Valuation in the Presence of Stocks of Commodities

Exploring the temporal single-system interpretation of Marx

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Abstract

This paper seeks to stimulate debate on how to value commodities following the temporal single-system interpretation (TSSI) of Marx's value theory. I suggest that, although both Andrew Kliman and Alan Freeman follow a TSSI of Marx, their approaches to calculating the value of commodities differ. To illustrate this difference, I consider a simple model of the economy with stocks of unsold commodities being carried forward from one period to the next. I conclude that this difference in approach indicates just how much exciting research remains to be done within the TSSI of Marx.

Introduction

All too often, articles dealing with the Temporal Single-system Interpretation (TSSI) of Marx are either fighting for its right to exist or attempting to eliminate the TSSI as a disturbing virus. As critics of the TSSI outnumber its supporters, most journals insist that articles supporting the TSSI should follow and address the agenda of the

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TSSI's critics. In contrast, I shall attempt to explore the TSSI. After briefly explaining the TSSI's sequential and non-dualistic nature, I shall consider how, within the TSSI, there is a difference between Kliman's and Freeman's interpretations of how Marx calculates commodities' values in the presence of stocks of commodities. The difference is not one of error; both interpretations are internally consistent. Rather, they simply interpret how Marx determines commodities' values differently. I conclude that this difference of approach indicates how research informed by the TSSI of Marx is not a matter of following a particular dogma, but rather is an open and exciting route to attempt to apply Marx's analysis of capitalism to understanding the world today.

The Temporal Single-system Interpretation of Marx

The TSSI of Marx abstractly imagines alternating sequential periods of production and circulation, and employs a non-dualistic concept of price and value. It is

> [n]on-dualistic (unitary, or redistributive) because it considers that prices and values *reciprocally determine* each other in a succession of periods of production and circulation. Prices are not determined independent of values but neither are values determined independently of prices. [Freeman and Carchedi 1996, x, emphasis in original]

We are no longer in Bortkiewicz's (1952, 1984) simultaneous and non-dualistic world; values are not simply determined by the technical conditions of production and seen as a separate concept/system from the price system.[2] Capitals buy inputs for the current period of

2 Bortkiewicz (1952, 1984) "discovered" that, if value is interpreted in a simultaneous and dualistic fashion, then it becomes an inconsistent or internally contradictory concept, as expressed by the failure of "Marx's" transformation "problem" to add up. Total value may be equated to total price (Winternitz 1948), or total profit may be equated to total surplus-value (as in Bortkiewicz's "solution"), or the price of wage goods may be equated to their value (Seton 1957). However, as Kliman (2007) makes clear, it is Bortkiewicz's simultaneous and dualistic concept of value which is internally inconsistent, not Marx's concept of value (if we interpret Marx, as the TSSI interprets Marx, as having a sequential and non-dualistic

production, in the preceding period of circulation, at prices established at the end of the preceding period of production; "The value of a commodity is expressed in its price before it enters into circulation, and it is therefore a pre-condition of circulation, not its result" (Marx 1976, 260).

These money prices express the socially determined values of inputs for the current production period, and represent the value appropriated (received) by the capitals that produced them last period, which may differ from their produced values at the end of production last period. Constant capital and variable capital now enter production with the former values, i.e. those based on the value appropriated.

> It should be clear that Marx's embodied labour theory is a theory of abstract, alienated labour. Because the embodiment of abstract, alienated labour is a peculiar social process, not a technological requirement as such, the abstract labour embodied in a commodity need not equal the amount of (concrete) labour needed to (re)produce it. Although exchange does not alter the quantum of value in existence, it does redistribute it. Because abstract labour is redistributed through exchange, some commodities *embody* more abstract labour than they would otherwise, some less. On the basis of this notion of labour embodiment, one can comprehend how the capital advanced to production does not cease to be a sum of value merely because it differs from the value of its material elements (means of production and subsistence). [McGlone and Kliman 1996, 32, emphasis in original]

Appropriated values and produced values may diverge, but the divergence is subject to the overall constraint that total appropriated value must equal total produced value, i.e. the price of total output must equal the value of total output (Marx 1981, Chap. 9). The TSSI thus defines the monetary expression of labor-time (MELT), established at the end of production each period along with the process of price formation, as the nominal price (or money expression) of total capital divided by the total produced value of capital in terms of labor-time. The MELT relates how many nominal units of money

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concept of value). Following the TSSI of Marx, all three of Marx's aggregate equalities hold in the transformation "problem" (Kliman and McGlone, 1988). Marx's sequential and non-dualistic concept of value is—and has always been—internally consistent.

represent one hour of labor-time. We may establish inputs' appropriated values, in terms of labor-time, by dividing their nominal price by the MELT that exists at the time of their purchase, i.e. the MELT established upon price formation at the end of production last period. We recalculate the MELT at the end of production each period, when that period's prices are formed, which enables us to express all end-of-period value magnitudes, produced and appropriated, in both nominal money and labor-time terms.

In summary, at the end of a production period, the value produced by a capital:

1. depends on the surplus labor-time added in production, and the value of inputs, as defined by their price at the end of the previous period;
2. will differ (by the tendency to profit rate equalisation) from the value that capital appropriates through price formation; and
3. may be expressed (as may its components) both in nominal units of money and in labor-time terms, through adjustment by the MELT appropriate to that point in the circuit of capital.

The Difference Over Stocks

To focus on commodity stocks, let us assume that the economy consists of a single sector that produces a single commodity with no input other than living labor, *L*. We have thus, for simplicity, abstractly assumed away all constant capital, both circulating or fixed. We assume that a stock, *U*, of our single commodity is carried forward from the prior period to the start of our current period. Production now occurs in our current period, resulting in an output, *Q,* of our single commodity.

What is the unit value of our single commodity in terms of labor-time? Let us first explore Kliman's approach to stock valuation, indeed valuation in general. When considering Marx's (1976, 317–18) example of a rise in price of cotton, Kliman (1999, 105; 2007, 21) states:

> it is clear that, because values are determined by current production conditions, when the value transferred to newly produced yarn rises, so must the value transferred to existing stocks of yarn.

> The phrase "currently needed to produce" reflects the idea that the value of newly-produced items determines the value of already-existing ones. If wheat harvested last year had a value of $4/bushel, while wheat harvested today has a value of £3/bushel, then any wheat that remains from last year likewise has a value of $3/bushel today.

For Kliman, the *current* unit value, in terms of labor-time, of our single commodity, at the end of production in the current period, is $v_c = L / Q$. Stocks of our single commodity, carried forward from the previous period, have a value of $v_p U$ (where v_p is the *past* unit value), at the start of the current period, to be replaced by $v_c U$ at the end of the current period. If $v_c \neq v_p$, we can clearly see that stocks have to be revalued. The total value of current output and carried-over stocks from last period, at the end of the current period, equals $v_c Q + v_c U$ or, equivalently, $L + v_c U$ (since $v_c Q = L$). Potential stock revaluation implies that the total value of current output plus stocks at the end of the current period does not equal the sum of the living labor applied in production during the period and the value of stocks at the start of the period, i.e.,

$$v_{\mathrm{c}}(Q+U) \neq L + v_{\mathrm{p}}U$$

unless $v_c = v_p$.

However, Kliman's method does ensure, and it is based on the concept that, the value of newly produced commodities is determined by the amount of labor actually expended in their production. In our simple model, this latter amount is *L*; in general, it is the used-up constant capital plus the living labor applied.

Let us now consider Freeman's (1996, 255–56) different treatment of stocks of commodities:

> Production begins with a definite quantity of each commodity possessing a definite value. … Total use value is the initial stock less what was consumed plus what was produced; while its exchange value is the initial stock less what was consumed, plus value transferred in production, plus the value product. Dividing the second by the first gives the new market value of the commodity, arising from the two sources of existing stocks and new product. … As before, there is a contradiction between the output and input values of C_{I}. The 50 units of output have an individual value given, as usual, by the sum of metamorphosed inputs (1400) and value product (300).

> Their unit individual value is therefore 1700 ÷ 50 = 34. If it were not for the [45] units of preserved stocks of C_I, this would be the market value. But these preserved stocks also contain the value with which they started, namely [1800], corresponding to the old unit value of 40. There is only one coherent way to resolve this contradiction, which is to estimate the new market (social) value of C_I as the average of the whole value contained in the whole stock of C_I.[3]

In the context of our simple model, we now calculate the unit value, in terms of labor-time, of our single commodity at the end of the current period as:

$$v_c = \frac{L + v_p U}{Q + U}$$

We carry the start-of-period value of stocks through to the end of the period to determine, together with the living labor performed in the current period, the total value of currently produced output and carried-over stocks. Our single commodity's unit value at the end of the current period is simply this total value divided by the sum of the newly produced units of our single commodity and stocks of our single commodity that have been carried over from the last period. Treating stocks in this way ensures that the increase in the total value of stocks and currently produced output, at the end of the current period, over and above the value of stocks at the start of the period, is precisely the living labor applied in that period, since it follows from the equation above that

$$v_c(Q + \mathrm{U}) = L + v_p U$$

However, if $v_c \neq v_p$, then the current period's produced output will not embody the living labor worked in that period, since

$$v_c Q = \left(\frac{L + v_p U}{Q + U}\right) Q \neq L$$

unless $v_c = v_p$.

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3 The figures in square brackets replace two incorrect figures in Freeman's text.

We thus have a clear difference between Freeman's approach and Kliman's approach to valuation in the presence of stocks. Let us assume that $v_p = 4$ (hours), U = 5 (physical units), L = 60 (hours), and Q = 20 (physical units). For Kliman,

$$v_{\mathrm{c}} = \frac{L}{Q} = \frac{60}{20} = 3 \text{ hours}$$

$$v_{\mathrm{c}}Q = 3 \cdot 20 = 60 \text{ hours} = L$$

So the total value of output does equal the value added by living labor. However,

$$v_{\mathrm{c}}(Q+U) = 3(20+5) = 75 \text{ hours} \neq L + v_{\mathrm{p}}U = 60 + 4 \cdot 5 = 80 \text{ hours}$$

That is, the total value of assets (output plus stocks) after production does not equal the total value of assets before production (stocks) plus the value added by living labor in production. In contrast, for Freeman:

$$v_{\mathrm{c}} = \frac{L + v_{\mathrm{p}}U}{Q+U} = \frac{60 + 4 \cdot 5}{20+5} = 3.2 \text{ hours}$$

$$v_{\mathrm{c}}Q = 3.2 \cdot 20 = 64 \text{ hours} \neq L$$

The total value of output does not equal the value added by living labor. However,

$$v_{\mathrm{c}}(Q+U) = 3.2(20+5) = 80 \text{ hours} = L + v_{\mathrm{p}}U = 60 + 4 \cdot 5$$

That is, the total value of assets (output plus stocks) after production does equal the total value of assets before production (stocks) plus the value added by living labor in production.

If we assume an absence of technological change (constant productivity, $v_c = v_p$), the difference between the two approaches would be hidden; both would satisfy $v_c Q = L$ and $v_c(Q+U) = L + v_p U$. But as soon as $v_c \neq v_p$, the approaches diverge, because of their different methods of valuation in the presence of stocks. In Kliman's approach, $v_c Q = L$ holds, but $v_c(Q+U) = L + v_p U$ does not. In Freeman's approach, $v_c(Q+U) = L + v_p U$ holds, but $v_c Q = L$ does not.

More generally, if we include constant capital, the value of newly produced output will always equal the constant capital transferred and the living labor added in the production of this output if we follow Kliman's interpretation. But if productivity changes and stocks are carried over into the next period, this equality will not hold true if we follow Freeman's interpretation. If we include circulating constant capital, our $v_c(Q+U) = L + v_p U$ equality becomes an equality between the total value of assets (output plus stocks) after production and the sum of the value added by living labor in production and the total value of assets before production (stocks plus constant capital).[4] Furthermore, recognizing that capitalists advance variable capital, the equality implies that the value of total capital at the end of production (output plus stocks) exceeds the total value of capital advanced at the start of production (stocks, plus variable and constant capital advanced) by the total surplus-value extracted from living labor in production. On Freeman's interpretation, this is so; the value of total capital will always grow by the surplus-value extracted in production. But if productivity changes and stocks are carried over into the next period, this is not so if we follow Kliman's interpretation.

In summary, both Kliman and Freeman employ a sequential and non-dualistic method, but differ in their concepts of what the produced unit values of commodities should be; they have contrasting methods of valuation in the presence of stocks.

A More Thorough Example

To focus on the question of valuing stocks we shall assume a very simple/abstract economy. We assume no fixed capital, and identical capitals that produce a single identical commodity. Capitalists carry

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4 If we were to include fixed capital, we would have to consider the question of how, on Kliman's interpretation and Freeman's interpretation, we should value remaining units of fixed capital at the end of production. We shall briefly consider this question in footnote 9, but a full treatment of this issue is beyond the scope of the current paper.

over stocks of our single commodity from one period to another. We assume that stocks do not perish, remaining identical in use-value to new units of output of our single commodity. Strictly speaking, capitalists have no reason to trade with each other. To impose the need to exchange commodities in circulation, let us assume that capitalists cannot use their own output or stocks as inputs or for their own consumption.[5]

Although we wish to consider only one complete period, starting with production and ending with instantaneous circulation, the TSSI's sequential nature requires us to define the situation at the end of the previous period, period $t-1$, in order to determine the values of inputs in our current period, period t. Given that Kliman's and Freeman's alternative approaches produce different results if stocks are carried over and productivity changes, we shall assume that no stocks are carried over to period $t-1$ from period $t-2$. We thus start from a common base; at the end of period $t-1$, there are no carried-over stocks from period $t-2$ to potentially be revalued.

In circulation at the end of period $t-1$, one part of total output is demanded and sold, and the other part becomes stocks to be carried over to period t. Demand at the end of period $t-1$ comes from three sources. Firstly, there are capitalists' purchases of our single com-

5 At our very abstract aggregate one-commodity level, money is merely acting as a unit of account; we do not model hoards or circulation of any paper or commodity money. Let us assume that our capitalists are identical and that, in each period, they entirely own their inputs and carried-over stocks. (They are in debt to no one). At the end of each period, there is an instantaneous process of circulation in which wages are advanced by capitalists and, as we assume that workers entirely spend their wages, this money returns to capitalists, while capitalists swap identical amounts of our single commodity between each other (each spends what he or she realizes from the others). If we abstractly imagined that capitalists and workers had deposits in a costless deposit credit-money system, all deposits would begin at zero when circulation starts. There would then be matching debits and credits during the instantaneous process of circulation, so that all deposits would be balanced at zero again at the end of the circulation process. In our abstract model, input price changes do not cause our capitalists to have to tie up or release any actual money capital. With only a single commodity (and no stocks of commodity money, which would represent the introduction of a second commodity), any price change is by definition neutral, i.e. it cannot affect the value of our single commodity in labor-time terms. For a discussion, following a TSSI of Marx, of the release and tie up of capital in response to input price changes for a particular capital, in a multi-commodity world, see Maldonado-Filho (1997).

modity for their own consumption. Secondly, capitalists' purchase our single commodity to apply it as constant-capital input for period *t*. Thirdly, at the end of period *t*–1, capitalists advance workers their wages for period t. We assume that the wages are entirely spent during circulation at the end of period *t*–1. Thus, the workers of the upcoming period consume part of last period's output.

With period *t* inputs defined, production proceeds in period *t*. The amount of labor-time agreed to in the wage bargain at the end of period *t*–1 is worked during production in period *t*.[6] Surplus labor-time equals the difference between the total labor-time and paid labor-time, or variable capital (as determined by the wage paid at the end of period *t*–1). At the end of period *t's* production, our single commodity has a produced unit value, with total capital equaling the value of newly produced output plus the value of stocks that were carried over from the previous period. With only one commodity, appropriated value cannot deviate from produced value, as there is no other commodity to deviate in the opposite direction. Price formation at the end of period *t*'s production will simply ensure that appropriated value equals produced value. Demand will now determine how many units of the commodity will be exchanged in circulation at the end of period *t* and how many units will be stocks that are carried over into period *t* + 1.

Table 1 provides the notation we shall employ. For example, $Y_t^{£*}$ represents the monetary expression of the appropriated value of total capital at the end of period *t's* production (conventionally, this is called M'_t). Y_t^0 represents the number of physical units of our commodity that make up total capital at the end of period *t*'s production. Y_t^{h} represents the total produced value of capital, measured in terms of labor-time, at the end of period *t*'s production. $Y_t^{\mathrm{h}*}$ represents the total appropriated value of capital in terms of labor-time at the end of period *t*'s production. Note that we apply

6 If production proceeds as planned, the labor-time promised in the wage bargain is delivered. If, for any reason, actual labor-time falls short of (or exceeds) the labor-time promised in the wage bargain, surplus labor-time is this reduced (or higher) level of labor-time minus the value of variable capital. We assume that all labor-time magnitudes are in units of average socially-necessary simple labor (having no specific skill and put to work at average intensity and under socially average conditions of production).

Table 1. Definitions

Variables

C	constant capital input at the start of the production period
D	demand during circulation at the end of the period
K	capitalists' personal consumption purchases during circulation at the end of the period
L	labor-power applied during the production period
Q	the output of our single commodity at the end of the production period
S	surplus-value produced by the end of the production period
U	stocks of our single commodity after circulation at the end of the period
V	variable capital input at the start of the production period
Y	total capital at the end of the production period

Ratios

m	the monetary expression of labor-time (MELT) at the end of the production period
p	the price of our single commodity at the end of the production period
r	the rate of exploitation of labor during the production period
ρ	the profit rate at the end of the production period
v	the unit value of our single commodity at the end of the production period

Superscripts and Subscript

h	indicates that a variable refers to produced value, in terms of labor-time
h*	indicates that a variable refers to appropriated value, in terms of labor-time
£	indicates that a variable refers to produced value, in nominal units of money
£*	indicates that a variable refers to appropriated value, in nominal units of money
o	indicates that a variable is expressed in physical units of our single commodity
t	indicates the time period to which a variable refers

Table 2. End of Period *t*–1

End of Production in Period t–1

Units	p	m	Value Produced Q	U	Y	v	Value Appropriated Q	U	Y	v
0			**27**	**0**	27		27	0	27	
£	**5**	1	135	0	135	5	135	0	135	5
h			**135**	0	135	5	135	0	135	5

Circulation at End of Period t–1

Units	p	m	C	V	K	U	D+U
0			**15**	**5**	**2**	5	27
£	**5**	1	75	25	10	25	135
h			75	25	10	25	135

no superscript to the MELT (the number of nominal units of money that represent one hour of labor-time, at the end of a period's production).

Table 2 shows the situation we assume for the end of period t–1. (Figures for the end of period t–1 are omitted if they are not needed in order to compute the figures of period t. E.g., we do not specify period t–1's surplus-value or profit rate.)

In Table 2, the nominal price of our single commodity and the MELT are positioned under the end of production, before produced and appropriated values, in order to stress that, according to the TSSI, prices and thus the MELT are formed at the end of production, when produced and appropriated values are formed, not after. This allows us to use the MELT to determine the monetary expression of the produced values: $Q_{t-1}^{£} = m_{t-1}Q_{t-1}^{h}$, $Y_{t-1}^{£} = m_{t-1}Y_{t-1}^{h}$, and $v_{t-1}^{£} = m_{t-1}v_{t-1}^{h}$.

The MELT also allows us to express appropriated values. In monetary terms, the appropriated values are $Q_{t-1}^{£*} = p_{t-1}^{£*}Q_{t-1}^{0}$, $Y_{t-1}^{£*} = p_{t-1}^{£*}(Q_{t-1}^{0} + U_{t-2}^{0})$, and $v_{t-1}^{£*} = p_{t-1}^{£*}$; in terms of labor-time, they are $Q_{t-1}^{h*} = Q_{t-1}^{£*} / m_{t-1}$, $Y_{t-1}^{h*} = Y_{t-1}^{£*} / m_{t-1}$, and $v_{t-1}^{h*} = p_{t-1}^{£*} / m_{t-1}$. If there were more than one commodity, then, for each commodity, value produced would be likely to differ from value appropriated, and the MELT would allow us to express this difference both in monetary and in labor-time terms.

Let us stress that we do not have two sets of prices or appropriated values here. In our model, we exogenously set the price at the end of production, which gives us appropriated values in terms of nominal units of money. Once we know both produced values in terms of labor-time, and appropriated values in terms of nominal money, at the end of production, we can calculate the MELT at the end of production. The MELT allows us to express produced values in nominal units of money, or express appropriated values in terms of labor-time, thus allowing us to compare produced and appropriated values in the same units (either nominal units of money or units of labor-time). Produced values in terms of nominal units of money thus do not represent a second set of prices or appropriated values.

In order to eliminate the question of stock revaluation at the end of production in period t–1, we assume that no stocks are carried over into period t–1 after circulation at the end of period t–2; thus $U^{\mathrm{o}}_{t-2} = 0$. To determine all of the values at the end of the production period, we only need to exogenously set the nominal price, the physical output, and the physical output's produced value in terms of labor-time (given in bold in Table 2, along with zero stocks from period t–2). Since there are no carried-over stocks from period t–2, the MELT at the end of production period t–1 (m_{t-1}) and the produced unit value of our commodity in terms of labor-time (v^{h}_{t-1}) are

$$m_{t-1} = \frac{Y^{£*}_{t-1}}{Y^{\mathrm{h}}_{t-1}} = \frac{Q^{£*}_{t-1}}{Q^{\mathrm{h}}_{t-1}} = \frac{p^{£*}_{t-1}Q^{\mathrm{o}}_{t-1}}{Q^{\mathrm{h}}_{t-1}} = \frac{135}{135} = 1$$

$$v^{\mathrm{h}}_{t-1} = \frac{Y^{\mathrm{h}}_{t-1}}{Y^{\mathrm{o}}_{t-1}} = \frac{Q^{\mathrm{h}}_{t-1}}{Q^{\mathrm{o}}_{t-1}} = \frac{135}{27} = 5$$

Total capital equals the total output of our single commodity. (Given the magnitudes of $Q^{£*}_{t-1}$ and Q^{h}_{t-1} that have been chosen, $m_{t-1} = 1$, so variables' monetary expressions equal their labor-time values.)

I have just explained how the MELT allows us to calculate the monetary expression of produced values and appropriated values in terms of labor-time. In our one-commodity model, appropriated values must equal produced values, obscuring the TSSI's non-dualistic nature.

Let us now consider circulation at the end of period *t*–1. As we assume that circulation is instantaneous, and that price is formed at the end of production before circulation, it would seem reasonable to assume that demand is also determined at the end of production, before circulation. Circulation merely records how titles to commodities change. We exogenously set the following, indicated in boldface in Table 2: capitalists' demand for constant-capital input in the next period, their personal consumption demand for the next period, and workers' demand (equal to capitalists' advances of variable capital) for the next period. Five physical units of stock will be carried over into period *t* (i.e., $U_{t-1}^{\mathrm{o}} = Q_{t-1}^{\mathrm{o}} - D_{t-1}^{\mathrm{o}}$). Note that $D_{t-1}^{\mathrm{h}*} + U_{t-1}^{\mathrm{h}*} = Y_{t-1}^{\mathrm{h}*}$ and that $D_{t-1}^{£*} + U_{t-1}^{£*} = Y_{t-1}^{£*} = Y_{t-1}^{£} = m_{t-1}Y_{t-1}^{\mathrm{h}}$; in other words, price formation at the end of production, and subsequent circulation, cannot alter the total value produced in production.

We now move to the start of production in period *t*. We have already set, in circulation at the end of period *t*–1, period *t*'s inputs in physical terms (C_t^{o} and V_t^{o}) and the level of stocks to be carried over into period *t* (U_{t-1}^{o}). Equations (1) through (3) show how $C_t^{\mathrm{h}*}$, $V_t^{\mathrm{h}*}$, and $U_{t-1}^{\mathrm{h}*}$ are given by their monetary expressions divided by the MELT of the end of period *t*–1 (which is equivalent to their appropriated unit value in terms of labor-time, at the end of period *t*–1, multiplied by their physical quantity):

$$C_t^{\mathrm{h}*} = \frac{C_t^{£*}}{m_{t-1}} = v_{t-1}^{\mathrm{h}*}C_t^{\mathrm{o}} = 5 \cdot 15 = 75 \tag{1}$$

$$V_t^{\mathrm{h}*} = \frac{V_t^{£*}}{m_{t-1}} = v_{t-1}^{\mathrm{h}*}V_t^{\mathrm{o}} = 5 \cdot 5 = 25 \tag{2}$$

$$U_{t-1}^{\mathrm{h}*} = \frac{U_{t-1}^{£*}}{m_{t-1}} = v_{t-1}^{\mathrm{h}*}U_{t-1}^{\mathrm{o}} = 5 \cdot 5 = 25 \tag{3}$$

Since $m_{t-1} = 1$, monetary and labor-time expressions of inputs are identical.

During production in period *t*, L_t^{h} units of labor are worked, and since $V_t^{\mathrm{h}*}$ is already given in equation (2), surplus labor is

$$S_t^{\mathrm{h}} = L_t^{\mathrm{h}} - V_t^{\mathrm{h}} = 50 - 25 = 25 \quad (4)$$

We can now calculate the rate of exploitation of labor:

$$r_t^{\mathrm{h}} = S_t^{\mathrm{h}} / V_t^{\mathrm{h}} = 25 / 25 = 100\% \quad (5)$$

Equations (1) through (5) hold in both Kliman's and Freeman's approaches. Note that the manner in which we treat stocks is irrelevant to the rate of exploitation of labor; any potential "waste" of surplus-value is of no concern to the workers who produce it.

Kliman's Approach

Table 3 gives the figures for period *t* according to Kliman's approach. In this approach, to calculate the single commodity's per-unit value produced in terms of labor-time at the end of production, we only need to refer to the total value in terms of labor-time and the physical quantity of total output. Total value in terms of labor-time is

$$Q_t^{\mathrm{h}} = C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}} = 75 + 25 + 25 = 125 \quad (6)$$

Equation (6) ensures that, according to Kliman's approach, the value of newly produced output always equals the constant capital transferred and the living labor added in the production of this output.

Using (6) and the physical quantity of total output, we obtain the per-unit value:

$$v_t^{\mathrm{h}} = \frac{Q_t^{\mathrm{h}}}{Q_t^{\mathrm{o}}} = \frac{v_{t-1}^{\mathrm{h}*}(\mathrm{C}_t^{\mathrm{o}} + V_t^{\mathrm{o}}) + \mathrm{S}_t^{\mathrm{h}}}{Q_t^{\mathrm{o}}} = \frac{125}{30} = 4.17 \quad (7)$$

Thus productivity improves in period *t*: $v_t^{\mathrm{h}} < v_{t-1}^{\mathrm{h}*} = v_{t-1}^{\mathrm{h}}$.

Table 3. Production in Period *t*: Kliman

Units	Start of Production			End of Production				
	C	V	U	L	S	r	p	m
o	**15**	**5**	**5**					
£	**75**	**25**	**25**	60	30		**5**	1.2
h	**75**	**25**	**25**	**50**	25	100%		

Units	End of Production (continued)									
	Value Produced					Value Appropriated				
	Q	U	Y	v	ρ (%)	Q	U	Y	v	ρ (%)
o	**30**	5	35		40	30	5	35		40
£	150	25	175	5	40	150	25	175	5	40

Stocks, held through period *t*'s production, do not enter production, and, as such, do not influence the unit value of our commodity. But because they are identical in use-value to the output of period *t*, they must share the same value produced, per unit, at the end of production. Using (7), equation (8) determines the value (value produced) of stocks in terms of labor-time at the end of period *t*'s production:

$$v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} = 4.17 \cdot 5 = 20.8 \tag{8}$$

Using equations (7) and (8), we can also calculate the produced value of total capital, in terms of labor-time, at the end of period *t*'s production:

$$\begin{aligned} Y_t^{\mathrm{h}} &= Q_t^{\mathrm{h}} + v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} = v_t^{\mathrm{h}} Q_t^{\mathrm{o}} + v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} \\ &= 4.17 \cdot 30 + 4.17 \cdot 5 = 125 + 20.8 = 145.8 \end{aligned} \tag{9}$$

Note that total capital does not necessarily increase by the S_t^{h} extracted in production:

$$Y_t^{\text{h}} - (\text{C}_t^{\text{h}*} + V_t^{\text{h}*} + v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}}) =$$
$$\text{C}_t^{\text{h}*} + V_t^{\text{h}*} + S_t^{\text{h}} + v_t^{\text{h}} U_{t-1}^{\text{o}} - \text{C}_t^{\text{h}*} - V_t^{\text{h}*} - v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}} =$$
$$S_t^{\text{h}} + v_t^{\text{h}} U_{t-1}^{\text{o}} - v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}} = S_t^{\text{h}} + (v_t^{\text{h}} - v_{t-1}^{\text{h}*}) U_{t-1}^{\text{o}} \neq S_t^{\text{h}}$$

unless $v_t^{\text{h}} = v_{t-1}^{\text{h}*}$.

Since we assume that price is established at the end of period *t*'s production (in our model, it is exogenously set at $p_t^{\text{£}*} = \text{£}5$), we can calculate the MELT at the end of period *t* as follows:

$$m_t = \frac{Y_t^{\text{£}*}}{Y_t^{\text{h}}} = \frac{p_t^{\text{£}*}(\text{Q}_t^{\text{o}} + U_{t-1}^{\text{o}})}{v_t^{\text{h}}(\text{Q}_t^{\text{o}} + U_{t-1}^{\text{o}})} \quad \text{or} \quad m_t = \frac{Q_t^{\text{£}*}}{Q_t^{\text{h}}} = \frac{p_t^{\text{£}*} Q_t^{\text{o}}}{v_t^{\text{h}} Q_t^{\text{o}}}$$
$$= \frac{175}{145.8} = \frac{150}{125} = 1.2 \tag{10}$$

We have defined the MELT as the total money price of capital divided by the total labor-time value of this capital. This is so if we follow Kliman's method. Yet as equations (7) and (10) make clear, one part of the value of that capital—the value of stocks at the start of period *t*—is irrelevant to our calculation of the MELT, because we revalue stocks at the value of newly produced output before we calculate Y_t^{h}. Thus, according to Kliman's interpretation, it is the ratio of the total money price of output to the total labor-time value of output, $Q_t^{\text{£}*} / Q_t^{\text{h}}$, that actually determines the magnitude of the MELT.

To calculate the value-produced profit rate in labor-time terms, we must include the start-of-period value of stocks as part of the total capital advanced. If $v_t^{\text{h}} \neq v_{t-1}^{\text{h}*}$, total capital will not grow by S_t^{h}, so the numerator of the profit rate must include, along with S_t^{h}, any change in the value of these stocks during production period *t*:

$$\rho_t^{\mathrm{h}} = \frac{Y_t^{\mathrm{h}} - (\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}})}{\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}} = \frac{S_t^{\mathrm{h}} + (v_t^{\mathrm{h}} - v_{t-1}^{\mathrm{h}*}) U_{t-1}^{\mathrm{o}}}{\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}} =$$

$$\frac{25 + (4.17 - 5)\cdot 5}{75 + 25 + 5\cdot 5} = \frac{20.8}{125} = 16.7\% \tag{11}$$

The value-produced profit rate in terms of labor-time is substantially lower than the physical profit rate:

$$\rho_t^{\mathrm{o}} = \frac{S_t^{\mathrm{o}}}{\mathrm{C}_t^{\mathrm{o}} + V_t^{\mathrm{o}} + U_{t-1}^{\mathrm{o}}} = \frac{10}{15 + 5 + 5} = 40\% \tag{12}$$

Note that the physical surplus product ($S_t^{\mathrm{o}} = Q_t^{\mathrm{o}} - C_t^{\mathrm{o}} - V_t^{\mathrm{o}}$) does not embody total surplus-value as understood by the TSSI, unless productivity is constant ($v_t^{\mathrm{h}} = v_{t-1}^{\mathrm{h}*}$).

Produced values expressed in terms of labor-time can instead be expressed in terms of money, simply by multiplying them by the relevant MELT. Conversely, by taking appropriated values expressed in terms of money, and dividing them by the relevant MELT, we obtain appropriated values expressed in terms of labor-time.[7] The value-appropriated profit rate in labor-time terms is thus

$$\rho_t^{\mathrm{h}*} = \frac{\dfrac{Y_t^{£*}}{m_t} - \dfrac{(\mathrm{C}_t^{£*} + V_t^{£*} + U_{t-1}^{£*})}{m_{t-1}}}{\dfrac{(\mathrm{C}_t^{£*} + V_t^{£*} + U_{t-1}^{£*})}{m_{t-1}}} = \frac{145.8 - 125}{125} = 16.7\% \tag{13}$$

Note that the advanced capital in equation (13) is divided by the end-of-period MELT of period t–1, i.e., the MELT when that capital was advanced. Finally, the appropriated rate of profit in nominal money terms is given by equation (14):

[7] Thus, $v_t^{\mathrm{h}*} = p_t^{£*} / m_t$, $Q_t^{\mathrm{h}*} = Q_t^{£*} / m_t$, $v_t^{\mathrm{h}*} U_{t-1}^{0} = (p_t^{£*} U_{t-1}^{0}) / m_t$, and $Y_t^{\mathrm{h}*} = Y_t^{£*} / m_t$.

$$\rho_t^{£*} = \frac{Y_t^{£*} - (C_t^{£*} + V_t^{£*} + U_{t-1}^{£*})}{(C_t^{£*} + V_t^{£*} + U_{t-1}^{£*})} = \frac{175 - 125}{125} = 40\% \qquad (14)$$

We see that all appropriated-value figures in Table 3 equal all produced-value figures, whether expressed in labor-time or in money terms, as we would expect for a one-commodity aggregate model.

Freeman's Approach

With Kliman's approach to valuation in the presence of stocks clear, let us move on to Freeman's alternative treatment of valuation in the presence of stocks. Since the difference between Kliman's approach and Freeman's approach emerges at the end of production in period *t*, we shall consider circulation at the end of period *t* after we have explored Freeman's approach to valuation in the presence of stocks. As I explained above, equations (1) through (5) hold for both approaches, with the difference between approaches emerging when we consider the produced value of our commodity in terms of labor-time at the end of period *t*'s production. Table 4 repeats Table 3, but also reports money and labor-time figures according to Freeman's approach, in italicized script.

Unlike Kliman, Freeman does not use equation (7) to calculate the value produced per unit in terms of labor-time, and then use this new unit value in equation (9) to revalue stocks. Instead, Freeman keeps the start-of-period value of stocks in terms of labor-time (U_{t-1}^{h*}) intact; and he uses it, together with the constant capital transferred in production and the living labor added ($C_t^{h} + V_t^{h} + S_t^{h}$), to establish the total value (value produced) of capital in terms of labor-time (Y_t^{h*}) at the end of the period:

$$Y_t^{h} = (C_t^{h*} + V_t^{h*} + S_t^{h}) + U_{t-1}^{h*} = (75 + 25 + 25) + 25 = 150 \qquad (9')$$

As noted above, this implies that the increase in the total value of capital during the period is equal to the surplus-value:

$$Y_t^{h} - (C_t^{h*} + V_t^{h*} + v_{t-1}^{h*} U_{t-1}^{o}) = S_t^{h}$$

Table 4. Production in Period *t*: Kliman & Freeman

Units	Start of Production			End of Production				
	C	*V*	*U*	*L*	*S*	*r*	*p*	*m*
o	**15**	**5**	**5**					
£ (K)	**75**	**25**	**25**	60	30		**5**	1.2
£ (F)	***75***	***25***	***25***	*58.13*	*29.17*		***5***	*1.17*
h (K)	**75**	**25**	**25**	**50**	25	100%		
h (F)	***75***	***25***	***25***	***50***	*25*	*100%*		

Units	End of Production (continued)									
	Value Produced					Value Appropriated				
	Q	*U*	*Y*	*v*	*ρ* (%)	*Q*	*U*	*Y*	*v*	*ρ* (%)
o	**30**	5	35		40	30	5	35		40
£ (K)	150	25	175	5	40	150	25	175	5	40
£ (F)	*150*	*25*	*175*	*5*	*40*	*150*	*25*	*175*	*5*	*40*
h (K)	125	20.8	145.8	4.17	16.7	125	20.8	145.8	4.17	16.7
h (F)	*128.6*	*21.4*	*150*	*4.29*	*20*	*128.6*	*21.4*	*150*	*4.29*	*20*

Note: (K) and (F) stand for Kliman and Freeman, respectively.

and thus Freeman's value-produced profit rate, in terms of labor-time, is

$$\rho_t^{\mathrm{h}} = \frac{Y_t^{\mathrm{h}} - (\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}})}{\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}}$$

$$= \frac{S_t^{\mathrm{h}}}{\mathrm{C}_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}} = \frac{25}{125} = 20\% \qquad (11')$$

Freeman's total value of capital in terms of labor-time, Y_t^{h}, is a little higher than Kliman's, because Freeman does not revalue stocks downward, to reflect the productivity improvement, before including them in Y_t^{h}. Freeman's value-produced profit rate in terms of labor-time is consequently a little higher than Kliman's. Although the denominator of the profit rate is the same in both approaches (see equations (11) and (11′)), Freeman's numerator is larger because he does not deduct any stock revaluation when computing the increase in the total value of capital during the period. As we saw by re-arranging equation (9′), above, in Freeman's approach, in contrast to Kliman's, the total value of capital expands by exactly S_t^{h} during the production period whether or not productivity changes.

In Freeman's approach, we do not need to know Q_t^{o} in order to calculate Y_t^{h} or ρ_t^{h}. These latter figures are unaffected by Q_t^{o}, the level of output produced in period *t*, and thus they are also unaffected by the unit value of the commodity, v_t^{h}.

Freeman calculates v_t^{h} by dividing total capital, in terms of labor-time, by the total number of units of our commodity in existence, whether the latter are new output or carried-over stocks:

$$v_t^{\mathrm{h}} = \frac{Y_t^{\mathrm{h}}}{Y_t^{\mathrm{o}}} = \frac{v_{t-1}^{\mathrm{h}*}(\mathrm{C}_t^{\mathrm{o}} + V_t^{\mathrm{o}} + U_{t-1}^{\mathrm{o}}) + \mathrm{S}_t^{\mathrm{h}}}{Q_t^{\mathrm{o}} + U_{t-1}^{\mathrm{o}}} = \frac{150}{35} = 4.29 \qquad (7')$$

Because Freeman carries over the start-of-period value of stocks, without revaluation, to form part of Y_t^{h}, his v_t^{h} figure is a little larger than Kliman's. Freeman and Kliman consequently have two different concepts of what the unit value of the commodity should be.

Let us be clear: like Kliman, Freeman *does* revalue stocks at the end of production. In both approaches, at the end of production, a unit of stock will have the same value at the end of production, v_t^{h}, as a unit of newly produced output. Thus, in both approaches, the produced value of stocks carried over from period *t*–1, at the end of production in period *t,* equals $v_t^{\mathrm{h}}U_{t-1}^{\mathrm{o}}$. But if productivity changes, Freeman and Kliman calculate different values for v_t^{h}, and this explains why their $v_t^{\mathrm{h}}U_{t-1}^{\mathrm{o}}$ figures differ.

Let us now compare the two approaches' figures for the labor-time value of newly produced output. According to Kliman, it is

$$Q_t^{\mathrm{h}} = v_t^{\mathrm{h}} Q_t^{\mathrm{o}} = C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}} \tag{15}$$

To find the value of newly produced output according to Freeman, we can deduct the value of stocks at the end of production from the value of total capital at the end of production:

$$\begin{aligned} Q_t^{\mathrm{h}} &= Y_t^{\mathrm{h}} - v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} = (C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}} + v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}) - v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} \\ &= (C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}}) + (v_{t-1}^{\mathrm{h}*} - v_t^{\mathrm{h}}) U_{t-1}^{\mathrm{o}} \end{aligned} \tag{15′}$$

Thus, while in Kliman's approach, the value of newly produced output equals the constant capital transferred plus the living labor added in the production of this output $(C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}})$, they are unequal, according to Freeman (unless productivity is constant, so that $v_{t-1}^{\mathrm{h}*} - v_t^{\mathrm{h}} = 0$).

With produced values in terms of labor-time established, let us calculate the MELT that is also established at the end of production when price is formed (according to the TSSI). We again exogenously set $p_t^{£*}$ at £5. Both interpretations hold that $m_t = Y_t^{£*} / Y_t^{\mathrm{h}}$ but, since they differ regarding how Y_t^{h} is determined if stocks are carried forward from the previous period, the MELT they calculate will differ if productivity changes. Thus, according to Kliman, the MELT at the end of period *t* is

$$m_t = \frac{Y_t^{£*}}{Y_t^{\mathrm{h}}} = \frac{175}{145.8} = 1.20 .$$

Freeman would say that the MELT is

$$m_t = \frac{Y_t^{£*}}{Y_t^{\mathrm{h}}} = \frac{175}{150} = 1.17.$$

And because Freeman's result for the MELT differs from Kliman's, so too do his figures for value appropriated in terms of labor-time, since these figures are determined in part by the MELT. But appropriated values continue to equal produced values, whether expressed in labor-time or in money terms—as they must because of our assumption of a single commodity. Finally both approaches compute the same monetary and physical rates of profit.

Circulation

Table 5 considers what takes place in circulation at the end of period *t*. In both approaches, circulation is "neutral"; i.e., the sum of value in existence is neither increased nor decreased in circulation, but merely redistributed. We exogenously set demand for period *t*+1*'s* inputs (*C* and *V*) and for capitalists' personal consumption (*K*), in order to then find the level of stocks that need to be carried over into period *t*+1. In physical terms, total demand is $D_t^o = C_{t+1}^o + V_{t+1}^o + K_t^o$, and carried-over stocks are the difference between the capital at the end of the period and total demand: $U_t^o = Y_t^o - D_t^o$.

Our single commodity has the money price and the appropriated value in labor-time terms that were determined at the end of production in period *t*; see Tables 3 and 4. It has the same price and appropriated value whether it is sold or whether it becomes a stock carried over into period *t*+1. And since circulation neither creates nor

Table 5. Circulation at End of Period *t:* Kliman & Freeman

units	*Y*	Demand (*D*)			*U* = *Y*–*D*
		C	*V*	*K*	
o	35	18	5	5	7
£ (K)	175	90	25	25	35
£ (F)	*175*	*90*	*25*	*25*	*35*
h (K)	145.8	75	20.8	20.8	29.2
h (F)	*150*	*77.1*	*21.4*	*21.4*	*30*

Notes: (K) and (F) stand for Kliman and Freeman, respectively. *Y* figures are those computed in Tables 3 and 4. Physical *C*, *V*, and *K* figures are given exogenously. Money figures are the physical figures multiplied by the unit price ($p_t^{£*} = £5$), and labor-time figures are the physical figures multiplied by the appropriated value ($v_t^{h*} = 4.17$, according to Kliman; $v_t^{h*} = 4.29$, according to Freeman), computed in Tables 3 and 4.

destroys value, it follows that $Y_t^{h*} = D_t^{h*} + U_t^{h*} = Y_t^{h}$ and that $Y_t^{£*} = D_t^{£*} + U_t^{£*} = Y_t^{£} = m_t Y_t^{h}$.

As Table 5 shows, Kliman and Freeman arrive at the same conclusions concerning demands and stocks in money terms, because the unit price of the commodity is the same in both approaches. However, their demands and stocks in labor-time terms differ because, as was shown in Tables 3 and 4, their figures for the commodity's unit value, and thus the MELT, are different.

Both Kliman and Freeman follow the TSSI's sequential and nondualistic method, so in both cases, this period's produced values depend on last period's appropriated values, and price is formed at the end of production. But Kliman and Freeman have different concepts of the commodity's unit value (i.e., value produced per unit), because of their contrasting methods of valuation in the presence of stocks. According to Kliman's method, a productivity improvement causes the value of total capital to fall short of the value of capital advanced plus the surplus-value extracted in production, but the value of newly produced output always equals the constant capital transferred plus the living labor added in the production of this output. According to Freeman's method, in contrast, the value of total capital always equals the value of capital advanced plus the surplus-value extracted in production, but a productivity improvement causes the value of newly produced output to exceed the constant capital transferred plus the living labor added in the production of this output.

Effects of a Productivity Regress

Let us now repeat the same example, but assume instead that productivity *regresses*. Above, the physical output of period *t* was $Q_t^{o} = 30$. In Table 6, it is $Q_t^{o} = 22$. All other exogenously given data are the same as before. Table 7 compares Kliman's and Freeman's procedures for computing the labor-time variables and the MELT of Table 6.

Once again, according to Kliman, the value of newly produced output in terms of labor-time equals the constant capital transferred plus the living labor added in the production of this output. Equation

Table 6. Production and Circulation in Period *t* Given Productivity Regress: Kliman & Freeman

Production

Units	Start of Production			End of Production				
	C	*V*	*U*	*L*	*S*	*r*	*p*	*m*
0	**15**	**5**	**5**					
£ (K)	**75**	**25**	**25**	44	22		**5**	0.88
£ (F)	***75***	***25***	***25***	*45*	*22.5*		**5**	*0.90*
h (K)	**75**	**25**	**25**	**50**	25	100%		
h (F)	***75***	***25***	***25***	***50***	*25*	*100%*		

Units	End of Production (continued)									
	Value Produced					Value Appropriated				
	Q	*U*	*Y*	*v*	*ρ* (%)	*Q*	*U*	*Y*	*v*	*ρ* (%)
0	**22**	5	27		8	22	5	27		40
£ (K)	110	25	135	5	8	110	25	135	5	8
£ (F)	*110*	*25*	*135*	*5*	*8*	*110*	*25*	*135*	*5*	*8*
h (K)	125	28.4	153.4	5.68	22.7	125	28.4	153.4	5.68	22.7
h (F)	*122.2*	*27.8*	*150*	*5.56*	*20*	*122.2*	*27.8*	*150*	*5.56*	*20*

Circulation

units	*Y*	Demand (*D*)			*U* = *Y*–*D*
		C	*V*	*K*	
0	27	15	5	4	3
£ (K)	135	75	25	20	15
£ (F)	*135*	*75*	*25*	*20*	*15*
h (K)	153.4	85.2	28.4	22.7	17.0
h (F)	*150*	*83.3*	*27.8*	*22.2*	*16.7*

Note: (K) and (F) stand for Kliman and Freeman, respectively.

Table 7. Computations of Labor-Time Variables and MELT

Kliman	Freeman
$Q_t^{\text{h}} = C_t^{\text{h}*} + V_t^{\text{h}*} + S_t^{\text{h}} = 75 + 25 + 25 = 125$	$Q_t^{\text{h}} = C_t^{\text{h}*} + V_t^{\text{h}*} + S_t^{\text{h}} + (v_{t-1}^{\text{h}*} - v_t^{\text{h}})U_{t-1}^{\text{o}} = 75 + 25 + 25 + (5 - 5.56)\cdot 5 = 122.2$
$v_t^{\text{h}} = \dfrac{Q_t^{\text{h}}}{Q_t^{\text{o}}} = \dfrac{v_{t-1}^{\text{h}*}(C_t^{\text{o}} + V_t^{\text{o}}) + S_t^{\text{h}}}{Q_t^{\text{o}}} = \dfrac{125}{22} = 5.68$	$v_t^{\text{h}} = \dfrac{Y_t^{\text{h}}}{Y_t^{\text{o}}} = \dfrac{v_{t-1}^{\text{h}*}(C_t^{\text{o}} + V_t^{\text{o}} + U_{t-1}^{\text{o}}) + S_t^{\text{h}}}{Q_t^{\text{o}} + U_{t-1}^{\text{o}}} = \dfrac{150}{27} = 5.56$
$v_t^{\text{h}} U_{t-1}^{\text{o}} = 5.68 \cdot 5 = 28.4$	$v_t^{\text{h}} U_{t-1}^{\text{o}} = 5.56 \cdot 5 = 27.8$
$Y_t^{\text{h}} = Q_t^{\text{h}} + v_t^{\text{h}} U_{t-1}^{\text{o}} = v_t^{\text{h}} Q_t^{\text{o}} + v_t^{\text{h}} U_{t-1}^{\text{o}} = 125 + 28.4 = 153.4$	$Y_t^{\text{h}} = C_t^{\text{h}*} + V_t^{\text{h}*} + S_t^{\text{h}} + U_{t-1}^{\text{h}*} = 75 + 25 + 25 + 25 = 150$ $(U_{t-1}^{\text{h}*} = v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}} = 5 \cdot 5 = 25)$
$\rho_t^{\text{h}} = \dfrac{S_t^{\text{h}} + (v_t^{\text{h}} - v_{t-1}^{\text{h}*})U_{t-1}^{\text{o}}}{C_t^{\text{h}*} + V_t^{\text{h}*} + v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}}} = \dfrac{25 + (5.68 - 5)\cdot 5}{75 + 25 + 5 \cdot 5} = 22.7\%$	$\rho_t^{\text{h}} = \dfrac{S_t^{\text{h}}}{C_t^{\text{h}*} + V_t^{\text{h}*} + v_{t-1}^{\text{h}*} U_{t-1}^{\text{o}}} = \dfrac{25}{75 + 25 + 5 \cdot 5} = 20\%$
$m_t = \dfrac{Y_t^{£*}}{Y_t^{\text{h}}} = \dfrac{p_t^{£*}(Q_t^{\text{o}} + U_{t-1}^{\text{o}})}{v_t^{\text{h}}(Q_t^{\text{o}} + U_{t-1}^{\text{o}})} = \dfrac{p_t^{£*} Q_t^{\text{o}}}{v_t^{\text{h}} Q_t^{\text{o}}} = \dfrac{Q_t^{£*}}{Q_t^{\text{h}}} = \dfrac{110}{125} = 0.88$	$m_t = \dfrac{Y_t^{£*}}{Y_t^{\text{h}}} = \dfrac{Q_t^{£*} + p_t^{£*}\mathrm{U}_{t-1}^{\text{o}}}{C_t^{\text{h}*} + V_t^{\text{h}*} + S_t^{\text{h}} + \mathrm{v}_{t-1}^{\text{h}*}\mathrm{U}_{t-1}^{\text{o}}} = \dfrac{110 + 25}{75 + 25 + 25 + 5 \cdot 5} = 0.9$

(6) ensures that this result holds true, no matter what happens to productivity. But because of the productivity regress, carried-over stocks increase in value in terms of labor-time. The value of total capital in terms of labor-time exceeds the value of the capital advanced plus the surplus-value extracted in production, thus boosting the profit rate in terms of labor-time. Since the unit value of output has risen while its price has remained constant, by assumption, the MELT falls to £0.88 per hour of labor-time.

As I pointed out when we considered a productivity improvement, Freeman's figures for Y_t^{h} and ρ_t^{h} do not depend on Q_t^{o}, so they are also unchanged now that productivity regresses. Once again, total capital in terms of labor-time, Y_t^{h}, increases by the total surplus-value extracted from labor in production. Freeman's equation (9′) ensures that this result holds true, no matter what happens to productivity. Since Y_t^{h} is unchanged, while carried-over stocks have appreciated in value ($v_t^{\mathrm{h}} U_{t-1}^{\mathrm{o}} > v_{t-1}^{\mathrm{h}*} U_{t-1}^{\mathrm{o}}$), the other component of the value of capital—the value of newly-produced output, Q_t^{h}—must decline. This causes it to fall short of $C_t^{\mathrm{h}*} + V_t^{\mathrm{h}*} + S_t^{\mathrm{h}}$; in other words, the output now embodies less value, in terms of labor-time, than the constant capital transferred plus the living labor added in the production of this output. As in Kliman's approach, the MELT falls, since the unit value of output has risen as a result of the productivity regress, while the price of output is assumed to remain constant.

According to both Kliman and Freeman, the productivity regress causes the physical profit rate, ρ_t^0, to fall below the profit rate in terms of labor-time, ρ_t^{h}. The latter, according to Freeman, remains constant at 20%, while for Kliman, it is boosted to 22.7% as a result of the productivity regress.

Summing Up

To sum up, if we follow Kliman's approach, we appear to contradict Marx's insistence that surplus labor is the sole source of profit. Total capital in terms of labor-time grows by more than the surplus labor-time extracted in production in that period if productivity regresses, whereas it fails to expand by the surplus labor-time extracted in production in that period if productivity improves. Alternatively if we follow Freeman's approach, total capital in terms of labor-time does expand by the surplus labor-time extracted in production in that period, no matter if productivity improves or regresses. So is there a problem with Kliman's approach?

However, following Kliman's approach does ensure that, whether productivity improves or regresses, the value of newly produced commodities equals the constant capital transferred plus the living labor added to them in production in that period. If we follow Freeman's approach, when productivity improves (regresses), the value of newly produced commodities exceeds (falls short of) the constant capital transferred plus the living labor added to them in production in that period. So is there a problem with Freeman's approach?

I suggest that neither approach has a problem; rather, they simply interpret how Marx determines commodities' values differently. If, like Kliman, we interpret Marx as having held that commodities' values are determined solely by the values of newly produced commodities, then we must accept the need to revalue stocks. The values of stocks that are not being used as inputs into current production change so that they are the same as the values of newly produced commodities.[8] This stock revaluation changes the value of total capital, but can we really imagine that this is a creation or destruction of value, by some source of value other than labor? It is simply a change in the value of commodities that are not participating in the formation of values as determined by current production conditions.

Conversely, Freeman interprets Marx as having held that commodities' values are determined not only by the values of newly produced commodities, but by the values of existing units of those commodities as well. We simply have two interpretations of how Marx determines commodities' values, both of which follow a sequential and non-dualistic method; i.e., both follow a TSSI of Marx.

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8 Although I do not have the space here to fully explore fixed capital, I should note that if we were to include fixed capital, applied but not used up in the current period, it would need to be revalued, according to Kliman's interpretation, in order to reflect the value of newly produced units of fixed capital. In contrast, if we follow Freeman's interpretation, the new value of fixed capital would depend not only on the value of newly produced units of fixed capital, but also on the value of existing units of fixed capital.

Conclusion

I hope to have shown, by means of the above discussion of how to value commodities in the presence of stocks, how the TSSI of Marx represents an exciting and open approach to researching Marx's economics. Quite simply, Laibman's labeling of the TSSI as the dogma of "new orthodox Marxists (NOMists)" (Laibman 1999, 253) could not be more mistaken. Questions such as how to treat fixed capital or stocks of commodity money, or how changes in demand or prices may change commodities' socially-determined values, require further research and are open for further research. Personally, I have employed a sequential and non-dualistic approach in order to begin to consider how we might integrate the productive economy and the financial system (Potts 2005). I do not claim to have done more than scratch the surface of this critical area of research; Potts (2009a and 2009b) represent the current status of that scratch. However, I am already convinced that simultaneous approaches generate concepts of value that are too rigidly stuck in their simultaneous limitations to allow them to be integrated with the dynamic behavior of the financial system. For example, Fine, Lapavitsas, and Milonakis (1999) consider questions of value theory and then monetary/financial questions in distinct sections, reflecting mainstream economics' separation of the "real" and the monetary/financial system. To conclude, I believe that the TSSI of Marx has proven its right to exist, so let us explore how it can help us understand our world.

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