Simultaneous Valuation vs. the Exploitation Theory of Profit

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Despite their other differences, all interpretations of Marx's value theory agree that it identifies the exploitation of workers, i.e., the extraction of surplus-labor, as the sole source of profit. Proponents of the various interpretations, moreover, all claim to have replicated this feature of his value theory. Yet the mathematics of their systems often tells a different story. As I will show, in those systems in which the prices and values of inputs are determined simultaneously with the prices and values of outputs, the extraction of surplus-labor is insufficient and, generally, unnecessary for the existence of positive profit. In these "simultaneist" interpretations, then, surplus-labor is not the sole source of profit.

It is well known that, when joint products are produced, the standard interpretation is incompatible with Marx's theory of profit (see Steedman (1977)). As section 1 will show, however, even without joint production, all simultaneist interpretations (not only the standard one) are incompatible with his theory.[i]

Because theorists have failed to study the problem in a general setting, this incompatibility has not received attention. In some special cases — those in which, in every period, a positive physical surplus of every good is produced or the aggregate price of the net product is positive — simultaneist interpretations do imply that surplus-labor and positive profit go hand in hand. Yet section 2 will demonstrate that this result cannot be generalized. I will argue, moreover, that these special cases impose conditions that are much more restrictive and less plausible than is usually thought. In particular, economies can easily reproduce themselves physically without fulfilling either

of these conditions.

Even under the most general conditions, on the other hand, surplus-labor becomes both necessary and sufficient for positive profit once simultaneous valuation is eschewed. This will be shown in section 3. Section 4 contains a brief summary and conclusion.

1. The Incompatibility

A. The Fundamental Marxian Theorem

In the standard interpretation of Marx's value theory, distinct price and value systems exist, and the inputs and outputs in each are valued simultaneously. Another distinctive feature of this interpretation is that it construes wages in the price system as the price of the wage goods workers receive, and wages in the value system as the value of these wage goods.

Employing this interpretation, Okishio (1993a, 1993b) discovered a set of theorems which Morishima (1973) later dubbed the "fundamental Marxian theorem" (FMT). The FMT is often said to have shown that surplus-labor is necessary and sufficient for positive profit when no joint products are produced (see, e.g., Howard and King (1994, pp. 230, 239)). Yet the FMT relies crucially on a very restrictive condition: in every period, a positive physical surplus of every good is produced.[ii]

Physical surplus is output net of both consumed inputs and workers' consumption, and, in this interpretation, profit is simply the vector of physical surpluses valued at end-of-period (replacement) prices. Using the usual input-output notation,[iii] the column vector of physical surpluses is $\mathbf{f} = (\mathbf{I} - \mathbf{A} - \mathbf{b}\ell)\mathbf{x}$, so profit is

$$\pi = \mathbf{pf} \qquad (1)$$

where **p** is a row vector of unit prices. Unit values are defined as the row vector of vertically integrated labor coefficients **?**, so surplus-labor, s, is the living labor extracted minus the value of wage goods: $s = \ell\mathbf{x} - \mathbf{?b}\ell\mathbf{x}$. But since $\mathbf{?} = \ell(\mathbf{I} - \mathbf{A})^{-1}$, it follows that $\mathbf{?}(\mathbf{I} - \mathbf{A}) = \ell$ and thus that $\ell\mathbf{x} = \mathbf{?}(\mathbf{I} - \mathbf{A})\mathbf{x}$. Surplus-labor can thus be expressed as $s = \mathbf{?}(\mathbf{I} - \mathbf{A})\mathbf{x} - \mathbf{?b}\ell\mathbf{x} = \mathbf{?}(\mathbf{I} - \mathbf{A} - \mathbf{b}\ell)\mathbf{x}$, or simply as

$$s = \mathbf{?f}. \qquad (2)$$

In the standard interpretation, then, profit and surplus-labor are simply the same vector of physical surpluses valued in two different ways. When all elements of **f** are positive, i.e., when a positive physical surplus of every use-value is produced, it is obvious that the FMT holds. Both π and s must then be positive, given only that no prices or values are negative and that some of both are positive. Because all physical surpluses are positive, it does not matter that prices and values differ, or by how much; a set of strictly positive physical surpluses valued according to either must be positive.

It is, however, equally obvious that the FMT fails to hold unless all physical surpluses are positive. Once there is a negative physical surplus of some good, it matters that values and prices differ. The total "worth" of the physical surplus vector can then be negative when valued at prices and positive when valued at values, or vice-versa.[iv] Assume, for instance, a two-good economy, in which $f_1 = -1$ and $f_2 = 2$. If $?_1 = 19$ and $?_2 = 10$, then $s = 19(-1) + 10 \cdot 2 = 1$. Yet if $p_1 = 21$ and $p_2 = 10$, then $\pi = 21(-1) + 10 \cdot 2 = -1$. If, however, $?_1 = 21$ and $p_1 = 19$, then $s = -1$ but $\pi = 1$. This proves that, under

the standard interpretation, surplus-labor is neither sufficient nor necessary for profit to exist.

B. The "New Interpretation" and Simultaneous Single-System Interpretations

In the past two decades, other simultaneist interpretations of Marx's value theory have also emerged. In the context of the present paper, the key difference between the standard interpretation, on the one hand, and both the "New Interpretation" (e.g., Duménil (1983), Foley (1982)) and the simultaneous single-system interpretations (e.g., Lee (1993), Moseley (1993), Ramos and Rodriguez (1996))[v], on the other, concerns their definitions of wages and surplus-labor.

Rather than defining wages as the price or value of wage goods, the latter interpretations construe wages as the sum of money paid to workers. To assess whether surplus-labor is extracted, money wages are converted into the equivalent sum of labor-time (or living labor is converted into a monetary equivalent). The ratio of the aggregate net product, $(\mathbf{I} - \mathbf{A})\mathbf{x}$, valued at end-of-period (replacement) prices, to living labor,

$$\mathsf{s} = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{x}/\ell\mathbf{x} \qquad (3)$$

is used to convert monetary sums into labor-time sums. I call this ratio s to indicate the "simultaneist monetary expression of labor-time." It is alleged to be the ratio between the monetary and labor-time measures of value added.

In these interpretations, profit is thus defined as the vector of physical net products, valued at end-of-period prices, minus the wage bill:

$$\pi = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{x} - w\ell\mathbf{x}, \qquad (4)$$

where w is the money wage per unit of living labor extracted, and surplus-labor is defined as living labor minus the labor-time equivalent of the money wage:

$$s = \boldsymbol{\ell x} - (1/\mathsf{s})\, w\boldsymbol{\ell x}. \qquad (5)$$

Multiplication of (5) by s yields $\mathsf{s}s = \mathsf{s}\boldsymbol{\ell x} - w\boldsymbol{\ell x} = \mathbf{p}(\mathbf{I} - \mathbf{A})\mathbf{x} - w\boldsymbol{\ell x}$, or, equivalently,

$$\pi = \mathsf{s}s. \qquad (6)$$

This result has led proponents of the "New" and single-system interpretations to claim that they yield an exact correspondence between surplus-value and profit. Not only is surplus-labor necessary and sufficient for positive profit, but the magnitudes of surplus-labor and profit are strictly proportional.

Yet it simply does not follow from this proportionality that surplus-labor is sufficient for positive profit. Indeed, it is not sufficient. Equation (3) implies that, if the net product valued at end-of-period prices is negative, then so is s. Profit is therefore negative although surplus-labor is positive.

Another perverse implication of these interpretations is that, when the price of the net product and therefore s are negative, equation (5) implies that a fall in the money wage rate will lead to a fall, rather than a rise, in the amount of surplus-labor extracted! This paradox, like the one above, discloses a serious conceptual flaw in the claim that the price of the net product is, by definition, the monetary value added by living labor.[vi]

The proportionality of surplus-labor and profit also fails to imply that surplus-labor is necessary for profit to exist. As Dmitriev (1974) discovered, if we imagine a fully automated economy that produces a positive net product of all goods — and if, in addition, prices in such an economy exist and are positive — then profit as

defined above is positive, even though no labor or surplus-labor is extracted.

Apart from this case, the interpretations in question do imply that, when the price of the net product happens to be positive, positive profit and positive surplus-labor will coexist. The relevant issue, however, is not *whether* they coexist, but *why*. Unless a theory denies that profit could be positive if no human labor were employed — and those under consideration seem not to do so[vii] — then we must conclude that it admits the possibility of positive profit without surplus-labor.

2. Reproduction

A. General Issues

Perhaps the main reason that the obvious points made in section 1 have not received attention is that theorists have been interested in economies that are able to reproduce themselves physically. Negative physical surpluses, or a negative price of the net product, have been thought to imply an economy incapable of long-run reproduction, and have therefore been deemed unworthy of theoretical consideration.

In response, four things may be noted. First, the appeal to physical reproducibility is either an evasion of the issue at hand or the result of a logical fallacy. Even were it true that, *if* an economy is capable of reproduction, *then* both surplus-labor and profit are positive according to the simultaneist interpretations, it would not follow that surplus-labor is either necessary or sufficient for positive profit. Analogously, *if* I am a man, *then* I am both male and adult. Yet not all males are adults, nor are all adults males.

Second, the existence of negative net products of some goods is not as implausible a condition as is sometimes suggested, and negative physical surpluses are even more plausible. Net products can be negative even in very productive economies. The Hawkins-Simon conditions, which stipulate that more of a good is produced than is consumed (directly and indirectly) in its own production, do not preclude negative net products, the existence of which also depends on scales of production. Imagine, for instance, that good A is the only input into the production of itself and of good B, and that each process requires ½ unit of good A per unit of output. Thus, only ½ unit of A is needed to produce 1 unit of A. Yet if 10 units of good A and 12 units of good B are produced, then the net product of A equals $10 - (½)10 - (½)12 = -1$.

Third, to exclude negative physical surplus and negative net product cases as unworthy of theoretical consideration is to deem all actual economies unworthy of theoretical consideration. All actual economies produce some negative net products, because some goods (386 computers, for instance) are used as inputs without being reproduced. The economies sustain themselves and even grow by producing, instead, similar but not identical goods (586 computers).[viii]

As was noted above, when they claim that surplus-labor is sufficient for positive profit, simultaneist theorists must postulate either that all net products are positive or that the aggregate price of the net product is positive. Since this postulate is violated in every actual economy, it follows that the claim does not apply to the real world. It is impossible, moreover, for simultaneists to construct comparable theorems to cover real-world situations, because simultaneous valuation is impossible when some inputs are not reproduced as outputs. To compute the aggregate price of the net product, one takes

the gross price of the outputs and subtracts the *replacement* cost of the inputs, i.e., the vector of inputs pre-multiplied by their end-of-period prices. Yet inputs that have been used up without being reproduced do not have end-of-period prices, so this is impossible.

One could, of course, use their prices when they entered production, but then one would not be valuing inputs and outputs simultaneously. The only other alternative is to impute end-of-period prices to the inputs by trying to establish an equivalence between them and goods which have replaced them as outputs. Yet any attempt to homogenize heterogeneous things is not only conceptually dubious; it also leads to arbitrary results. One estimate may conclude that the price of the aggregate net product is positive, while another, even slightly different, estimate may conclude that it is negative. The truth of a theorem that surplus-labor is necessary and/or sufficient for positive "profit" would then depend on the idiosyncrasies of the estimators!

B. Reproducibility Without "Reproducibility"

Fourth, even if we ignore non-reproduced inputs, it is very probable that actual economies, even highly productive ones that do reproduce themselves over time, fail to satisfy the received definitions of "reproducibility" (e.g., Roemer, 1981, p. 19). To satisfy such definitions, they must produce non-negative physical surpluses of all goods *in each and every period*. In reality, however, reproducibility merely requires that non-negative surpluses be produced over some longer time span (and that initial reserve stocks be of sufficient size).[ix] Assume, for instance, a two-good economy in which the physical surpluses of goods A and B are –1 and 10, respectively, in period 1, and 10 and –1 in period 2. Over these two periods, 9 more units of each good have been produced

than were consumed. Given an initial stock of A of at least 1 unit, there is no technological barrier to this economy's expanded reproduction.

Since the length of a "period" is arbitrary, one may of course lengthen the period and thereby include such cases among those in which all physical surpluses are non-negative in every period. Yet, once one does so, simultaneist theorems that surplus-labor is necessary and sufficient for positive profit will become false. Prices may change during the lengthened period in such a way that, for instance, even though surplus-labor is extracted in both sub-periods, profit may be negative in each of them and therefore over the lengthened period as a whole.

This is demonstrated in Table 1. It assumes a zero wage rate in order to apply to all simultaneist interpretations; their definitions of surplus-labor are identical in this case. If one were to assume a very low real wage consisting of equal amounts of the two goods, the same qualitative results would obtain. Table 1 also assumes that technical coefficients are unchanged over the two days, so that the 2% variations in each sector's daily output are due to changes in activity levels alone.

(Table 1 goes here.)

Examining the lengthened period, Day 1+2, we seem to have an example of strictly positive physical surpluses — 40,002 units of each good are produced, while only 40,000 units are used up in production. Surplus-labor (equal to living labor because wages are zero) is likewise positive, over the whole period, and in each sector on each day. Each sector's profit for Day 1+2 is negative, however, because the cost of its inputs exceeds the total price of its outputs.

Yet, given that physical surpluses are positive, how is this possible? The

solution to this riddle lies in the slight (2%) variations in each sector's price and activity levels over the two sub-periods, the latter of which causes a 2% physical deficit of good 2 on Day 1, and a 2% physical deficit of good 1 on Day 2. Because, on each day, the size of one sector's deficit is about equal to the other's surplus, and because the unit price of the deficit sector is slightly (2%) higher than that of the surplus sector, the deficit is worth more than the surplus. Profit is thus negative on each day.

It is also worth noting the plethora of equilibrium conditions packed into this example. The wage rate is equalized across sectors. Techniques of production are not changing, and it is consistent with the example to assume that all firms in a sector use the same technique. Over the two-day period, the two sectors' physical growth rates are equal, as are their profit rates. By any reasonable definition, everything is in equilibrium except that prices vary slightly. [x]

Hence, taking a "period" to be some reasonable length of time, say two days (or hours or minutes), surplus-labor is insufficient for positive profit to exist, even when (a) daily physical deficits are very small in percentage terms, (b) physical deficits are non-existent over two days, and (c) profit rates are equal. The theorems that claim the opposite are therefore true only if a non-negative surplus of each use-value is produced in each and every period, *no matter how short one takes a period to be*. (A period in this context can be no longer than the length of time during which prices remain constant; but they can change from one instant to the next.)

This is the only way to rescue these theorems' formal validity. The shorter the period, however, the less likely it is that all physical surpluses will be positive. For very short periods, it is almost inconceivable that this is the case. Many factories and offices

shut down overnight, but night in one part of the world is midday in another. Some business is therefore always using up some input that its supplier is not reproducing at that moment. Hence, if the theorems in question are valid, they are irrelevant, because their key premise is never true, while if they are relevant, they are invalid.

Table 2 presents a similar example that applies to the standard interpretation. Surplus-labor is zero, but profit is positive. The unit value of each commodity is 1, so the top set of figures represent both physical and value magnitudes. Again, wage rates are equal, techniques are not changing, the two sectors grow at the same rate (zero) in physical terms, and their (positive) two-day profit rates are equal (1%). This economy can also sustain itself *ad infinitum*, both in a physical sense — two-day surpluses of both goods are non-negative — and because the capitalists are making profits. Even in this tableau, in which the economy is in simple reproduction and everything is in equilibrium except that relative prices vary modestly, the standard interpretation implies that surplus-labor is unnecessary for positive profit.

(Table 2 goes here.)

3. The Temporal Single-System Interpretation

The results to this point have been negative. It might therefore be thought that, no matter how Marx's value theory is interpreted, surplus-labor turns out not to be the sole source of profit. This is not the case. The temporal single-system interpretation (see, e.g., Ernst (1982), Kliman and McGlone (1988), Giussani (1991-92), Freeman and Carchedi, eds. (1996), Maldonado-Filho (1997), Ramos (1997)) does imply that

surplus-labor is both necessary and sufficient for profit to exist, and under completely general conditions. More precisely, although negative (positive) *nominal* profit and positive (negative) surplus-labor can coexist, *real* profit is positive (negative) when surplus-labor is positive (negative).

This interpretation, like the newer simultaneist interpretations, construes surplus-labor as living labor minus the labor-time equivalent of the money wage:

$$s = \ell\mathbf{x} - (1/\tau_t)w_t\ell\mathbf{x}. \qquad (7)$$

Note, however, that variables are now defined within historical time. In particular, τ, the "temporalist monetary expression of labor-time," differs from s because the relation between money and labor-time is allowed to vary between the time of input and the time of output.

Nominal profit is interpreted as $\pi^N = \mathbf{p_{t+1}x} - \mathbf{p_tAx} - w_t\ell\mathbf{x}$ (inputs enter production at time t, output is produced at time t+1). Real profit, however, deflates sales revenue ($\mathbf{p_{t+1}x}$) in order to adjust for changes between times t and t+1 in the amount of money that represents one unit of value. Thus

$$\pi^R = (1/[1+i])\mathbf{p_{t+1}x} - \mathbf{p_tAx} - w_t\ell\mathbf{x}, \qquad (8)$$

where $i = (\tau_{t+1} - \tau_t)/\tau_t$ is the rate of inflation in the monetary expression of labor-time.

τ_{t+1} is defined as the ratio of the money price of output to the sum of dead and living labor needed to produce it:

$$\tau_{t+1} = \mathbf{p_{t+1}x}/([1/\tau_t]\mathbf{p_tAx} + \ell\mathbf{x}) = \tau_t\mathbf{p_{t+1}x}/(\mathbf{p_tAx} + \tau_t\ell\mathbf{x}), \qquad (9)$$

where $[1/\tau_t]\mathbf{p_tAx}$, the labor-time equivalent of the price of means of production, is interpreted as dead labor. Since $1+i = \tau_{t+1}/\tau_t$, (9) implies that $\tau_{t+1}/\tau_t = 1+i =$

$\mathbf{p_{t+1}x}/(\mathbf{p_tAx} + \tau_t\ell\mathbf{x})$. Substituting this result into (8), one obtains

$$\pi^R = (\mathbf{p_tAx} + \tau_t\ell\mathbf{x}) - \mathbf{p_tAx} - w_t\ell\mathbf{x} = \tau_t\ell\mathbf{x} - w_t\ell\mathbf{x}, \qquad (10)$$

which implies that

$$\pi^R = \tau_t s. \qquad (11)$$

This looks very similar to the proportionality between surplus-labor and profit that was derived from the newer simultaneist interpretations. Yet whereas s, the simultaneist monetary expression of labor-time, need not be positive, examination of (10) shows that, if prices and the initial condition τ_0 are positive, then all subsequent terms of the τ-series must also be positive. The proportionality between surplus-labor and real profit, *together* with this result on τ, imply that surplus-labor is both necessary and sufficient for real profit to be positive.

4. Conclusion

Due to their static character, simultaneist interpretations of Marx's value theory grant value no role in explaining the dynamics of capitalism. Although proponents of simultaneist interpretations have acknowledged this, they seem untroubled by it. They contend that the "core of the explanatory power of the labor theory of value lies in the analysis of exploitation" rather than the dynamic analysis (Duménil and Lévy (1997:16)) and, invoking the FMT and similar theorems, they have argued that their interpretations do imply that exploitation of workers is the sole source of profit.

This paper has demonstrated, to the contrary, that simultaneism and the exploitation theory of profit are incompatible. The FMT holds only when all physical

surpluses are positive (or profit rates are equal) in every period, and similar theorems pertaining to more recent simultaneist interpretations hold only when the aggregate price of the net product is positive in every period — *no matter how brief the period.* These conditions have been shown to be implausible and completely unnecessary for reproduction. A choice between simultaneous valuation and the exploitation theory of profit must therefore be made.

Marx's value theory thus seems to be far more of a "package deal" than has hitherto been recognized. The attempts to fragment it into dynamic and static aspects, and to reject the former while embracing the latter, have not succeeded. When his value theory is given a static interpretation, not only do his explanations of dynamic issues, such as the tendency of the profit rate, seem to be false, so does his explanation of the origin of profit, a putatively static issue. Conversely, the temporal single-system interpretation, which vindicates the internal consistency of his value theory in other respects, also vindicates the logical coherence of the exploitation theory of profit.

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Table 1

POSITIVE SURPLUS-LABOR WITH NEGATIVE PROFIT

PHYSICAL QUANTITIES

Day	Sector	Inputs: Good 1	Inputs: Good 2	Inputs: Living Labor	Output
	1	10,100	10,100	1.01	20,201.01
1	2	9,900	9,900	0.99	19,800.99
	tot.	20,000	20,000	2.00	
	1	9,900	9,900	0.99	19,800.99
2	2	10,100	10,100	1.01	20,201.01
	tot.	20,000	20,000	2.00	
	1	20,000	20,000	2.00	40,002.00
1+2	2	20,000	20,000	2.00	40,002.00
	tot.	40,000	40,000	4.00	

PRICES AND PROFITS

Day	Sector	Unit Price	Costs: Good 1	Costs: Good 2	Costs: Total	Total Price	Profit
	1	0.99	9,999	10,201	20,200	19,999.00	-201.00
1	2	1.01	9,801	9,999	19,800	19,999.00	199.00
	tot.		19,800	20,200	40,000	39,998.00	-2.00
	1	1.01	9,999	9,801	19,800	19,999.00	199.00
2	2	0.99	10,201	9,999	20,200	19,999.00	-201.00
	tot.		20,200	19,800	40,000	39,998.00	-2.00
	1		19,998	20,002	40,000	39,998.00	-2.00
1+2	2		20,002	19,998	40,000	39,998.00	-2.00
	tot.		40,000	40,000	80,000	79,996.00	-4.00

NOTES: The equalized wage rate = 0. Over the two-day period, the profit rate is equalized, positive physical surpluses are produced, and balanced expanded reproduction occurs. “Total Price” and “Profit” figures are rounded to two decimal places.

Table 2

ZERO SURPLUS-LABOR WITH POSITIVE PROFIT

PHYSICAL AND VALUE QUANTITIES

Day	Sector	Wage Bundles Good 1	Wage Bundles Good 2	Living Labor	Surplus-Labor	Output
	1	29,997	29,997	59,994	0	59,994
1	2	9,999	9,999	19,998	0	19,998
	tot.	39,996	39,996	79,992	0	
	1	9,999	9,999	19,998	0	19,998
2	2	29,997	29,997	59,994	0	59,994
	tot.	39,996	39,996	79,992	0	
	1	39,996	39,996	79,992	0	79,992
1+2	2	39,996	39,996	79,992	0	79,992
	tot.	79,992	79,992	159,984	0	

PRICES AND PROFITS

Day	Sector	Unit Price	Wage Costs Good 1	Wage Costs Good 2	Wage Costs Total	Total Price	Profit
	1	$10401/9999$	31,203	29,997	61,200	62,406	1206
1	2	1	10,401	9,999	20,400	19,998	-402
	tot.		41,604	39,996	81,600	82,404	804
	1	$9601/9999$	9,601	9,999	19,600	19,202	-398
2	2	1	28,803	29,997	58,800	59,994	1194
	tot.		38,404	39,996	78,400	79,196	796
	1		40,804	39,996	80,800	81,608	808
1+2	2		39,204	39,996	79,200	79,992	792
	tot.		80,008	79,992	160,000	161,600	1,600

NOTES: The wage rate is equalized. Over the two-day period, the profit rate is equalized, physical surpluses are non-negative, and simple reproduction occurs.

NOTES

[i] Because they refrain from asserting any relationship between surplus-labor and profit (measured in terms of money or a numéraire), the interpretation of Wolff, Callari, and Roberts (1984) is an exception.

[ii] The analysis below pertains to the most general versions of the FMT, which do not postulate that profit rates are equal in every period. Those versions which rely on this postulate are likewise very restrictive. As the examples in Tables 1 and 2, below, show, even if profit rates are equalized over two "one-day" periods, the FMT does not hold.

[iii] $\mathbf{A} = [a_{ij}]$ is a square matrix of input-output coefficients; a_{ij} is the amount of good i used to produce one unit of good j. $\mathbf{b}$ is a column vector of wage goods per unit of living labor, ℓ is a row vector of living labor requirements per unit of output, and $\mathbf{x}$ is a column vector of outputs. $\mathbf{I}$ is the identity matrix.

[iv] When no physical surpluses are negative, but some are zero, and some prices and/or values are zero, the aggregate worth of the physical surplus vector can be zero when valued at prices and positive when valued at values, or vice-versa.

[v] Note, however, Ramos's (1997) critique of simultaneism.

[vi] See Kliman (1997) for other criticisms of this concept.

[vii] Foley (1997:19, emphasis in original), in particular, has noted that his interpretation "is completely general, in that it is consistent with *any* theory of price formation …."

[viii] I am indebted to Alan Freeman for emphasizing this crucial point.

[ix] Roemer (1981, p. 19) first notes that reproduction requires that no stock be run down to zero. He then points out that one way of "assuring" this — i.e., one *sufficient* condition for reproducibility — is to postulate that, in every period, all physical surpluses are non-negative. Immediately thereafter, however, he pronounces this postulate a "requirement" for reproducibility — i.e., a *necessary* condition — which it certainly is not!

[x] To keep the example simple, stocks have been excluded from the denominator of the profit rate, and prices have been measured in units of a third asset, not produced in this economy. Yet qualitatively identical results, including equalized two-day profit rates, can also be obtained if stocks are included and numéraire prices are used.