

**THE CONSERVATION OF VALUE
A REJOINDER TO ALAN FREEMAN**
Preliminary Draft

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Version: March 27, 1997.

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RÉSUMÉ

LA CONSERVATION DE LA VALEUR RÉPONSE A ALAN FREEMAN

Cette étude a pour objet la critique des valeurs séquentielles d'Alan Freeman. Dans cette analyse, la valeur est conservée de période en période indépendamment du changement technique et du déséquilibre, en contradiction avec la conception traditionnelle où les valeurs sont réestimées à chaque période selon les conditions de production existantes. Notre désaccord principal est que les valeurs séquentielles vident le concept de dévaluation du capital de son contenu, dès lors que l'économie est considérée globalement. La dévalorisation des marchandises est possible dans le cadre de Freeman, mais la perte de valeur est toujours compensée par un gain correspondant pour une autre marchandise. Cette étude attire également l'attention sur un certain nombre de propriétés troublantes des valeurs séquentielles, en particulier la compatibilité de valeurs croissantes et du progrès de la productivité du travail. Le traitement inhabituel du capital fixe, qui est assimilé à une matière première impérissable, soulève également de sérieuses objections.

ABSTRACT

THE CONSERVATION OF VALUE A REJOINDER TO ALAN FREEMAN

This paper is a critique of Alan Freeman's sequential values. In this approach value is conserved from period to period independently of technical change and disequilibrium, contrary to the traditional view that values are reevaluated at each period depending on the existing conditions of production. Our main disagreement is that sequential values dismiss the notion of devaluation of capital, when the economy is considered globally. Devaluation is possible for individual commodities in Freeman's framework, but the loss of value is always compensated by a corresponding gain for another commodity. The paper also points to a number of puzzling properties of sequential values, in particular the compatibility of increasing values with rising labor productivity. The unusual treatment of fixed capital, in which fixed capital is assimilated to an imperishable raw material, also raises serious objections.

MOTS CLEFS : Théorie de la valeur travail, capital fixe, dévalorisation.
KEYWORDS : Labor theory of value, fixed capital, devaluation.
J.E.L. Nomenclature : B14.

Introduction

Beginning with the first formalizations of the “transformation problem” in the early 20th centuries, there has been, at least, a broad agreement on what was precisely described as Marx’s “mistake” in *Capital*. Inputs and outputs should be evaluated at the same values in the equations accounting for the determination of labor values. This is equivalent to saying that the value of inputs must be estimated on the basis of their present conditions of production, independently of the amount of labor actually required for their production in their past history. What is transferred to the outputs of the period is the present value of inputs. This view, that we will denote as the “traditional” view, is now challenged by a group of researchers (FREEMAN A., CARCHEDI G. 1996). Instead, the contention is that inputs transfer to outputs their historical labor content — value is always conserved. Value is only destroyed by final consumption. Values computed along such lines are labeled “sequential values”.

This debate over the computation of values has important bearings on the “transformation problem”. Since inputs are not reevaluated under the present conditions of production in the sequential value approach, the alleged “contradictions” in Marx’s analysis disappear. Consequently, what we will call “the value conservation principle” restores in a straightforward manner Marx’s demonstration, allowing, in particular, for the satisfaction of the famous two equalities on aggregates: The value of gross output is equal to its price, and total surplus-value is equal to total profit. This approach contradicts other interpretations of the theory of value, in particular, that given nearly twenty years ago by Gérard Duménil and Duncan Foley (DUMÉNIL G. 1980 and FOLEY D. 1982), still known as the “new” interpretation (see FOLEY D. 1997). This approach is faithful to the traditional definition of values, and the equalities among aggregates obtain in a different manner.

This paper is a rejoinder to Alan Freeman’s demonstration in FREEMAN A. 1996, on the basis of a common recognition of the importance of Marx’s labor theory of value, and a same dedication to the restoration of Marx’s framework. The discussion can be conducted at three distinct levels:

1. All advocates of the traditional approach are under violent attack in Freeman’s analysis. The first sentence of his paper reads: “[...] the simultaneous equations approach [*i.e.*, *the traditional definition of values*] of General Equilibrium theory [...]” (p. XXX). We are familiar with this criticism: it means that all Marxist economists who used in the past, or are still using, the traditional conception of values, are actually neo-classicals. We disagree! In our view, it is erroneous to contend that the traditional computation of value assumes that the economy is in an equilibrium and that there is no technical change, and it is specifically wrong to contend that this computation assumes that the economy is in a *Walrasian* equilibrium.
2. The main divergence concerns obviously the *value conservation principle* itself. It is based on an abusive analogy with physics. Value is not a quantum of energy or an electric charge. The value conservation principle leads to an inappropriate treatment of the effects of technical change and of disequilibrium on values.
3. There is, however, another facet to this controversy, that of internal criticism. The new theory is still incomplete (in its treatment of fixed capital). It assumes far more

equilibrium that it acknowledges. It leads to properties which contradict basic insights, and will certainly not help Marxist economists in their investigation of contemporary capitalism.

The discussion below abstracts from a number of important issues, such as the treatment of money or unemployment.

The paper divides into four sections. Section 1 introduces the framework of sequential values, in comparison to that of traditional values, and discusses some of its puzzling properties. The main criticism of the value conservation principle is presented in section 2. Section 3 is devoted to the treatment of fixed capital and some of its deficiencies. Last, section 4 vindicates the traditional approach.

1 - Sequential Values and the Productivity Paradox

This section recalls the main elements of Freeman's analysis. The definition of sequential values and their relationship to Marx's distinction between individual and market values are introduced in the first section. The second section discusses the common points and differences between the traditional and sequential formalisms. The next section contends that Freeman's equations assume more equilibrium than is explicit. The last section provides an example of some of the paradoxical properties of this framework, such as the possible rise of values with a growing labor productivity.

1.1 Creation and Destruction of Value - Individual and Market Values

In many respects, Freeman's view of the creation and destruction of value is traditional:

1. Circulation does not *per se* create or destroy value, but redistributes it within the economy.
2. Value is increased in production, by the amount of socially necessary labor time incorporated. The value of inputs is transferred to that of outputs.
3. Value is destroyed in final consumption.

At this very general level of analysis, there should be a basic agreement.¹

The core of the controversy lies, however, in the notion of "transfer". Transfer is given by Freeman a very general meaning, in particular transfer over time, with which we disagree. According to Freeman, value, once created, can *only* be destroyed in final

1. One could, however, question several options in Freeman's analysis. It would, for example, be more appropriate to contend that value is destroyed when commodities are purchased by final consumers, since goods in the hands of consumers can no longer be called commodities in the strict sense. Value is a social relationship, not the property of a good independent of its link to the market.

consumption. One consequence of this value conservation principle is that inputs are estimated, within value equations, at their value as outputs of the previous period. This is the meaning of the term “sequential” as opposed to “simultaneous”.

A problem is posed to the value conservation approach by the possible coexistence on a market of commodities produced at different periods. This issue is not discussed clearly by Freeman, but we can interpret his view from his equations.

Consider, for example, a stock of inventories of unsold commodities transferred to the next market simultaneously to a new round of production. If technology changes, the two categories of the same commodity, according to their distinct origin, coexist on the market. Following the value conservation principle, the two categories of goods have different “individual values” (since the value of inventories transferred are not reevaluated under the present technique). In this framework, it seems logical to compute the average of individual values, as Freeman does. The same procedure holds in the case of the transmission over time of a stock of raw materials, and of a stock of fixed capital whose service life is larger than one period.

This procedure echoes Marx’s notions of “market value” and “individual value” (MARX K. 1894, Ch. 10). When various amounts of a commodity are produced by different techniques, the value of this commodity, its market value, is the weighted average of the individual values. However, in Marx’s analysis, the notion of market value applies at a particular point in time, when technology is heterogeneous. Freeman uses the same notion in a temporal framework, averaging values inherited from several periods.

1.2 A Comparison of the Formalisms of Traditional and Sequential Values

It seems useful to introduce the presentation of the formalism of traditional and sequential values, by a few remarks concerning the use of formalization in economics in general. The difficulty is to actually translate an economic analysis (with words) into equations, and the adequation between the two approaches must be carefully controlled. The problem of the appropriate degree of complexity is crucial. A formal framework must be simultaneously simple and apt to generalization :

1. Outright complexity must be avoided. Complexity often hides implicit important assumptions. Moreover, it is typically distorted in one specific direction. (One aspect of the model is abusively developed while others are treated simplistically.)²
2. A model must be susceptible to generalization, *i.e.*, made more concrete in one or another direction.

These principles have straightforward implications concerning value analysis. It is quite appropriate to begin the investigation with a simple linear model of single production, in discrete time—the simplest manner of modeling production. Most basic problems can be addressed in this framework, the difference between the traditional and sequential computations of values, in particular. Then, the analysis will have to pass the test of generalization, for example, its extension to joint production. As has been amply shown by a

2. Marx’s analysis in *Capital* provides a clear example of the power of abstraction. A problem is usually treated originally, in the simplest (most abstract) possible framework. The same principles should apply to modeling.

few decades of controversies over joint production, the typical feedback, when difficulties are met, is the discovery that basic concepts have not been correctly defined.³

The above principles suggest that we begin our investigation of sequential values in the simple and natural framework of standard sequential analysis in discrete time, in which production and circulation periods follow one another :

$$\cdots \rightarrow \begin{array}{c} \text{Production} \\ \text{Period } t-1 \end{array} \rightarrow \begin{array}{c} \text{Circulation} \\ \text{Period } t-1 \end{array} \rightarrow \begin{array}{c} \text{Production} \\ \text{Period } t \end{array} \rightarrow \begin{array}{c} \text{Circulation} \\ \text{Period } t \end{array} \rightarrow \cdots$$

Freeman favors a continuous time framework, but there is nothing specific to sequential value analysis which requires the use of continuous time.

Contrary to his claim, Freeman's continuous time framework is not more general than conventional discrete time models :

1. The conventional discrete time framework assumes that the *period of production* is equal to the *unit time period*, and that the *circulation period* is equal to zero. Any other assumptions, for example the consideration of a production period equal to a multiple of the unit time period and a circulation period different from zero, would require the consideration of inventories of goods in process and finished goods. If, for example, the production of a ship takes one year and the unit time period is one day, a stock of goods in process must be considered at each period, the unfinished ship after 1, 2, ..., or 364 days.
2. Freeman criticizes the assumption, within discrete time models, of a same production period in each production process, but his own assumption is even stronger : production periods in his approach are not only of the same duration, but equal to zero. Freeman's model does not solve any of the problems concerning the strictly positive duration of production and circulation periods.⁴

The description of technology is conventional. n goods exist (with $i = 1, \dots, n$). Each production process is denoted with the same subscript as the good produced. Returns to scale are constant. A production process is represented by a row vector of physical inputs, and an amount of labor (a scalar) :

$$A_i, L_i \rightarrow 1 \text{ unit of good } i$$

We use the following notation :

- A Matrix of physical input coefficients
- L Column vector of labor input coefficients
- Λ Column vector of traditional values
- λ Column vector of sequential values

These variables all change over time and must be indexed with the superscript t .⁵

3. See, for example, in DUMÉNIL G., LÉVY D. 1988 and 1989, the definition of values and the discussion of the condition for the existence of positive values. We substitute of the notion of non-reductivity to that of productivity, and the reference to Marx's concepts of individual and market values.

4. The consideration of production and circulation periods different from zero in a continuous time framework would be possible (with production and circulation period of a given duration beginning at any instant), but the model would combine differential equations and a relation of recursion. It would be far more complex.

5. The notation x^t refers to variable x in period t , whereas $(x)^t$ denotes x power t .

The equations for the two definitions of values are :

$$\Lambda^t = A^t \Lambda^t + L^t \quad (1)$$

$$\lambda^t = A^t \lambda^{t-1} + L^t \quad (2)$$

The second equation corresponds to Freeman's equation 13. Two differences can be noted :

1. Λ^t is only a function of the technology in period t .⁶ λ^t further depends on values manifested during the previous circulation period.
2. Traditional values are attached to a *production* period, and sequential values, to a *circulation* period. Due to this relation to circulation, sequential values are congenial to prices. This is explicit in Freeman's analysis: "From one period to the next, p changes [...] $p^t \rightarrow p^t + \Delta_p p^t$, a quantity we have hitherto called λ [...]" (p. XXX). The use of prices, p^{t-1} instead of λ^{t-1} in the equation which corresponds, within his general formalism, to equation 2 is well in line with this conception of the link between values and prices.

There is no significant differences between the two approaches in the treatment of joint production. Problems and solutions are identical. Additional notation is required in the modeling of joint production, B_i and B , respectively denoting the vectors and matrix of the amount of the various goods produced. Production in process i can be represented as :

$$A_i, L_i \rightarrow B_i$$

In this framework, the conventional neo-Ricardian definition of values is based on the following equation :

$$B^t \Lambda^t = A^t \Lambda^t + L^t \quad (3)$$

A number of technical problems may arise when these equations are solved, in particular negative values may obtain. In FREEMAN A. 1996, joint production is not treated, since the matrix X is assumed to be diagonal. The simplest generalization of the sequential approach to joint production is :

$$B^t \lambda^t = A^t \lambda^{t-1} + L^t$$

A common point between the two approaches is that the above difficulties, in particular, the possible existence of negative values, exist in the two formalisms. Sequential values are no exception in this respect.⁷

In DUMÉNIL G., LÉVY D. 1988 and 1989, we rejected the conventional resolution of equation 3, in reference to the distinction between individual and market values.⁸ When a same good is produced in several manners, as in joint production, the relevant framework is that of market value. Each joint production process is disaggregated into as many single

6. If a commodity is produced by different techniques, values refer to the average of individual values, and (A^t, L^t) denotes the average, not the best available, technology.

7. Consider, for example, the technology in period 1 :

$$A^1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, L^1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{and} \quad B^1 = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$$

the new values, λ^1 , during circulation 1 can be derived from those inherited from the previous periods, λ^0 . With $\lambda^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, one obtains $\lambda^1 = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

8. See also FLASCHEL P. 1983.

production processes as commodities produced. The difficulty lies in allocating inputs to the various commodities. A problem of indeterminacy is posed, that the theory of value, in the strict sense, cannot solve. Exactly the same procedure is used in FREEMAN A. 1991.

1.3 The Equation of Sequential Values Assumes Market Clearing

Freeman is certainly right to present his views in the simple formalism of his sections 3 and 4, but he should not claim that his equation does not assume equilibrium in opposition to equation 1: his equation 13 assumes equilibrium on the market.

What a disequilibrium sequential value equation would look like? If markets do not clear (one aspect of disequilibrium), it becomes necessary to consider stocks of inventories of unsold commodities. These inventories are transferred to the next market, and are part of the supply on the next market. If Q_i^{t-1} and D_i^{t-1} denote respectively the supply and demand of good i on market $t-1$, with for example $Q_i^{t-1} > D_i^{t-1}$, a stock of inventories $S_i^t = Q_i^{t-1} - D_i^{t-1}$ is held. With the sequential approach, the new output, Y_i^t , and the inventories, S_i^t , inherited from the previous market, do not have the same individual value. The individual value of the commodity, as produced in period t , will be denoted $\lambda_{i,1}^t$ and, as transferred in the stock of inventories, $\lambda_{i,2}^t$. Individual sequential values are determined by:

$$\begin{aligned}\lambda_{i,1}^t &= A_i^t \lambda^{t-1} + L_i^t \\ \lambda_{i,2}^t &= \lambda_i^{t-1}\end{aligned}$$

Weighting these two values by the corresponding amounts (Y_i^t produced and S_i^t inventories transmitted from the previous market $t-1$), one obtains:

$$\lambda_i^t = \frac{Y_i^t \lambda_{i,1}^t + S_i^t \lambda_{i,2}^t}{Y_i^t + S_i^t} = \frac{Y_i^t (A_i^t \lambda^{t-1} + L_i^t) + S_i^t \lambda_i^{t-1}}{Y_i^t + S_i^t}$$

which is different from equation 2, unless market is in equilibrium in $t-1$, *i.e.*, if S_i^t equals zero.

1.4 A Productivity Paradox?

The adoption of the formalism of sequential values may have unexpected consequences, which question their ability to account for basic features of capitalism, in particular the analysis of technical change and historical tendencies. Consider, for example, a case of “pure” technical progress in which the amounts of physical inputs and living labor both diminish over time. Using the traditional definition of values, one has:

$$\Lambda^t = (I - A^t)^{-1} L^t = \sum_{k=0}^{\infty} (A^t)^k L^t$$

If A^t or L^t diminishes with time, then Λ^t also declines. This is not always true for sequential values.

This puzzling property is already evident in an economy with a single commodity. Technology is as follows:

$$a \text{ units of good} + l \text{ units of labor} \rightarrow 1 \text{ unit of good}$$

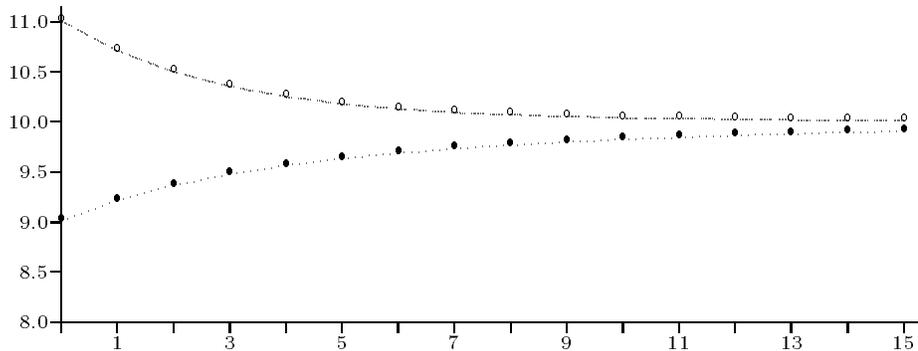
Two cases are considered below, that of two successive periods, and that of an infinite trajectory :

1. In period 1, technology is $a^1 = 0.5$ and $l^1 = 1$. Both technical coefficients are diminished in period 2: $a^2 = 0.44$ and $l^2 = 0.96$. With $\lambda^0 = 1.2$, it is easy to determine λ^1 and λ^2 . One obtains: $\lambda^1 = 1.6$ and $\lambda^2 = 1.664$. Value has been increased: $\lambda^0 < \lambda^1 < \lambda^2$.⁹
2. Still within the same economy with a single good, assume that the amount of physical input is maintained, $a^t = a$, and that of labor reduced at each period: $l^t = \alpha + \beta(\gamma)^t$, with $\gamma < 1$. Sequential values can be determined explicitly:

$$\lambda^t = \frac{\alpha}{1-a} + \left(\lambda^0 - \frac{\alpha}{1-a} + \frac{\beta}{a-\gamma} \right) (a)^t - \frac{\beta}{a-\gamma} (\gamma)^t$$

If $a > \gamma$ and $\lambda^0 + \frac{\beta}{a-\gamma} < \frac{\alpha}{1-a}$, then λ^t always *increases* with time, instead of declining. Figure 1 illustrates this property for $a = 0.9$, $\alpha = 1$, $\beta = 0.1$, $\gamma = 0.7$, and $\lambda^0 = 9$.

Figure 1 Sequential (•) and traditional (◦) values, with rising labor productivity



When labor productivity rises, traditional values (◦) always decline, whereas sequential values (•) may rise.

2 - The Devaluation of Capital

This section is devoted to the criticism of Freeman's value conservation principle, on which sequential values are based. The first two sections address the issues of the devaluation of capital, in relation to either technical change or disequilibrium. A third section briefly discusses Marx's view of these problems.

9. A different λ^0 would yield different results. For example, $\lambda^0 = 1.6$ gives $\lambda^0 < \lambda^2 < \lambda^1$ and $\lambda^0 = 2.4$ gives $\lambda^2 < \lambda^1 < \lambda^0$!

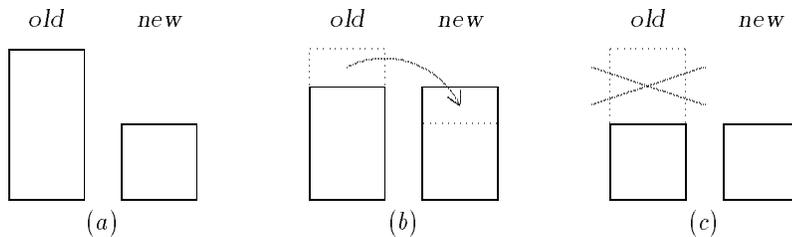
2.1 Technical Change

This section focuses on the problems met in the determination of the value of a commodity produced in the past, under the assumption of technical progress, as less and less labor is required for its production in the present period than was the case in previous periods (what is known as the progress of labor productivity). The difference between the two perspectives is particularly evident in this respect :

1. Within the traditional conception of value, the extra labor embodied in the past is no longer acknowledged as socially necessary labor time on the present market, and the corresponding value vanishes. The capital, in which such obsolete goods exist as components of commodity or productive capital, is devalued.
2. In the sequential value approach, one must distinguish between the individual values of commodities produced under distinct historical conditions and the market value of this commodity, the average of all individual values (section 1.1). The value conservation principle implies that *all individual values are conserved*. The market value can be smaller or larger than the individual values of these various components. Some capitals may be devalued, other reevaluated. What is clear, is that *globally* there is no devaluation (see figure 2).

This property cannot be accepted, since it reduces an important phenomenon, the devaluation of capital in relation to technical change, to a problem of allocation of value among various capitals. On the contrary, devaluation is a threat for capitalists considered globally.

Figure 2 Two Views of the Effects of Technical Change



(a) Individual values of commodities produced with distinct techniques at two different points in time ; (b) Transfer of value and equalization, according to the value conservation principle ; (c) Devaluation of commodities produced in the past, as in the traditional approach.

2.2 Disequilibrium and Crises

The title of the book in which Freeman's study was published refers explicitly to disequilibrium. The introduction of the chapter lists eight aspects of disequilibrium. We fully applaud to the importance conferred on this notion, but violent forms of disequilibrium, which are not considered in Freeman's formalism, actually question his approach, in particular the value conservation principle. This section briefly discusses two examples of such

developments, the destruction of commodity and productive capital, and the problem of the transmission of the value of fixed capital during crises.

Disequilibrium represents a constant threat on commodity or productive capital in capitalism. Each interruption of the circulation of capital is risky. A crisis is a dramatic and general manifestation of disequilibrium. It may lead to the destruction of raw materials, commodities, or fixed capital. In our opinion, these destructions translate directly into devaluations, actual losses of values. The value of commodities which have been destroyed is not transferred to those which survived the crisis, but vanishes.

A more difficult issue is that of the capacity utilization rate, a crucial aspect of disequilibrium in capitalism. Productive capacity is not fully used. Even if we accept the existence of a normal capacity utilization rate different from 100%, say 80%, this level is not constantly reached. These fluctuations mirror day to day maladjustments, as well as business-cycle fluctuations.¹⁰ The central problem, however, is that of the conservation of value during crises. If machines lie idle during a considerable period of time what becomes of their value, is it destroyed, conserved, or transferred?

Overall, the value conservation principle certainly suffers a number of exceptions related to disequilibrium and, in particular, crises. This is an important feature of actual capitalism. It accounts for much of the violence of adjustments in capitalism.

2.3 Marx and the Value Conservation Principle

It is very difficult to claim that Marx was an advocate of the value conservation principle. He never alluded to a compensation (a transfer of value) when capital is devalued, and repeatedly pointed to the opposite property :

But in addition to the material wear and tear, a machine also undergoes what we might call a moral depreciation. It loses exchange value, either because machines of the same sort are being produced more cheaply than it was, or because better machines are entering into competition with it. In both cases, however young and full of life the machine may be, its value is no longer determined by the necessary labour-time objectified in it, but by the labour-time necessary to reproduce either it or the better machine. It has therefore been devalued to a greater or lesser extent¹¹.

A commodity represents, say, 6 working hours. If an invention is made by which it can be produced in 3 hours, the value, even of the commodity already produced, falls by half. It now represents 3 hours of socially necessary labor instead of the 6 formerly required. It is therefore the quantity of labour required to produce it, not the objectified form of that labour, which determines the amount of the value of a commodity¹².

Apart from all the accidental circumstances, a large part of the existing capital is always being more or less devalued in the course of the reproduction process,

10. There is no simple manner of formalizing the utilization of productive capacity in Freeman's fixed capital framework. This remark echoes the fact that this important mark of disequilibrium is not listed in his eight points.

11. MARX K. 1867, Ch. 15, p. 528.

12. MARX K. 1867, Ch. 19, p. 677.

since the value of commodities is determined not by the labour-time originally taken by their production, but rather by the labour-time that their reproduction takes, and this steadily decreases as the social productivity of labour develops¹³.

Marx's discussion of the devaluation of capital during crises is well known. The physical components of capital are at issue, but less than the value embodied in them :

The chief disruption [*in a crisis*], and the one possessing the sharpest character, would occur in connection with capital in so far as it possesses the property of value, i.e. in connection with capital *values*. The portion of capital value that exists simply in the form of future claims on surplus-value and profit, in other words promissory notes on production in their various forms, is devalued simultaneously with the fall in the revenues on which it is reckoned¹⁴.

3 - Fixed Capital

Up to this point, Freeman's framework was considered to diverge from the traditional analysis in only one respect, the definition of value. The problem is more complex concerning fixed capital, since the *description of technology* is also at issue. To the traditional conception in which two machines of different ages are treated as two distinct commodities, Freeman substitutes a view in which they are considered as two distinct quantities of a same good. The purpose of this section is to discuss the relationship between Freeman's modeling of fixed capital and his particular definition of values.

The first section below recalls the standard framework of fixed capital in linear models of production and the traditional computation of values. The next section is devoted to Freeman's analysis, his modeling of fixed capital and the determination of sequential values. A last section discusses the compatibility of sequential values and the standard modeling of fixed capital. *A priori* a general theory of values should be compatible with any "reasonable" modeling of fixed capital :

1. The traditional definition of values is compatible with the two frameworks, the standard model of fixed capital as well as that of Freeman.
2. The problems faced by the sequential definition of values in the traditional modeling of fixed capital seem insuperable. Freeman had to build an alternative framework to render his interpretation compatible with the existence of fixed capital.

3.1 The Standard Modeling of Fixed Capital and the Traditional Computation of Values

We consider here a simple model, which can be easily generalized. Fixed capital is represented by a machine which can be used over two production periods. Its use-value

13. MARX K. 1894, Ch. 24, p. 522.

14. MARX K. 1894, Ch. 15, p. 362.

remains unaltered — the new and one-period old machines produce the same amount, b , of the output — but it must be discarded after two periods. There is no physical inputs other than the machine, and only labor is required. Therefore, technology for the production of each commodity is described by two alternative processes, as follows :

$$\begin{aligned} a \text{ new machines} + l \text{ units of labor} &\rightarrow b \text{ units of the good} + a \text{ old machines} \\ a \text{ old machines} + l \text{ units of labor} &\rightarrow b \text{ units of the good} \end{aligned}$$

The traditional values, Λ_1 and Λ_2 , of the new and old machines can be easily determined in the subsector producing the machine itself, which can be isolated from the rest of the economy :

$$\begin{aligned} a\Lambda_1 + l &= b\Lambda_1 + a\Lambda_2 \\ a\Lambda_2 + l &= b\Lambda_1 \end{aligned} \tag{4}$$

It follows that $\Lambda_2 = \frac{1}{2}\Lambda_1$, *i.e.*, half of the value of the machine is transferred during each production period. The determination of $\Lambda_1 = \frac{2l}{2b-a}$ has little interest, except to recall the condition : $b > a/2$.

3.2 Freeman's Treatment of Fixed Capital and the Determination of Sequential Values

Freeman's line of argument in *Age doesn't matter* (p.XXX) is difficult to follow. He first considers the example of an imperishable raw material: cooper. We certainly agree that, for this particular category of good, it would be possible to abstract from age. However, it is impossible to follow Freeman when he extends this assumption — not a simplifying assumption, but the establishment of a new approach — to all constant capital, circulating or fixed. To us, age matters. This seems obvious for some perishable inputs, say cheese, but is also true for fixed capital.

In Freeman's approach, machines are treated like imperishable raw materials. Old and new machines are different quantities of a same good. During a production process, a "fraction" of machines is consumed. For example, if the initial number of machines is two, and if their service life is of two periods, one new machine exists after production in Freeman's approach, when the traditional modeling states that two old machines emerge from the production process.

Although we believe Freeman's approach is not the best possible approximation, the treatment is coherent and can be formalized. To be closer to Freeman's analysis, we assume that only one other commodity exists in the economy, a total of two commodities, the machine and a consumption good. Each commodity is produced by a distinct production process using machines and labor. In each process, the fraction of the stock of machines which has not been consumed is conserved (half of the machines if the service life is 2 periods) :

$$\begin{aligned} a \text{ machines} + l \text{ units of labor} &\rightarrow b \text{ machines} + \frac{a}{2} \text{ machines} \\ a' \text{ machines} + l \text{ units of labor} &\rightarrow b' \text{ units of consumption good} + \frac{a'}{2} \text{ machines} \end{aligned}$$

or :

$$\begin{aligned} (a, 0), l &\rightarrow \left(\frac{a}{2} + b, 0\right) \\ (a', 0), l' &\rightarrow \left(\frac{a'}{2}, b'\right) \end{aligned}$$

The issue is now the computation of values. There is no problem to determine traditional values in this framework.¹⁵ Here is how we understand Freeman's computation of sequential values. Since the machine is the output of several processes, the relevant framework is that of individual and market values. The term $\lambda_{1,1}^1$ denotes the individual value of the machine in the first process, and $\lambda_{1,2}^1$, its individual value in the second process. There is no specific difficulty concerning the first process, which allows for the determination of $\lambda_{1,1}^1$ as a function of λ_1^0 which is given :

$$a\lambda_1^0 + l = \left(\frac{a}{2} + b\right) \lambda_{1,1}^1 \quad (5)$$

The second process is similar to a case of joint production : In addition to the output of consumption good, b' , we also find $a'/2$ machines. Freeman substitutes for this joint production process, two processes of single production. The first one produces the consumption good with half of the stock of machines, and labor. The second half is conserved without alteration :

$$\begin{aligned} \left(\frac{a'}{2}, 0\right), l' &\rightarrow (0, b') \\ \left(\frac{a'}{2}, 0\right), 0 &\rightarrow \left(\frac{a'}{2}, 0\right) \end{aligned}$$

As can be easily checked, summing the two processes, one obtains the original process. To each single production process corresponds one value equation :

$$\frac{a'}{2}\lambda_1^0 + l' = b'\lambda_2^1 \quad (6)$$

$$\frac{a'}{2}\lambda_1^0 = \frac{a'}{2}\lambda_{1,2}^1 \quad (7)$$

From equation 6, one can directly determine the value of the consumption good :

$$\lambda_2^1 = \frac{\frac{a'}{2}\lambda_1^0 + l'}{b'} \quad (8)$$

For the production good, one must combine equation 5 to equation 7. The computation of the average of these individual values provides the value of λ_1^1 :

$$\lambda_1^1 = \frac{\left(\frac{a}{2} + b\right) \lambda_{1,1}^1 + \frac{a'}{2} \lambda_{1,2}^1}{\frac{a}{2} + b + \frac{a'}{2}} = \frac{a\lambda_1^0 + l + \frac{a'}{2}\lambda_1^0}{\frac{a}{2} + b + \frac{a'}{2}} \quad (9)$$

15. One finds :

$$\Lambda_1 = \frac{2l}{2b - a} \quad \text{and} \quad \Lambda_2 = \frac{b}{b'} \frac{2l' + \frac{a'l - al'}{b}}{2b - a}$$

Equations 9 and 8, giving the values of λ_1^1 and λ_2^1 , are identical to Freeman's equations 23 and 24, for $a = 70$, $l = 300$, $b = 50$, $a' = 20$, $l' = 200$, and $b' = 100$.

3.3 Sequential Values in the Standard Modeling of Fixed Capital

The purpose of this section is to discuss the compatibility of Freeman's definition of value with the traditional modeling of fixed capital: Can the sequential value approach apply in this framework? In what follows two such attempts have been tried without success. It is difficult to contend that this extension is impossible, but it appears, at best, uneasy. Problems arise in the dynamic properties of sequential values, and this is the viewpoint adopted in this section.

The problematic character of sequential values in the traditional modeling of fixed capital can already be demonstrated in a very simple model in which only one good is produced, and in the absence of technical change. This good can be used either as consumption good or production good. In this latter case, it can be used during two periods, with the same use-value during the two periods (as assumed in the previous section). However, two "goods" must be distinguished in the formalism. The first good is that which has just been produced, and the second is the production good produced one period earlier. Thus, technology can be described as:

$$\begin{aligned} a \text{ new machines} + l \text{ units of labor} &\rightarrow b \text{ new machines} + a \text{ old machines} \\ a \text{ old machines} + l \text{ units of labor} &\rightarrow b \text{ new machines} \end{aligned}$$

or :

$$\begin{aligned} (a, 0), l &\rightarrow (b, a) \\ (0, a), l &\rightarrow (b, 0) \end{aligned}$$

The sequential value approach does not provide any clue concerning the treatment of this case. The value conservation principle is not sufficient, and an additional assumption must be made.

A first assumption could be that the individual value of the good is the same in the two processes. This assumption seems quite natural, since the two processes use the same amounts of inputs (a units of consumption good and l units of labor) to obtain the same output (b units of output). The value equations can be easily written under this assumption:

$$\begin{aligned} a\lambda_1^t + l &= b\lambda_1^{t+1} + a\lambda_2^{t+1} \\ a\lambda_2^t + l &= b\lambda_1^{t+1} \end{aligned}$$

Subtracting the second equation from the first, one obtains a relation of recursion :

$$\begin{aligned} \lambda_1^{t+1} &= \frac{a}{b}\lambda_2^t + \frac{l}{b} \\ \lambda_2^{t+1} &= \lambda_1^t - \lambda_2^t \end{aligned} \tag{10}$$

The fixed point can be determined from :

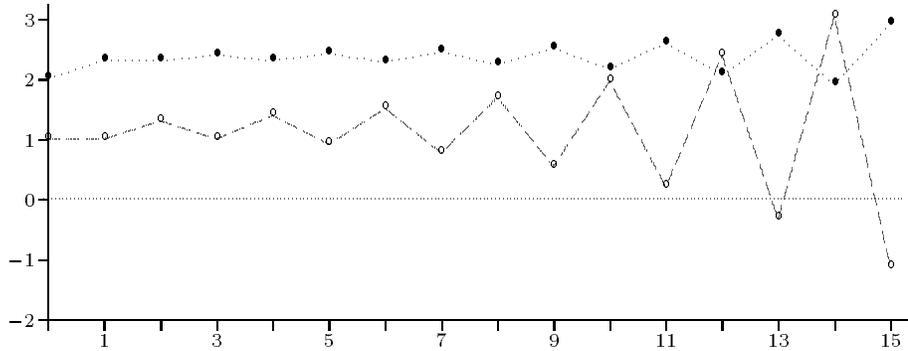
$$\begin{aligned} \lambda_1^* &= \frac{a}{b}\lambda_2^* + \frac{l}{b} \\ \lambda_2^* &= \lambda_1^* - \lambda_2^* \end{aligned}$$

One obtains :

$$\lambda_2^* = \frac{\lambda_1^*}{2} = \frac{l}{2b-a}$$

i.e., traditional values ($\lambda_i^* = \Lambda_i$). The problem is that the recursion is always *unstable*.¹⁶ This instability is manifested in the fact that, beginning with any initial values other than traditional values, one of the two values becomes necessarily negative (see figure 3).

Figure 3 Negative sequential value in a simple fixed capital model



The determination of sequential value in the usual fixed capital framework raises considerable problems. In the example in this figure, the sequential values of a new machine and a one-period old machine fluctuate over time. Two puzzling properties are observed. The value (\circ) of the older machine becomes recurrently larger than that (\bullet) of the new machine, and sometimes negative.

An alternative to the above assumption could be that the production good transfers its value in proportion to time. If λ_1^t is the value of the new production good, it depreciates by half of its value at each period and the other half remains in the old machine: $\lambda_2^{t+1} = 1/2\lambda_1^t$. Under this assumption, the first process can be written :

$$\frac{a}{2}\lambda_1^t + l = b\lambda_1^{t+1}$$

and the second (with $\lambda_2^t = \frac{1}{2}\lambda_1^{t-1}$) :

$$\frac{a}{2}\lambda_1^{t-1} + l = b\lambda_1^{t+1}$$

These two equations are incompatible, except if $\lambda_1^t = \lambda_1^{t-1}$, *i.e.*, if the values are constant over time (and equal to traditional values).

16. The recursion 10 can be written :

$$\begin{pmatrix} \lambda_1^{t+1} - \lambda_1^* \\ \lambda_2^{t+1} - \lambda_2^* \end{pmatrix} = M \begin{pmatrix} \lambda_1^t - \lambda_1^* \\ \lambda_2^t - \lambda_2^* \end{pmatrix} \quad \text{with} \quad M = \begin{pmatrix} 0 & \frac{a}{b} \\ 1 & -1 \end{pmatrix}$$

The polynomial characteristic of the Jacobian matrix is :

$$P(\mu) = \det(\mu I - M) = \begin{vmatrix} \mu & -\frac{a}{b} \\ -1 & \mu + 1 \end{vmatrix} = \mu(\mu + 1) - \frac{a}{b}$$

One root is always smaller than -1 .

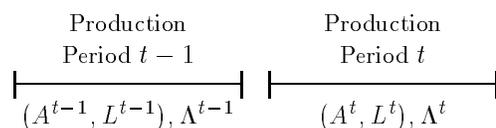
These difficulties, met in an extremely simple formalism, basically question the claim that sequential values provide a general theory of values. They appear rather quite dependent on a specific modeling of fixed capital, on which important reservations can be made.

4 - The Traditional Approach under Attack

The first section below challenges Freeman's criticisms of the traditional approach concerning technical change and disequilibrium. The second section contrasts Freeman's view of the explanatory power of the labor theory of value to ours. A last section addresses the issues of equilibrium and dynamics, on which we also diverge.

4.1 Freeman's Criticisms

Freeman's first criticism is that the traditional interpretation is incompatible with the existence of technical change. Consider two successive production periods, with distinct techniques :



The technique during the first period is (A^{t-1}, L^{t-1}) , and values are Λ^{t-1} . During the second period they become (A^t, L^t) , and Λ^t . *A priori*, Λ^t differs from Λ^{t-1} . Since the outputs of the first period are precisely the inputs of the second, they have two distinct values—a fact that Freeman cannot accept. To him the traditional interpretation is not compatible with technical change. In the same vein, the use of simultaneous equations implies, still following Freeman, that equilibrium prevails.

Freeman would be right if values were prices. A commodity cannot have a price as the output of one production period, and another as an input of the next period, since there is only one transaction. Similarly, if equation 1 were a price equation, it would express an equilibrium, a fixed point in a relation of recursion. But values are *not* prices.

A commodity can have a value, when considered in relation to the conditions of production in one period, and another one, when considered in relation to the conditions of production of the next period. Semantically, the expression “the value of a commodity” is an abbreviation for “the value of a commodity in the conditions of production prevailing at this particular, present or past, instant”. In other words, the reference to value independently of specific conditions of production is irrelevant, and a commodity has as many distinct individual values as conditions of production.

4.2 The Use and Abuse of the Labor Theory of Value

We believe we can agree with Freeman on the following. The labor theory of value is an analytical tool to understand the functioning of capitalism. It is fundamental in a sense, since it provides the basis for the development of the theory of *capital*. Capital, following Marx, is “value” in a process of self-engrossment. “Value in process” refers to the circulation of value-capital through its various forms of money, commodity, and productive capital, as in Volume II of *Capital*. “Self-engrossment” stands here for the theory of surplus value. This is where the labor theory of value is unescapable.

The core of the explanatory power of the labor theory of value lies in the analysis of exploitation in capitalism. Neither Walrasian equilibrium nor Sraffa’s framework allow for its understanding. Several components of the analysis are combined: (1) Only labor creates value; (2) Value can be created in one point of the productive system and realized somewhere else (and this explains why the notions of prices or physical bundles are not sufficient); (3) Through their wages, workers recover a purchasing power over a fraction of the total value they have created in one period.

The disagreement with Freeman concerns the *extension* of the explanatory power of the labor theory of value. The analogy with physics is very misleading (DUMÉNIL G., LÉVY D. 1997). It is true, in physics, that Maxwell’s equations and Newton’s gravitation equations provide the foundations on which the physics and chemistry should be established. But the attempts to ground all economics on the labor theory of value (or any other fundamental principle) are misdirected. The labor theory of value is not the necessary foundation for the analysis of every mechanism in capitalism.¹⁷ For example, the gravitation of prices around prices of production must be established independently of the theory of value. Contrary to Freeman, we believe that other theories also exist independently of labor value, such as the theory of crisis or of historical tendencies. In particular, the labor theory of value does not provide the framework to account for disequilibrium and dynamics in capitalism.

The statement that the analysis of disequilibrium in capitalism is alien to the explanatory power of the labor theory of value does not mean that disequilibrium is not a central aspect of capitalism. We have been working for years on this issue. One aspect of this work was, in particular, to translate in equations Marx’s description of behaviors in competition, building disequilibrium microeconomics based on adjustment behaviors (DUMÉNIL G., LÉVY D. 1993).

4.3 Equilibrium, Disequilibrium, and Dynamics

A final disagreement concerns the use of equilibrium in economic theory, that Freeman does not like. This issue touches upon a large number of interesting aspects of economic theory. We will not repeat here basic principles that we presented in other works, and that we used as guidelines in actual theoretical and empirical research, but limit our comments here to three points where we obviously diverge with Freeman :

17. Even if this were the case, additional assumptions would be required. For example, the tendency for the rate of profit to fall cannot be proven independently of assumptions on technical change.

1. *The usefulness of a theory of equilibrium.* That the economy is always in disequilibrium does not dismiss the theory of equilibrium as irrelevant. A well-known example is the theory of prices of production.¹⁸
2. *All theories of equilibrium are not equivalent.* Walrasian equilibrium is apologetic, and does not provide a faithful account of the working of capitalism. Classical (Marx's) theory of long-term equilibrium is an important tool in the analysis of capitalism. One must, in particular, distinguish between an *ex post* conception of equilibrium in which equilibrium is the fixed point of a realistic disequilibrium dynamic system (with stocks and flux relationships and transactions out of equilibrium), and an *ex ante* equilibrium in which no such process can be defined.
3. *Dynamics and disequilibrium are not synonymous.* The appropriate framework of analysis must combine disequilibrium and dynamics, but a dynamic framework can assume equilibrium as is the case in a sequence of Walrasian temporary equilibria (or in the equations of sequential values in the absence of inventories and other disequilibria).

18. The existence of structural change does not refute the theory of classical long-term equilibrium. Market prices gravitate around a target (prices of production) moving over time (DUMÉNIL G., LÉVY D. 1995).

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Contents

Introduction	1
1 - Sequential Values	2
1.1 Creation and Destruction of Value - Individual and Market Values	2
1.2 A Comparison of the Formalisms of Traditional and Sequential Values	3
1.3 The Equation of Sequential Values Assumes Market Clearing	6
1.4 A Productivity Paradox?	6
2 - The Devaluation of Capital	7
2.1 Technical Change	7
2.2 Disequilibrium and Crises	8
2.3 Marx and the Value Conservation Principle	9
3 - Fixed Capital	10
3.1 The Standard Modeling of Fixed Capital and the Traditional Computation of Values	10
3.2 Freeman's Treatment of Fixed Capital and the Determination of Sequential Values	11
3.3 Sequential Values in the Standard Modeling of Fixed Capital	13
4 - The Traditional Approach under Attack	15
4.1 Freeman's Criticisms	15
4.2 The Use and Abuse of the Labor Theory of Value	15
4.3 Equilibrium, Disequilibrium, and Dynamics	16
References	18
Contents	19