate between the two spaces and satisfies the two macro constraints, although only one constraint is effective. The other two solutions generate a monetary profit rate which is not equal to the general profit rate in the abstract space. Worse still, the other two solutions cannot satisfy the two macro constraints so dear to Marx and his followers. This implies that, even if the macro constraint of value conservation is preserved, some monetary surplus value would be created or lost during the transformation process, an awkward situation, since the core of the labor theory of exploitation is to explain how the monetary surplus value is created in the value space. Samuelson's sharp criticism of Marx's theory of labor value would still remain valid since he pretends that there is no logical connection between the two spaces: the monetary values would be independent of the labor values and, hence, there is no point in having two value spaces to explain what is going on in the real world.

The aim of this paper is to demonstrate that, even if Marx's solution can be modified, his basic conclusions remain valid. It will be shown in particular by the three following points:

(i) Marx's solution requires the specification of one macro constraint despite that his results satisfy the two macro equalities and a common general profit rate. This conclusion is already well known in the literature, but needs to be restated since it is a very crucial point.

(ii) The Duménil-Foley-Lipietz solution requires one macro constraint and satisfies the two macro equalities if the value added is chosen, although the general profit rate is different between the two spaces. The result is not valid anymore if the gross value is chosen as the macro constraint. The Morishima solution yields a similar result, even with the value added.

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[3]See in particular Marx (1967) Book III, part 1 and 2.

[4]Marx (1967), in Book III, chapter 11, p. 165, says ... Since the price of production may differ from the value of a commodity, it follows that the cost- price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it.... Our present analysis does not necessitate a closer examination of this point.

[5]The dynamic approach put forward by M. Naples (1989) is an interesting approach which sould be pursued. After all, even neoclassical economists have developed a dynamic general equilibrium approach; but I still consider that Marx's static equilibrium approach has to be addressed adequately in this debate.

[6]For a survey of the neo-ricardian debate, see Mandel and Freeman (1984) and Steedman (1981). Also, a good criticism of the neo-ricardian approach is found in Naples (1989) and Shaikh (1982).

(iii) The new alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices and will produce results which satisfy the two macro constraints. This solution contradicts the neo-ricardian approach based on a composite commodity as a standard of value, since neo-ricardians argue that there exists a linear relation between profit and wage and, once one of them is specified, the other is automatically determined. Moreover, the average profit rate cannot be determined here in the monetary space only: it is equal to the general profit rate determined in the abstract value space**, and simultaneity excludes independence between the two spaces**.

Definition of a two sector economy

In order to illustrate the various solutions, a two sector economy will be chosen as an example. The value equations in the abstract space are written:

$$\theta_1 x_1 = c_1 + v_1 + pl_1$$
$$\theta_2 x_2 = c_2 + v_2 + pl_2.$$

It is assumed that the turnover of capital is unity for all types of capital c and v, that the exchange rate between one dollar and one unit of social labor is unity, that there exists different organic compositions of capital γ_i and exploitation rates e_i specific to each sector. Hence, the profit rate of each sector will be $r_i = e_i /(\gamma_i +1)$, i=1,2. The unit value of commodity in each sector is θ_i while the output quantities are respectively x_1 and x_2. The gross value measured in abstract labor units is $\theta_i x_i$. Since Marx defines the exploitation rate as the ratio of unnecessary labor to necessary labor for the production of value added, let μ_i be the necessary labor time for one hour of labor and $(1-\mu_i)$ the unnecessary labor time. The relation between the wage rate and the exploitation rate in the labor value space is $\mu_i = (1+e_i)^{-1}$. Assuming that sector I produces consumption goods, the real wage rate in the abstract space is μ_i /θ_1, i=1,2 .

The equivalence of Marx's equations in the Leontief form is

$$q_1 x_1 = a_{11} q_1 x_1 + a_{21} q_2 x_1 + l_1 x_1$$
$$q_2 x_2 = a_{12} q_1 x_2 + a_{22} q_2 x_2 + l_2 x_2$$

The transposed matrix of coefficients $\{a_{ji}\}$ has the usual meaning, i.e. a_{ji} is the quantity of commodity j which is necessary for producing one unit of commodity i while the element l_i is the quantity of living labor neceesry for producing one unit of commodity i. The identification of each component is based on the definition of gross value made by the two components constant capital and value added:

$$c_1 = a_{11} q_1 x_1 + a_{21} q_2 x_1$$
$$c_2 = a_{12} q_1 x_2 + a_{22} q_2 x_2$$
$$v_1 + pl_1 = l_1 x_1$$
$$v_2 + pl_2 = l_2 x_2.$$

These last two equalities can be re-written as

$$v_1 + pl_1 = \mu_1 l_1 x_1 + (1-\mu_1) l_1 x_1 = (1 + e_1)^{-1} l_1 x_1 + e_1 (1 + e_1)^{-1} l_1 x_1$$
$$v_2 + pl_2 = \mu_2 l_2 x_2 + (1-\mu_2) l_2 x_2 = (1 + e_2)^{-1} l_2 x_2 + e_2 (1 + e_2)^{-1} l_2 x_2.$$

The price of production equations can be written in the monetary space according to the marxian notation

$$p_1 x_1 = c'_1 + v'_1 + \pi_1$$
$$p_2 x_2 = c'_2 + v'_2 + \pi_2$$

When transcripted into the Leontief notation, these equations are

$$p_1 x_1 = (1+r) (a_{11} p_1 x_1 + a_{21} p_2 x_1 + w_1 l_1 x_1)$$
$$p_2 x_2 = (1+r) (a_{12} p_1 x_2 + a_{22} p_2 x_2 + w_2 l_2 x_2)$$

The term by term identification is

$$c'_1 = a_{11} p_1 x_1 + a_{21} p_2 x_1$$
$$c'_2 = a_{12} p_1 x_2 + a_{22} p_2 x_2$$
$$v'_1 = w_1 l_1 x_1$$
$$v'_2 = w_2 l_2 x_2$$
$$\pi_1 = r (a_{11} p_1 x_1 + a_{21} p_2 x_1 + w_1 l_1 x_1)$$
$$\pi_2 = r (a_{12} p_1 x_2 + a_{22} p_2 x_2 + w_2 l_2 x_2).$$

Marx's solution

Marx's solution is based on the hypothesis of the non transformation of costs and on the macro constraint that the gross value in the social space must equal the gross value in the monetary space. Translating these hypotheses in the mathematical model, the implication is:

$$c_1 = c'_1 , \quad v_1 = v'_1$$
$$c_2 = c'_2 , v_2 = v'_2$$
$$p_1x_1 + p_2x_2 = q_1x_1 + q_2x_2$$

It is immediately observed that Marx's second macro constraint, -the sum of profits is equal to the sum of surplus values- is already implied by the previous constraints. Indeed,

$$c'_1 + v'_1 + p_1 + c'_2 + v'_2 + p_2 = c_1 + v_1 + pl_1 + c_2 + v_2 + pl_2.$$

Hence,

$$p_1 + p_2 = pl_1 + pl_2.$$

Therefore, Marx's solution does not imply two independent macro constraints, but only one. In that respect, his solution is not different from the other "algebraic" solutions proposed by neo-ricardians, Morishima or by Duménil-Foley-Lipietz.What is interesting, however, is that Marx's particular choice of assumption, -the non-transformation of costs- yields two remarkable results:

(i) the sum of profits equals the sum of surplus values;

(ii) the general or average rate of profit is the same in both spaces, although it is different on a sectoral basis in the social labor space.

This last result is easily deducted from the definition of the general profit rate in both spaces:

value space	monetary space
$r = S\ pl_i / S(c_i + v_i)$	$r' = S\ p_i / S(c'_i + v'_i)$

Since, by hypothesis, $S(c_i + v_i) = S(c'_i + v'_i)$ and, by deduction, $Spl_i = Sp_i$, it follows that $r = r'$.

The monetary value table associated to Marx's solution is easily calculated after solving a system of three simultaneous equations:

(1) $p_1x_1 = (1 + r)(c_1 + v_1) = (1 + r)b_1$

(2) $p_2x_2 = (1 + r)(c_2 + v_2) = (1 + r)b_2$

(3) $p_1x_1 + p_2x_2 = q_1x_1 + q_2x_2 = a$

where b_i and a are constant terms the value of which are determined in the abstract space. the only endogeneous variables are p_1 , p_2 and r. The solution is

$$p_1 = (a/x_1)(1- b_2 /(b_1 + b_2))$$
$$p_2 = (a/x_2)(b_2 /(b_1 + b_2))$$
$$r = (a-(b_1 + b_2))/(b_1 + b_2).$$

The other algebraic solutions

The rejection of the non transformation of costs assumption was judged by some marxists as a betrayal of Marx's insistence on the principle of conservation of value and that no monetary surplus value can be created or destroyed in the transformation process as long as it is analysed in a static framework. The demonstration that the simultaneous solution put forward by the general equilibrium approach would require only one macro constraint did not please at all Marx's disciples since it lead to the rejection of one important constraint. For instance, if the Duménil-Foley-Lipietz solution is calculated by standardizing with respect to the gross value instead of the value added, the sum of profits is no more equal to the sum of surplus values and their solution is not qualitatively different from Morishima's solution or from other neo-ricardian solutions. We can easily show that by choosing the value added constraint, the non-transformation of one component will impose equality on the other component between the two spaces. Indeed, the definition of value added is

abstract space	monetary space
$va = v_1 + pl_1 + v_2 + pl_2$	$va = v'_1 + p_1 + v'_2 + p_2$

The non transformation of variable capital implies $v_1 = v'_1$ and $v_2 = v'_2$. Hence, $pl_1 + pl_2 = p_1 + p_2$. In the Leontief form, these macro constraints are written as

$$va = l_1x_1 + l_2x_2 = w_1l_1x_1 + p_1 + w_2l_2x_2 + p_2.$$

Hence,

$$l_1x_1 - w_1l_1x_1 + l_2x_2 - w_2l_2x_2 = p_1 + p_2.$$

Since $m_i = w_i$ by assumption,

$$(1-m_1)l_1x_1 + (1-m_2)l_2x_2 = p_1 + p_2$$

or

$$pl_1 + pl_2 = p_1 + p_2.$$

Therefore, the assumption of equality of value added between the two spaces is identical to the assumption of equality between the sum of surplus values and the sum of profits. It follows that if the gross value is chosen as the macro constraint between the two spaces, the sum of surplus values is not equal anymore to the sum of profits. Indeed,

abstract space	monetary space
$gv = c_1+v_1+pl_1+c_2+v_2+pl_2$	$gv = c'_1+v'_1+p_1+c'_2+v'_2+p_2$

Assuming that $v_1 = v'_1$ and $v_2 = v'_2$, the macro constraint is reduced to

$$c_1 + c_2 + pl_1 + pl_2 = c'_1 + c'_2 + p_1 + p_2.$$

Unless (c_1+c_2) is assumed equal to $(c'_1+c'_2)$, (pl_1+pl_2) will be different from (p_1+p_2). It necessarily follows that the measurement of the average monetary profit rate will diverge from the general profit rate calculated in the social space, because both the numerator and the denominator of the money profit rate will be different from those already established in the social space. But what would be the effect of imposing the same general profit rate on the now familiar algebraic solutions? This will be the topic of the next section.

The general profit rate constraint

Let usassume the Duménil-Foley-Lipietz solution with the macro constraint of equalizing the gross value between the two spaces. The price equations are

$$p_1\, x_1 = (1+r)\,(a_{11}\, p_1\, x_1 + a_{21}\, p_2\, x_1 + w_1\, l_1\, x_1)$$
$$p_2\, x_2 = (1+r)\,(a_{12}\, p_1\, x_2 + a_{22}\, p_2\, x_2 + w_2\, l_2\, x_2)$$
$$p_1x_1 + p_2x_2 = \theta_1x_1 + \theta_2x_2 = a.$$

Since by hypothesis $w_1 = \mu_1$ and $w_2 = \mu_2$, these equations can be rewritten as

$$p_1 = (1+r)\,(a_{11}\, p_1 + a_{21}\, p_2 + \mu_1\, l_1)$$
$$p_2 = (1+r)\,(a_{12}\, p_1 + a_{22}\, p_2 + \mu_2\, l_2)$$
$$p_1x_1 + p_2x_2 = a.$$

The endogeneous variables are p_1, p_2 and r. The solution is obtained by solving a non linear system of second degree equations with respect to one of the p and then find the corresponding value for r.

If the equality of the general or average profit rate is imposed between the two spaces, r becomes exogeneous in the solution and the macro constraint of the equality of gross value between the two spaces is redundant. Indeed, the price equations becomes a two linear equation system which can be solved easily with respect to the two unknown prices. However, this solution is of no interest since none of Marx's macro constraints are satisfied.

Obviously, it is necessary to modify the assumption of an exogeneous wage rate if one wishes to re-establish the relevance of Marx's macro constraints with the hypothesis of a common general profit rate to the two spaces. Hence, it will be assumed here that the wage rate in the monetary space is determined by competition as any other price and that it can be different from its value determined in the abstract space. Incidentaly, it is assumed here, contrary to the neo-ricardian and marxist current of thought, that the wage rate is not fixed in the monetary space by a subsistence basket equivalent to the real wage, although such an hypothesis is currently admitted by Marx and a number of other people in the social labor space.. One can find many quotations in Capital which can support this viewpoint. Take for instance the excerpt of Book III, as quoted by A. Freeman (1995,p.60):

> « The 20v can similarly diverge from this value, if the spending of wages on consumption involves commodities whose prices of production are different from their values. The worker must work for a greater or lesser amount of time in order to buy back these commodities (to replace them) and must therefore *perform more or less necessary labour* than would be needed if the price of production of their necessary means of subsistence did coincide with their values » (underlined by Freeman).

One now needs to specify a non homogeneous system of three equations. The first two equations will be the price equations and the last will be the equality of the gross value between the two spaces. We do not need to specify the other macro constraint because it is already implicitly contained in the equality of the general profit rate between the two spaces. Indeed,

abstract space monetary space

$$r = Spl_i/S(q_i x_i - pl_i) \quad = \quad r' = Sp_i/S(p_i x_i - p_i)$$
$$1/r = Sq_i x_i/Spl_i \quad = \quad 1/r' = Sp_i x_i/Sp_i$$

If, by hypothesis, $Sq_i x_i = Sp_i x_i$, then $Spl_i = Sp_i$. Therefore, the general profit rate constraint, combined with one of the two macro constraints, gives an analogous result to Marx's no cost transformation assumption. A unique monetary wage rate can be assumed for both sectors or a differenciated wage rate can also be assumed provided that one of the two wage rates can be fixed exogeneous and equal to the subsistence wage of the social or abstract value of its sector. After all, if there are only three equations, a unique solution requires only three unknown variables. If an n equation system is specified, one could either assume a homogeneous labor force and determine endogeneously a unique monetary wage rate, or assume a homogeneous work force in (n-1) sectors, determine endogeneously the corresponding monetary wage rate and fix exogeneously the wage rate in the other sector equal to the wage rate already determined in the social labor space for that sector, or any other combination, such as an (n-k) sectors of homogeneous labor force the unique monetary wage rate of which would be determined endogeneously with the other prices and, in the k other sectors, the monetary wage rate would be fixed exogeneously equal to the wage rates already calculated in the social labor space. This new approach is a major change with respect to the Duménil-Foley-Lipietz solution where it is asummed that it is the wage rate instead of the profit rate which is determined simultaneously with the production prices. The improvment is also felt in terms of calculus, since a linear solution is obtained instead of a non linear one, as simple as the one that was already found with the Marx solution. Hence, the new system is written as

$$p_1 x_1 = d(a_{11} p_1 x_1 + a_{21} p_2 x_1 + w_1 l_1 x_1)$$
$$p_2 x_2 = d(a_{12} p_1 x_2 + a_{22} p_2 x_2 + w_2 l_2 x_2)$$
$$p_1 x_1 + p_2 x_2 = a.$$

Let us assume here that the labor force is heterogeneous and, consequently, the wage rate in the first sector is fixed to $w_1 = \mu_1$ and, in the second sector, it is determined endogeneously with the other prices. Then, the system is written as

$$p_1 x_1 = d(a_{11} p_1 x_1 + a_{21} p_2 x_1 + \mu_1 l_1 x_1)$$
$$p_2 x_2 = d(a_{12} p_1 x_2 + a_{22} p_2 x_2 + w_2 l_2 x_2)$$
$$p_1 x_1 + p_2 x_2 = a.$$

The endogeneous variables are p_1, p_2 et w_2, while the other variables $d = (1+r)$, μ_1 and a are exogeneous and fixed equal to their values already found in the social labor space.

A numerical exemple

In order to illustrate the application of this new solution, a case already elaborated in another article will be used (Loranger, 1996). Tables 1 and 2 contain basic information of a two sector economy, where the measurement unit is one hour of social or abstract labor.

Table 1

sector	constant capital c	variable capital v	surplus value pl	total qX
I	3 240	2160	1 080	6 480
II	2 760	1380	2 070	6 210
Total	6 000	3540	3 150	12 690

It is assumed that the turnover of capital is unity for all types of capital, that the exchange rate between one dollar and one unit of social labor is unity, that sector I produces 4000 units of consumption goods and sector II 2000 units of production goods. It is also assumed that output of sector II is chosen as an input for both sectors and that output of sector I is chosen as an input of that sector in the proportion of 1/5. By inspecting the figures contained in the table, the following results can easily be deducted.

Table 2

sector	quantity	unit value	exploit-ation rate	organic rate	profit rate[7]	wage rate
	x	q	e	g	r	m

[7]Since the turnover of capital is assumed equal to unity for all types of capital, r=e/(g+1).

I	4 000	1.62	0.5	1.5	0.20	.667
lI	2 000	3.105	1.5	2.0	0.50	.40

The values of the technical coefficients of matrix A' and of the labor force vector l are

$$A' = \begin{pmatrix} 0.20 & 0.1565 \\ 0.00 & 0.4348 \end{pmatrix} \qquad l = \begin{pmatrix} 0.81 \\ 1.725 \end{pmatrix}$$

The average profit rate is r = 3150/9540 = 0.3302 and the gross value of production is a = 12690. The system is now written as

(1) $p_1 = 1.3302(.20p_1 + .1565p_2 + (.81)0.667)$

(2) $p_2 = 1.3302(.4348p_2 + 1.725w_2)$

(3) $2p_1 + p_2 = 6.345.$[8]

After re-organizing terms, the system to be solved is

(1) $.7340p_1 - .2082p_2 = 0.7184$

(2) $.4216p_2 - 2.2946w_2 = 0$

(3) $2p_1 + p_2 = 6.345.$

The solution is

$p_1 = 1.773; p_2 = 2.80; w_2 = 0.514$

The corresponding table for the monetary values is[9]

Table 3

sector	**constant capital**	**variable capital**	**profits**	**total**
I	3172	2160	1760	7092
II	2435	1773	1390	5598
Total	5607	3933	3 150	12 690

It is verified that the sum of prices is equal to the sum of social values 12690 and that the sum of profits is equal to the sum of surplus values 3150. The general profit rate is the same in both spaces and equal to .3302. Money prices and wages differ from their values on a sectorial basis:

abstract value	monetary value
$q_1 = 1.796$	$p_1 = 1.667$
$q_2 = 2.754$	$p_2 = 3.012$
$m_1 = .667$	$w_1 = .667$
$m_2 = .40$	$w_2 = .514.$

Conclusion

The proposed new solution diverges from the Duménil-Foley-Lipietz solution on the hypothesis of the money wage rate which is assumed here as an endogeneous variable. The money wage level is determined by competition on the labor market. It is no more assumed to be fixed by the value of a subsistence basket of goods, although it may be so in the social labor space. If the labor force is assumed heterogeneous, one wage rate (which can be applied to many sectors) is determined endogeneously, while the other (or many others) is fixed exogeneously equal to the value already found in the abstract space. This result is in contradiction with the neo-ricardian approach which asumes that the monetary wage rate is fixed at the subsistence level and is a linear relation with the average profit rate. The real wage constraint in the Morishima solution is replaced by the average profit rate constraint simultaneously determined

[8] The macro constraint is $4000p_1 + 2000p_2 = 12690$. Dividing by 2000, one gets the previous equation.

[9] The detailed calculus for the various entries in the table are

$c'_1 = (.20(1.773) + .1565(2.80))4000 = 3172$

$c'_2 = (.4338(2.80))2000 = 2435$

$v'_1 = .81(.6667)4000 = 2160$

$v'_2 = 1.725(.514)2000 = 1773$

$\pi_1 = .3302(3172 + 2160) = 1760$

$\pi_2 = .3302(2435 + 1773) = 1390$

in the social labor space. This hypothesis is fundamental, since it imposes the interdependance between the two spaces, while the neo-ricardians assume Samuelson's hypothesis of independence between the two spaces and, therefore, argue only within the monetary space. This result is fully compatible with the Frobenius-Perron theorem which states that there is a unique profit rate associated with the dominant characteristic root of a matrix formed by all the technical coefficients of the system[10]. This solution is an alternative to the Marx solution because it fully transforms the cost of production and maintains the two macro constraints which link the two interdependent spaces. hence, Samuelson's devastating critique is ill founded!

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[10]This statement may appear ambiguous since the Frobenius root comes from a matrix usually constructed in the monetary space with a unique profit rate for all sectors. It can be shown that it is possible to construct a different matrix in the labor space which would include an exploitation rate in addition to the other technical coefficients. The average profit rate is then a weighted average of the sectorial profit rates.

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