

Marx's Reproduction Schemes as an Unbalanced-Growth Model

Andrew Kliman¹

Abstract

Although Marx's reproduction schemes are commonly construed as balanced-growth models, this paper argues that they can be understood as depicting a process of unbalanced growth. When the schemes of simple and expanded reproduction are compared, they imply that the transition from simple to expanded reproduction requires production of means of production to grow faster than production of consumer goods. On this interpretation, the reproduction schemes emerge as an early and accurate analysis of the "take-off" process. The paper defends this interpretation exegetically, and argues that it eliminates an apparent incompatibility, between the schemes and the non-equilibrium character of the bulk of *Capital*, that arises when the schemes are construed as balanced-growth models.

The Question of Internal Inconsistency

Drawing heavily on the works of equilibrium theorists such as Dmitriev, Bortkiewicz, Leontief, and Sraffa, mainstream "Marxian economics" has interpreted Marx as an equilibrium theorist. More recently, however, research in an alternative, non-equilibrium inter-

¹ Department of Economics, Pace University, 861 Bedford Road, Pleasantville, NY 10570, USA. e-mail: akliman@pace.edu.

pretation of Marx's value theory, the temporal single-system interpretation (TSSI), has identified fatal flaws in the equilibrium interpretations (see, e.g., Freeman and Carchedi 1996, Kliman 2007).

The most telling flaw is that the equilibrium interpretations fail to make Marx's value theory make sense. They give rise to many unsolvable "internal inconsistencies," and therefore imply that Marx's theory of the origin of profit, his law of the tendential fall in the rate of profit, and other crucial aspects of *Capital* must be rejected or corrected. Yet all such apparent inconsistencies disappear under the TSSI. Hence, the so-called *internal* inconsistencies are in fact *external* inconsistencies, not inconsistencies within Marx's own text, but inconsistencies between the equilibrium interpretations and the text itself.

According to a standard principle of exegetical interpretation, the relative adequacy of different interpretations depends on the degree to which they make the text make sense (if that is possible). Apparent self-contradictions in the text should be regarded as *prima facie* indications of the interpreter's misunderstanding (see, e.g., Warnke 1993, 21). This principle implies that the equilibrium interpretations of Marx should be rejected as inadequate. By creating inconsistencies in his text, these interpretations fail to make it make sense, although it can be read such that it does make sense, as the TSSI shows (see Kliman 2007, esp. Chap. 4).

Yet there remains another allegation of internal inconsistency in *Capital*, bearing on the question of equilibrium, which TSSI research has scarcely tackled. That is the burden of the present paper. The problem is this: if Marx was not an equilibrium theorist, as proponents of the TSSI hold, then what does one make of the schemes of simple and expanded reproduction in *Capital*, Vol. 2? These schemes, it has been argued, are equilibrium models in the sense of being balanced-growth models—output of Department I, which produces means of production, and output of Department II, which produces consumer goods, continually grow at the same rate. Thus, if other aspects of *Capital* do indeed have a non-equilibrium character, then there is a profound inconsistency between the reproduction schemes and the rest of the work.

Meghnad Desai has made this point forcefully:

How could one reconcile this picture of an economy in perpetual balanced growth with Marx's prediction[,] elsewhere in his work[,] of a capitalist economy riddled with crises and liable to breakdown

as a result of increasing contradictions[,] including a falling rate of profit despite growth and accumulation? ... Was this another example of a glaring inconsistency between different parts of *Capital*, as had been argued in the case of the value–price relationship by Böhm-Bawerk? [Desai 1990, 339]

The contrast between the dynamic implied in the Falling Rate of Profit and in the SER [scheme of Expanded Reproduction] is a blatant one. ... how is one to integrate it [the SER] back into the general Marxian model of disequilibrium dynamics in a monetary capitalist economy? ... It is this contradiction between the picture of capitalism ridden by crises and faced with a long run tendency of the rate of profit to fall in Vol. III[,] and the smooth expansion of Volume II[,] which is *the central unsolved problem of Marxian dynamics*. It is of much greater importance than the value–price contradiction of Vol. I and Vol. III [Desai 1979, 152, 156, emphasis in original]

Desai’s view of the reproduction schemes as balanced-growth models is widely, almost universally, accepted nowadays. This does not mean that everyone thinks that Marx intended to depict capitalism as a system in “perpetual balanced growth.” Many interpreters argue the opposite—he studied the case of balanced growth in order to highlight how implausible it is, how difficult it would be to achieve it in reality. But whether they think that Marx intended to affirm, or to deny, that growth under capitalism is balanced, recent interpreters almost invariably agree that the reproduction schemes themselves depict a process of balanced growth.

I will argue that the schemes need not be interpreted in this manner. They can plausibly be understood as an unbalanced-growth model. The next section provides analytical support for this contention. In the third section, I show that this unbalanced-growth interpretation, which seems almost unknown these days, was traditionally not uncommon. The fourth section shows that Marx himself discussed unbalanced growth as an implication of his schemes. In the fifth section, I discuss the significance of the schemes when they are interpreted in this manner. A brief summary and conclusions follow in the sixth section.

If the unbalanced-growth interpretation is plausible, it puts to rest the allegation that Marx’s discussion of reproduction and the remainder of *Capital* are necessarily internally inconsistent. Marx can be regarded as a *consistent* non-equilibrium theorist.

There is another interpretation which—if it were correct—would also put this allegation of inconsistency to rest. I refer to the view that Marx depicted balanced growth only in order to highlight its implausibility. The problem is that no textual support for this interpretation seems to exist. To understand why not, we need to distinguish between two senses of “balance” or “equilibrium” that seem at times to be conflated in the secondary literature:²

- *Balance of Supply and Demand*: The quantity of each good supplied (or, more precisely, currently produced) is equal to the quantity demanded.
- *Balanced Growth*: Departments I and II grow at the same percentage rate.

These two meanings of balance have nothing to do with one another. The first refers solely to a moment in time—in Marx's schemes, to the end of a given year—while the second refers to a process taking place over time. It is thus possible that, in each department, supply and demand are in balance at each moment in time (or, more realistically, on average) while, over time, the growth rate of Department I exceeds that of Department II.³

With this distinction in mind, we can say the following. Although Marx's schemes depict a continual balance between supplies and demands, he does at times refer to the difficulties involved in achieving such a balance (see, e.g., Marx 1981, 571, 593, 596). In contrast, he seems never to comment on obstacles to balanced growth. Indeed, he does not discuss at all whether balanced growth can be achieved—which is one fact that casts doubt on the balanced-growth interpretation of his schemes.

² Dunayevskaya (1943, Sect. II.1 (b)) and Howard and King (1975, 193) are among the authors who explicitly distinguish between these different meanings of balance. Others such as Harris (1978, 260–61) distinguish between them implicitly, by invoking them as distinct assumptions in their mathematical treatments of expanded reproduction.

³ The possibility that supply grows faster in Department I than in Department II, in other words, does not imply a chronic, secular shortfall in effective demand. Demands as well as supplies can grow in an unbalanced way—investment demand can grow faster than consumer demand—and thus supply and demand in each department can remain in balance over time. My argument is that this is the situation that Marx's reproduction schemes depict.

Unbalanced Growth in the Reproduction Schemes

In Marx's scheme of simple reproduction, Departments I and II grow at the same (zero) rate. In his numerical examples of expanded reproduction, they eventually grow at the same (positive) rate. If we counterpose the two schemes, consider them as two distinct, self-contained models, we arrive immediately at the balanced-growth interpretation. Marx has evidently presented us with two different growth models, in each of which growth is balanced.

Yet what if, instead of *counterposing* the two schemes, we *compare* them? They can then be seen to depict a process of transition, from simple reproduction to expanded reproduction. Given the assumptions embedded in the schemes, this transition can take place only if growth is unbalanced.

The schemes assume that there is no technological progress (Marx 1981, 469), so growth (expanded reproduction) can occur only if a greater volume of means of production is obtained. In principle, these additional means of production could be obtained by importing them, or acquiring them from domestic non-capitalist producers, but Marx's schemes assume a closed and purely capitalistic economy. Marx (1981, 581) also assumes away any accumulation or disaccumulation of stocks, so the additional means of production that are needed in order for growth to occur cannot be obtained from warehouses and storerooms. They can be obtained only by *first being produced* by Department I. Since there is no technological progress, however, Department I can produce the additional volume of means of production only by *first obtaining* additional means of production. We seem to be caught in a vicious circle. Apparently, Department I must increase its output before it can obtain the additional inputs that it needs in order to increase its output!

There is, however, one remaining source of the additional means of production that Department I needs—Department II. Under conditions of simple reproduction (no growth), Department I's output of means of production, at the end of one year, is just enough to replace the means of production that have been used up, during that year, by itself and by Department II. Imagine, however, that some of these newly produced means of production that Department II would have obtained (in order to produce on the same scale next year) are instead diverted to Department I. This gives Department I the additional means of production it needs. Its output in the second year is conse-

quently greater than in the first year, and this provides the economy with the increased volume of means of production that it needs in order to grow.

Yet since means of production have been diverted away from Department II, and since (by assumption) there is no technological progress, Department II's output in the second year is smaller than in the first year. Thus the expansion of Department I comes at the expense of Department II. Given Marx's assumptions, in other words, the transition from simple to expanded reproduction requires unbalanced growth. Department I must grow relative to Department II.

Initially, the contraction of Department II is absolute as well as relative. In order for the transition from simple to expanded reproduction to occur, Department II must initially become smaller than it was under simple reproduction. Subsequently, Department II can grow, but if the two departments grow at the *same* rate, as they eventually do in Marx's examples of expanded reproduction, the relationship between them remains unchanged, and so the relative imbalance persists. Hence, Department I remains relatively larger than Department II under expanded reproduction than under simple reproduction.

The foregoing has simply applied the technique of comparative analysis that is well known to every economist, and indeed to every introductory student of economics. I have compared two equilibrium positions—simple and expanded reproduction—with one another, and deduced from this comparison the changes that must take place in order for the transition from the first equilibrium to the second to take place. Given the prevalence of this technique within economics, it is somewhat surprising that modern interpreters do not employ it when assessing the implications of Marx's schemes, but instead construe the two equilibrium positions as distinct and self-contained models.

The following example illustrates the above analysis. In Year 1, the economy is in a state of simple reproduction. The output of means of production (W_I), 500, is only large enough to replace the means of production used up in the economy during the year (Total c), so no growth of the aggregate economy is possible.

In Years 2 and 3, the economy is in a state of expanded reproduction in which growth is balanced. Between Years 2 and 3, each department grows by 20%. Whereas the two departments were of equal size in Year 1, Department I is now 50% larger than Department II. Under Marx's assumptions, this relative increase in Department I's

Table 1. Transition from Simple to Expanded Reproduction

Year 1: Simple Reproduction				
Dept.	<i>c</i>	<i>v</i>	<i>s</i>	<i>W</i>
I	250	100	150	500
II	250	100	150	500
Total	500	200	300	1000

Year 2: Transition to Expanded Reproduction				
Dept.	<i>c</i>	<i>v</i>	<i>s</i>	<i>W</i>
I	300	120	180	600
II	200	80	120	400
Total	500	200	300	1000

Year 3: Expanded Reproduction				
Dept.	<i>c</i>	<i>v</i>	<i>s</i>	<i>W</i>
I	360	144	216	720
II	240	96	144	480
Total	600	240	360	1200

Note: *c*, *v*, and *s* stand for used-up constant capital, variable capital, and surplus-value. $W = c + v + s$ stands for total output. All figures are in value (money) terms. Technology and per-unit prices (= values) are constant throughout, and there is no fixed capital.

size is necessary if expanded reproduction is to take place. Given unchanging technology, 20% more means of production must be available in Year 3 than in Year 2 (Total *c* of 600 as against 500) if the economy is to grow by 20% between these periods. In the absence of foreign trade, non-capitalist producers, and drawing-down of stocks, these means of production can be supplied only from Department I's output of Year 2, so W_1 of Period 2 must equal 600. Given the other data of the example, this is possible only if Department I is exactly 50% larger than Department II.

How does Department I become relatively larger? The answer is that it must grow faster than Department II between Years 1 and 2. Department I's share of total output must increase from 50% to 60%, while Department II's must decline from 50% to 40%.

Taking the three years as a whole, the 20% aggregate growth of the economy is effected by Department I growing by 44% while Department II contracts by 4%. The absolute decline in Department II's size over the course of the three years is an incidental feature of this example.⁴ What is important, and necessary under Marx's assumptions, is that Department II undergoes a relative decline.

Before moving on, a comment on the unrealistic character of the example is in order. The example may give one the impression that the issue of unbalanced growth is much ado about very little. The transition to a 20% growth rate required only a 50% rise in the relative size of Department I. Since 20% growth is unrealistically fast, it might seem that a transition to a reasonable growth rate would require only a modest increase in the relative size of Department I. Moreover, the example seems to suggest that the transition from simple to expanded reproduction is of only brief duration—one "year."

However, these features of the example stem from its specific assumptions, especially the assumption that means of production last only one year (i.e., that there is no fixed capital). If, on the other hand, a relatively large share of capital were fixed, a larger increase in the relative size of Department I would be needed in order to sustain a much smaller growth rate, and the transition from simple to expanded reproduction would be of considerably longer duration.⁵

To illustrate this point, I will continue to use the Year 1 data of Table 1, but now assume that the figures for used-up constant capital, c , represent only 10% of the total constant capital in each department. I will also assume that Department I's growth rate is only 5% instead of 20%.

The results are as follows. First, Department I now eventually becomes 200% larger than (i.e., three times as large as) Department II, instead of only 50% larger, even though Department I grows much more slowly than before. Second, Department II experiences a much longer absolute decline. Instead of contracting during only during Year 2, it continues to contract through Year 10. Third, Department

⁴ In contrast, as noted above, the absolute decline in its size between Years 1 and 2 is—given Marx's assumptions—a necessary feature of the transition from simple to expanded reproduction.

⁵ Although Marx's numerical examples of expanded reproduction abstracted from fixed capital, his reproduction schemes *per se* do not. Marx (1981, 524–45) discussed fixed capital in great detail in connection with simple reproduction, and the scheme of simple reproduction exhibits exactly the same properties whether or not fixed capital is included.

II's output remains below its starting level (W_{II} in Year 1 = 500) through Year 21. Finally, its growth rate no longer adjusts immediately to Department I's, but approaches it only gradually and asymptotically. Through Year 28, Department II's growth rate remains more than 10% below Department I's. (See Figures 1 and 2. For additional discussion, see the Mathematical Appendix.)

The Secondary Literature

Except for Dunayevskaya (1943; 1988; 1991), modern commentators seem invariably to construe Marx's reproduction schemes as balanced-growth models. Some simply state, without qualification, that the schemes exhibit balanced growth, steady state properties, etc. (e.g., Harris 1978, 33). Others, such as Mandel, suggest that Marx modeled a process of balanced growth in order to show how exceptional it is:

the reproduction schemas show that equilibrium, not to speak of equilibrated growth, is the exception and not the rule under capitalism. ... [They are] an extreme simplification intended to bring out the underlying assumptions of equilibrium (or equilibrated, proportionate growth) under conditions of capitalist production. [Mandel 1981, 25, 28, emphases omitted]

Finally, some commentators (e.g., Dixon 1999, 980) present both possibilities without taking sides. But whatever their view of Marx's intentions, they all regard the schemes of reproduction themselves as balanced-growth models.

This was not always the case. Many, though not all, participants in the pre-World War I discussion of the schemes understood them as depicting the faster growth of Department I as against Department II.⁶

Lenin (1964, 162), for instance, argued that the disproportionate growth of production and consumption under capitalism "is expressed, as Marx demonstrated clearly in his schemes, by the fact that the production of the means of production can and must outstrip the production of articles of consumption." And earlier, he wrote,

⁶ Representatives of the contrary position include Boudin and Kautsky. According to Luxemburg (1968, 320n), they regarded the idea that Department I grows faster than Department II as Tugan Baranowski's "delusion."

Figure 1. Growth Rates

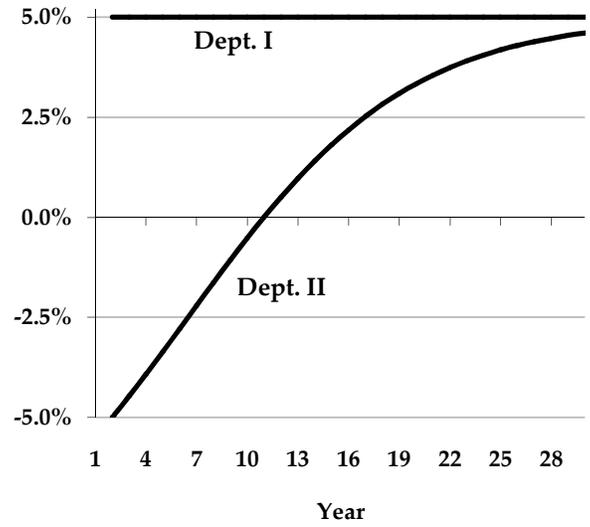
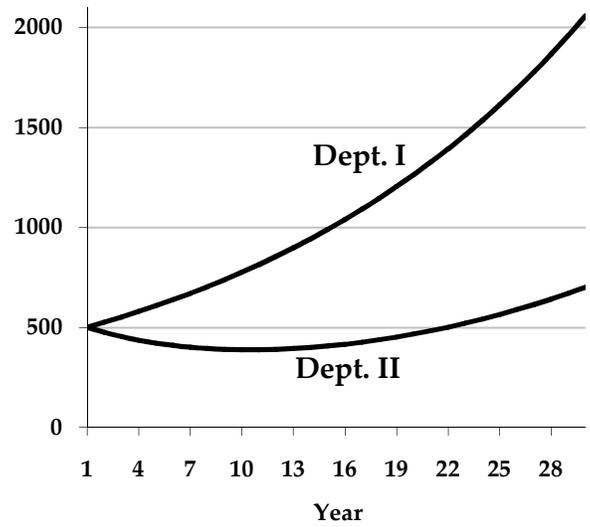


Figure 2. Levels of Output



the main conclusion from Marx's theory of realization is the following: ... the increase in means of production outstrips the increase in articles of consumption. ... the department of social production which makes the means of production must grow faster than the one which produces the means of consumption. Thus, for capitalism, the growth of the internal market is to a certain extent 'independent' of the growth of individual consumption. [Lenin 1972, 54, 56]

This interpretation has fallen so out of favor that modern commentators seldom acknowledge its existence. Perhaps they are not aware of it. Rosdolsky is a rare exception; he mentions the unbalanced-growth interpretation, but only to reject it as absurd. After quoting Lenin's claim that Marx's schemes show production of means of production outstripping production of articles of consumption, Rosdolsky (1989, 475) dismisses it with one quick sentence: "In actual fact, however, Marx's schemes show nothing of the kind, since, in both examples in Volume II, Department II develops at exactly the same speed as Department I."

Lenin wrote extensively on Marx's theory of reproduction and on capitalist development in Russia, and he was no lightweight either as a theorist or as an interpreter. Did he really commit the elementary blunder, which Rosdolsky attributes to him, of failing to notice that Marx's examples show the two departments eventually growing at the same rate? Or did Rosdolsky fail to notice that one may *compare* the schemes of simple and expanded reproduction instead of *counterposing* them, in which case Lenin's conclusions are sound?

Bulgakov and Tugan Baranowski were other pre-World War I authors who interpreted the schemes in a manner broadly similar to Lenin. Bulgakov argued that, in his reproduction schemes, "Marx ... has shown that the growth of consumption is fatally lagging behind that of production, and must do so whatever 'third persons' one might invent" (Bulgakov, quoted in Luxemburg 1968, 305). This implies that the growth of Department II, which produces the consumer goods, lags behind the growth of Department I, which produces means of production.

Tugan Baranowski (quoted in Luxemburg 1968, 311–12) drew the even stronger conclusion that Marx's reproduction schemes "prove conclusively [... that it is] possible that the volume of social consumption as a whole goes down while at the same time the aggregate social demand for commodities grows." Total production and Department I thus grow while production in Department II falls.

It is a curious fact that even Luxemburg, who criticized the reproduction schemes on the ground that production must ultimately be production for the sake of consumption (Luxemburg 1968, esp. Chap. 25), had the same general view. Not only did she contend that Department I grows faster than Department II in reality. She also seemed to suggest that Marx's reproduction schemes were intended to show that it grows faster:⁷

With the progress of capitalist development Department I *goes on growing* at the expense of Department II. *It was Marx himself who, as we all know, set up this law in which he grounded the schematic exposition of reproduction*, though in the further development of his diagram he ignored subsequent alterations for simplicity's sake.

. . . the quicker growth of Department I as against Department II is beyond dispute It is the foundation also of Marx's fundamental law that the rate of profit tends to fall. [Luxemburg 1968, 316, 320, emphasis added].

The final clause of the first passage refers to the fact that, in Marx's numerical examples of expanded reproduction, the two departments eventually grow at the same rate. Luxemburg is thus suggesting that, while Department I "goes on growing" faster than Department II in reality, Marx's presentation ignored this fact "for simplicity's sake."

Thirty years later, Dunayevskaya (1943, Sect. II.1 (b)) set out what is perhaps the most extensive and sophisticated textual analysis of the reproduction schemes as a depiction of unbalanced growth.⁸ She made three major points. Their analytical implications will be explored below. I discuss them here only in order to show that Dunayevskaya was in each case comparing, not counterposing, simple and expanded reproduction, and that this comparative method is what allowed her to conclude that the schemes depict unbalanced growth.

⁷ The first quotation below suggests that Marx used the reproduction schemes to "ground" the "law" that Department I grows faster than Department II. Later, however, Luxemburg (1968, 329, 341) alleges that the scheme of expanded reproduction contains contradictions and that it is in fact "downright impossible to achieve a faster expansion of Department I ... within the limits of Marx's diagram." The only way I am able to reconcile these statements is to conclude that Luxemburg held that Marx intended the schemes to show that Department I's growth outstrips Department II's, but that he failed to show this successfully.

⁸ See also Dunayevskaya (1988, 130-31; and 1991, *passim*) for briefer discussions of the issue of unbalanced growth.

First, she drew attention to the conditions for reproduction that Marx derived. Simple reproduction requires a “balance” (equality) between the new value generated in Department I ($v_1 + s_1$) and Department II’s demand for constant capital. Expanded reproduction, however, requires that the new value generated in Department I exceed Department II’s demand for constant capital. Dunayevskaya argued that this implied disproportionate growth.

Second, she quoted Marx’s statement that the material basis for expanded reproduction is a change in the “destination” of the various components of social output.⁹ Relatively more of the output of Department I is used by it, and less by Department II, than under simple reproduction. “The changed destination,” Dunayevskaya wrote, “is nothing more than the disproportionate growth of constant capital.”¹⁰

Finally, Dunayevskaya addressed the equalization of the two departments’ growth rates in Marx’s examples of expanded reproduction, taking issue with the notion that this implied balanced growth. The balance between the two departments, she suggested, is merely the outcome of an earlier disproportionate growth of constant capital relative to variable capital (and, presumably, of means of production relative to consumer goods).¹¹

Marx’s Discussion of Unbalanced Growth

I have argued that Marx’s schemes of reproduction may be viewed as a model of unbalanced growth and that this interpretation was traditionally not uncommon. But what was Marx’s own understanding of the schemes?

It is clear that his main purpose in developing the schemes was not to model balanced growth (nor to model unbalanced growth). The

⁹ The passage is Marx (1981, 582). Dunayevskaya cited a different translation.

¹⁰ It is true that Marx’s reproduction schemes assume a constant ratio of constant to variable capital, c/v , within each department. Yet if c/v is greater in Department I than in Department II, the *economy-wide* c/v will indeed increase when Department I grows faster. Dunayevskaya may have had this case in mind. I think it is more likely, however, that she was extrapolating from the schemes to the case of labor-saving technological change (see footnote 11, below).

¹¹ Here, and perhaps also in her preceding comment, Dunayevskaya is employing Lenin’s method of trying to understand the reproduction schemes in light of Marx’s analysis of accumulation in *Capital*, Vol. 1, in which he maintained that the ratio of constant to variable capital tends to increase over time.

main purpose of the schemes, and of the final part of *Capital*, Vol. 2 as a whole, was to refute the attempt of Adam Smith (and of the classical economists who accepted his analysis) to “conjure the constant part of capital out of commodity value” (Marx 1981: 449).¹² Smith’s theory implies that all accumulated surplus-value (i.e., all property income that is reinvested) must “ultimately” become additional variable capital, and thereby accrue to workers through increased employment, higher wage rates, or both.¹³ The issue is important practically as well as historically because, as Dunayevskaya (1991, 33–34) noted, Smith’s theory is an early version of trickle-down economics: “If, as Smith maintained, the constant portion of capital ‘in the final analysis’ dissolved itself into wages, then the workers need not struggle against the ‘temporary’ appropriation of the unpaid hours of labor. They need merely wait for the product of their labor to ‘dissolve’ itself into wages.”

The reproduction schemes helped Marx to present his refutation of Smith’s theory in a clear and simple way. The schemes’ division of all output and inputs into means of production and means of consumption allowed Marx easily to track the destination of investment. He showed that additional variable capital is only one portion of new investment; the remainder—in every period, and therefore ultimately as well—is additional constant capital (see esp. Marx 1981, 469, 470, 478, and footnote 13, above).

Of course, Marx could have shown this by using the scheme of expanded reproduction alone. But he decided first to analyze simple reproduction because “[t]he main difficulties arise not in the treatment of accumulation, but already in that of simple reproduction,” and also because previous theorists had concentrated on simple reproduction (Marx 1981, 446).

¹² See esp. Chap. 19 of Vol. 2. The same issue is discussed extensively in Part 1 of the *Theories of Surplus-Value* (Marx 1963, 97–151) and, more briefly, in *Capital*, Vol. 1 (Marx 1977, 734–38).

¹³ “There can be no greater error than the one repeated after Adam Smith by Ricardo and all subsequent political economists, namely the view that ‘the portion of revenue so said to be added to capital, is consumed by productive labourers’. According to this, all surplus-value that is transformed into capital becomes variable capital. However, in actual fact the surplus-value, like the value originally advanced, divides up into constant and variable capital, into means of production and labour-power” (Marx 1977, 736). The interior quotation closely paraphrases a statement of Ricardo’s.

There also seems to be no evidence that depiction of balanced growth was even a subsidiary aim of Marx's. His schemes do not assume that the two departments grow at the same rate. In his numerical examples of expanded reproduction, he instead assumed that a constant share of Department I's surplus-value is accumulated; even this assumption seems to have been made as a matter of convenience, not as any theory of investment behavior. Owing to this assumption, the growth rates of the two departments, which are at first unequal, eventually equalize. The eventual state of balanced growth is thus no more than an unintended *by-product* of Marx's assumption, rather than an *assumption* in its own right.

In fact, balanced growth is nowhere discussed in the text. Marx did not even comment on the fact that growth rates in his numerical examples eventually equalized. The reason he was inattentive to this feature of the examples is not that he lacked interest in growth rates, nor that he failed to compute and compare them. On the contrary, he commented at one point that reproduction between two periods in one of his examples requires that Department II accumulate more quickly than Department I (Marx 1981, 588). At the end of that example, he computed the growth rates of capital and surplus-value for the aggregate economy, but not for each department separately (Marx 1981: 588). In the midst of another example, he noted that Department I grew by one-twelfth, and Department II by one-ninth, between two successive periods (Marx 1981, 594). This time, at the end of the example, he did compute the absolute growth of capital in each of the two departments, but he did not take the ratios that would have allowed him to compare their growth rates (which were unequal over the three-year span he computed) (Marx 1981, 595).

In sum, the textual evidence suggests that Marx did not intend to develop a balanced-growth model, and that he evinced no interest in, and probably failed to notice, the fact that his examples eventually settled into a state of balanced growth.

In contrast, the evidence that follows indicates that Marx (1) repeatedly compared simple and expanded reproduction instead of counterposing them as distinct models, (2) regarded them as occurring in temporal succession, and (3) recognized that the transition from simple to expanded reproduction involves unbalanced growth. This is the case even though the concept of unbalanced growth does not appear explicitly in the text, and even though, as I noted above, Marx did not develop the reproduction schemes primarily in order to depict unbalanced growth.

Perhaps the most important point is that Marx regarded simple reproduction and expanded reproduction as occurring in temporal succession. The capitalist economy, which at one stage was in an inherited state of simple reproduction, moves to a state of expanded reproduction.

That Marx regarded these as successive stages is suggested by the fact that his discussion of expanded reproduction follows that of simple reproduction. But there is also clearer evidence. Marx (1981, 572, emphasis added) explicitly analyzed the relation between simple and expanded reproduction as a transition from the first to the second:¹⁴

If we consider the level of reproduction [at the start of the transition from simple to expanded reproduction] on the part of department I in value terms, then we still find ourselves within the limits of simple reproduction, for no additional capital has been set in motion ... and no more surplus labour [is performed] than was performed on the basis of simple reproduction. The distinction here lies only in the form of the surplus labour applied It has been spent on means of production for Ic instead of IIc, on means of production for means of production instead of on means of production for means of consumption. ... *Thus in order to make the transition from simple reproduction to expanded reproduction, production in department I must be in a position to produce fewer elements of constant capital for department II, but all the more for department I. This transition, which can never be achieved without difficulty, is made easier by the fact that a number of the products of department I can serve as means of production in both departments.*

Marx (1981, 573) thus concludes that “the material substratum for expanded reproduction is produced in the course of simple reproduction.” In other words, the transition from simple to expanded reproduction requires no prior accumulation of capital. Expanded reproduction proceeds initially from the state of simple reproduction.¹⁵

¹⁴ That Marx analyzed the transition from simple to expanded reproduction is scarcely mentioned by later writers. A rare exception is Luxemburg (1968, 123, 145, 147, 150, 163, 318). Because she treated Marx's concept of a transition period only insofar as it constituted a solution—in her opinion, a false one—to the problem of how expanded reproduction can occur, her references to it are consistently unfavorable.

¹⁵ How it does so is illustrated in Table 1, above. As Marx suggests, the aggregate amounts of constant and variable capital are no greater in Year 2 than in Year 1; the transition from simple to expanded reproduction instead requires only that some constant capital and labor-power be reallocated, from Department II to Department I.

Marx's text contains a few additional passages which likewise state that the difference between simple and expanded reproduction lies in the reallocation of resources away from Department II and toward Department I. In the first of these passages, Marx (1981, 580–81) again suggests that the material basis of expanded reproduction

is in fact simple reproduction, the elements merely being grouped together differently from the above schema [of simple reproduction], in accordance with the needs of future expansion, say in the coming year. ...

[There is a] different arrangement of the elements of department I (as far as reproduction is concerned), an arrangement without which there could be no reproduction on an expanded scale at all.

Immediately thereafter comes the passage cited by Dunayevskaya. It occurs at the start of Marx's first schematic presentation of accumulation. He again stresses that

reproduction on an expanded scale ... has nothing to do with the absolute size of the product ... it simply assumes a different arrangement or a different determination of the functions of the various elements of the given product, and is thus in the first instance only simple reproduction, as far as its value goes. It is not the quantity, but the qualitative character of the given elements of simple reproduction that is changed, and this change is the material precondition for the ensuing reproduction on an expanded scale. [Marx 1981, 582]

As Dunayevskaya noted, the "different arrangement" (or "destination") of the elements is Marx's way of referring to the reallocation of means of production and workers from Department II to Department I.

To show that what distinguishes simple and expanded reproduction is not "the absolute size of the product," Marx (1981, 582) then constructed another numerical example. Although the total social product is of the same size as in his example of expanded reproduction, only simple reproduction takes place, owing to the different "functional arrangement of its elements."

This last set of passages does not refer explicitly to the *transition* from simple to expanded reproduction when analyzing the difference between them. Yet Marx does compare simple and expanded reproduction, and it is plausible that he had the transition from the first to

the second in mind when he wrote these passages. His analysis of the transition period and these passages are both contained in the chapter on expanded reproduction, all of which was written entirely during 1878, apparently during the latter half of the year (see Engels, 1981a, 86; Engels 1981b, 103–04), and these passages follow the text's analysis of the transition period.

Finally, there are a few passages in which Marx compared the conditions needed for reproduction to occur under simple and expanded reproduction. He compared them briefly two times (Marx 1981, 590, 593), and extensively in another case (Marx 1981, 595–97). The key point, as I noted above in connection with Dunayevskaya's interpretation of these passages, is that simple reproduction requires that the new value generated in Department I, $I(v+s)$, equal Department II's demand for constant capital, Ic . Under expanded reproduction, in contrast, "the two cannot balance one another" (Marx 1981, 597). $I(v+s)$ needs to be greater than Ic .

What is at issue is not Marx's phrase "cannot balance." The issue is rather that the formulae show in a straightforward way that Department I must be larger in relation to Department II when $I(v+s) > Ic$ than when $I(v+s) = Ic$. In other words, Department I must be relatively larger under expanded reproduction than under simple reproduction. If we compare the two formulae, they once again suggest that the move from simple to expanded reproduction requires that Department I grow faster than Department II.

Significance of the Schemes as an Unbalanced-Growth Model

When understood as a model of unbalanced growth, Marx's schemes of reproduction emerge as perhaps the first analysis of what Rostow (1960) popularized as the "take-off" into self-sustained growth. Marx's concept of the transition from simple to expanded reproduction is broadly similar to Rostow's concept. Although, as I noted above, the secondary literature has rarely dwelt on Marx's concept of the transition, the same issue has regularly been taken up by theorists of growth and development. For instance, in discussing what he called the "time pattern of growth," Kuznets (1959, 35–36) wrote

At some period in the transition from pre-modern conditions to modern economic growth ... there must have been a shift from lower

to higher rates of growth. Since such shifts are rarely, if ever, sudden, there must have been a substantial period, extending over several decades, during which the rate of growth was accelerated, i.e., the rate of increase was rising.

Theorists of economic development have also explored the unevenness of growth—the reallocation of resources from Department II to Department I and the concomitant growth of production for industry at the expense of production for consumption—which Marx had identified as crucial to the transition from simple to expanded reproduction under capitalism. There are two main reasons why they have been interested in this issue. First, in capitalist country after capitalist country, the transition period has in fact exhibited the very unevenness that Marx had highlighted. As Sir W. A. Lewis (1955, 235, emphasis added) wrote

The British, the Japanese and the Russian industrial revolutions all fit into [... the same] pattern. In each case the immediate result is that the benefits of rising productivity *do not go to the classes who would increase their consumption—peasants, wage earners—but into private profits or public taxation, where the proceeds are used for further capital formation.* More and more labour is taken into wage employment, but real wages are not allowed to rise as fast as productivity.¹⁶

The other main reason why development theorists have been interested in this issue is that, during the 20th century, the reallocation of resources from production for consumption to production for production was often not just a fact but also a conscious policy choice. Referring to “the costs involved in achieving faster growth through the reallocation of resources,” Elkan (1995, 52–53) questions

whether a restriction of consumption as a matter of policy is in fact the most efficacious way of promoting economic development in low-income countries [Yet the] idea of a trade-off between current and future consumption has been very powerful in influencing economic development policy. It lay at the heart of the Soviet

¹⁶ Dunayevskaya (1992: 35-70), in an analysis written in 1942-43, may have been the first writer to argue that the direction of Russian development under Stalin fit the classical capitalist mold. This point was the essential ground of her contention that Russia was a state-capitalist society.

Union's policy of forced industrialization in the 1920s and later, and has played a vital role in communist China and in other countries that seek to model themselves on either of these two countries. But it is also a very influential notion among economists and others in a great many low-income countries which are persuaded of the usefulness of economic planning without being socialist or communist.

This is again quite similar to what Marx's discussion of the transition suggests.

His analysis thus emerges as a remarkably accurate anticipation of future developments. That does not mean he was a prophet. The initial process of capitalist accumulation in Britain was a matter of past history by his time, a history he studied carefully. Indeed, a possible explanation for the accuracy of Marx's analysis of the transition from simple to expanded reproduction is that he was able to use the British case to discern crucial features of capitalist development in general.

As I noted in Section I, another important implication of the present interpretation is that it eliminates the apparent "glaring inconsistency between different parts of *Capital*" (Desai 1990, 339). The apparent inconsistency arises when the reproduction schemes are interpreted as balanced-growth models; they then seem to be worlds apart from the bulk of *Capital*, which is concerned with economic crises, uneven development, and other nonequilibrium processes. When they are considered as an unbalanced-growth model, on the other hand, the schemes become compatible with the rest of the work, and indeed appear as yet another instance of *Capital's* concern with nonequilibrium processes.

In particular, when understood as a model of unbalanced growth, the reproduction schemes complement Marx's (1977, 742) vision, articulated in *Capital*, Vol. 1, of the capitalist system as a system of production for the sake of production: "Accumulate, accumulate! That is Moses and the prophets! ... Accumulation for the sake of accumulation, production for the sake of production: this was the formula in which classical economics expressed the historical mission of the bourgeoisie in the period of its domination."

The transition from simple to expanded reproduction is "production for the sake of production" in a very specific sense. A greater share of the economy's means of production is employed by Department I to produce more means of production, and a smaller share is employed by Department II to produce consumer goods. Accordingly, production of means of production grows faster than production of

consumer goods. Thus the capitalist economy *increasingly* becomes a system of production for production's sake.

Marx himself was probably aware that his schemes imply production for production's sake. As we saw above, he distinguished simple from expanded reproduction by noting that, in the latter case, more surplus labor is expended on providing "means of production for means of production instead of on means of production for means of consumption" (Marx 1981, 572).

That the reproduction schemes exemplify Marx's concept of capitalist production as production for production's sake is not an original observation. Lenin (1972, 56) argued that the faster growth of Department I is "real 'production as an end-in-itself,'" and that this property of Marx's schemes "corresponds to the historical mission of capitalism and to its specific social structure." The same idea was crucial to Dunayevskaya's (esp. 1943; and 1991, Chap. 3) interpretation of the schemes, as well as to her Marxist-Humanist critique of capitalist production as production aimed at the accumulation of abstract wealth rather than human development.¹⁷ Critics of the schemes such as Luxemburg (1968, 329–35) have likewise understood them as implying that capitalist production is production for production's sake (which notion, however, they reject as absurd).

On the interpretation I have put forward here, while Marx did not intend the reproduction schemes to be models of balanced growth, neither did he develop them in order to analyze economic crisis. This does not make the schemes *incompatible* with Marx's theories of falling profitability and economic crisis. The schemes do not give rise to a falling rate of profit for the simple reason that they abstract from

¹⁷ Zarembka (2002, 27–35) perceptively stresses the integral relationship between her view of expanded reproduction and her humanist critique of capitalism, from an underconsumptionist and Althusserian perspective hostile to both. However, he wrongly suggests that Dunayevskaya, unlike other critics of underconsumptionist doctrine, did *not* oppose it on the ground that "necessary 'customers' (i.e. capitalists) are there to realize ever increasing production of means of production, without worrying about any connection to consumption" (Zarembka 2002, 35). Instead, he suggests, Dunayevskaya put forward the bizarre position that *all* means of production can be realized in kind, without needing to be sold in a market (Zarembka 2002, 32–33). He bases this reading on the manner in which she quotes Marx. But a mere two sentences prior to the quotation in question, she wrote, "pig iron is needed for steel, steel for machine construction, etc.—and ... therefore, so far as the capital *market* is concerned, the capitalists are their own best 'customers' and 'buyers'" (Dunayevskaya 1991, 35, emphasis added).

what are, in Marx's theory, the sources of falling profitability—labor-saving technological changes and the concomitant increases in productivity. Of course, this does not imply that the schemes portray the capitalist system as one in which technical change is absent or in which the law of the tendential fall in the rate of profit fails to operate.¹⁸ It merely implies that Marx was examining one issue at a time, temporarily disregarding one source of unevenness in order to focus on another.

Summary and Conclusions

Marx's schemes of simple and expanded reproduction are commonly regarded as distinct models of balanced growth. However, I have argued here that:

1. when one compares the two schemes instead of counterposing them, they can plausibly be understood as a model of unbalanced growth;
2. the unbalanced-growth interpretation was at one time not uncommon; and
3. Marx did not intend to model balanced growth, and he was aware of, and analyzed, the unbalanced-growth properties of the schemes.

I have also suggested that when the reproduction schemes are viewed as an unbalanced-growth model, Marx can be seen to have been an early and perceptive theorist of what Rostow later called the "take-off" process. Finally, I have argued that the unbalanced-growth interpretation resolves the apparent incompatibility between the reproduction schemes and the rest of *Capital* that arises when the schemes are conceived of as balanced-growth models.

Although I have stressed that the reproduction schemes *can* be understood as depicting unbalanced growth, I have not suggested that they *must* be understood in this way. If one insists upon considering the schemes as two distinct models, then each of them is indeed a

¹⁸ This is because the schemes need not be understood as any sort of portrayal of how the capitalist system operates (i.e., model). As I argued above, they were developed for the quite different purpose of showing that not all investment resolves itself into wage payments.

balanced-growth model. So can we not say that there are two plausible interpretations of the schemes, and that one is entitled to choose the interpretation one prefers?

I think not. As I noted at the outset, an elemental interpretive principle tells us that textual interpretations must try to understand the text as a coherent whole; apparent inconsistencies are *prima facie* indications that the interpreter has misunderstood the text. The balanced-growth interpretation of Marx's schemes should therefore be rejected, precisely because it implies that his discussion of reproduction is inconsistent with rest of *Capital*, while the unbalanced-growth interpretation resolves the apparent inconsistency.

On the other hand, it seems to me entirely legitimate to say that models which have the same analytical properties as Marx's schemes of reproduction are balanced-growth models. *But they are not Marx's models.* Just as a knife in the hands of an assassin is different from a knife in my kitchen that has the same physical properties—one is a weapon; the other is a cooking utensil—balanced-growth models differ from Marx's schemes of reproduction even though their analytical properties are the same. The purpose to which a thing is put affects what it is.

To some, this point may seem to be a quibble. I would suggest, however, that it is an important argument in defense of theoretical pluralism and against the “imperialistic” coöptation of earlier thinkers, and anachronistic rewriting of history, practiced by Whig historians.

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Mathematical Appendix

A. Definitions

W_1, W_2	value of annual output (in dollars)
\hat{c}_1, \hat{c}_2	used-up constant capital per dollar of output
\hat{v}_1, \hat{v}_2	variable capital per dollar of output
\hat{C}_1, \hat{C}_2	total constant capital per dollar of output
$W \equiv \frac{W_1}{W_2}$	“relative size of Department I”
g	Department I's growth rate

The subscripts 1 and 2 refer to Departments I and II, respectively.

Note that, if we multiply the variables with “hats” ($\hat{}$) by the relevant value of annual output, we obtain “raw” figures rather than per-dollar ones. For instance, since \hat{v}_1 is Department I's variable capital per dollar of output, $\hat{v}_1 W_1$ is Department I's variable capital.

B. The Dynamic Balance Condition

In Department I, supply and demand are assumed to be equal in every period. Thus the following condition must hold true:

$$W_{1t} = \hat{c}_1 W_{1t} + \hat{c}_2 W_{2t} + \hat{C}_1 (W_{1t+1} - W_{1t}) + \hat{C}_2 (W_{2t+1} - W_{2t}) \quad (1)$$

The left-hand side is Department I's supply. The first two right-hand side terms are the demands for means of production to replace those

which have been used up in Departments I and II, respectively, while the last two right-hand side terms are the two departments' demands for additional means of production. Note that, since Marx's schemes assume that technology and prices are constant, and that prices equal values, the variables with "hats" are constant over time.

C. The Fixed-Capital Example

The Year 1 figures in Table 1 imply that $\hat{c}_1 = \hat{c}_2 = 0.5$, and that $\hat{v}_1 = \hat{v}_2 = 0.5$. The fixed-capital example in the text assumes that used-up constant capital in each department is 10% of its total constant capital. Hence \hat{C}_1 and \hat{C}_2 are 10 times as large as \hat{c}_1 and \hat{c}_2 , and thus $\hat{C}_1 = \hat{C}_2 = 5$. Because the fixed-capital example assumes that Department I grows by a constant 5% per year,

$$W_{1t+1} = 1.05 W_{1t} \quad (2)$$

Equation (1) therefore becomes

$$W_{1t} = 0.5W_{1t} + 0.5W_{2t} + 5(1.05W_{1t} - W_{1t}) + 5(W_{2t+1} - W_{2t}) \quad (1')$$

so that

$$W_{2t+1} = 0.9W_{2t} + 0.05W_{1t} \quad (1'')$$

Given the initial conditions $W_{11} = W_{21} = 500$ (from Table 1) and equations (2) and (1''), the time paths and growth rates of W_1 and W_2 can be computed for all subsequent years.

D. The Fixed-Capital Case in General

If Department I's rate of growth (g) is constant, and equation (1) continually holds true, it can be shown, using standard methods for solving difference equations, that

$$W_{1t} = W_{11}(1+g)^{t-1} \quad (3)$$

$$W_{2t} = A \left(1 - \frac{\hat{c}_2}{\hat{C}_2} \right)^{t-1} + \left(\frac{1 - \hat{c}_1 - g\hat{C}_1}{\hat{c}_2 + g\hat{C}_2} \right) W_{11} (1+g)^{t-1} \quad (4)$$

where $A = W_{21} - \left(\frac{1 - \hat{c}_1 - g\hat{C}_1}{\hat{c}_2 + g\hat{C}_2} \right) W_{11}$.

If g is positive, then $(1+g) > \left(1 - \frac{\hat{c}_2}{\hat{C}_2} \right)$, which implies that, as t increases, the growth rate of W_2 asymptotically approaches g . In other words, the growth rate of Department II converges on that of Department I in the long run.

Dividing Equation (4) by Equation (3), we obtain

$$\frac{W_{2t}}{W_{1t}} = \left(\frac{A}{W_{11}} \right) \left(\frac{1 - \hat{c}_2/\hat{C}_2}{1+g} \right)^{t-1} + \left(\frac{1 - \hat{c}_1 - g\hat{C}_1}{\hat{c}_2 + g\hat{C}_2} \right) \quad (5)$$

If g is positive, then the first right-hand side term vanishes as t increases. Thus, in the long run, the reciprocal of (5), which is

$$W \equiv \frac{W_1}{W_2} \text{ ("the relative size of Department I")}, \text{ approaches}$$

$$W^* = \frac{\hat{c}_2 + g\hat{C}_2}{1 - \hat{c}_1 - g\hat{C}_1} \quad (6)$$

E. Properties of the Long Run

It is clear from Equation (6) that an increase in g implies an increase in W^* . But in the long run, g is the whole economy's growth rate, i.e., the growth rate of both departments. Thus (given the schemes' assumptions) *any increase in the economy's long-run growth rate requires an increase in the relative size of Department I*. The transition from simple to expanded reproduction is just a special case of this. For instance, if we once again assume that $\hat{c}_1 = \hat{c}_2 = 0.5$

and that $\hat{C}_1 = \hat{C}_2 = 5$, then the transition from simple reproduction ($g = 0$) to long-run growth of 2% ($g = 0.02$) requires an increase in the relative size of Department 1, from $\frac{0.5 + (0)5}{1 - 0.5 - (0)5} = 1$ to $\frac{0.5 + (0.02)5}{1 - 0.5 - (0.02)5} = 1.5$. Likewise, the transition from long-run growth of 2% to long-run growth of 5% requires an increase in the relative size of Department 1 from 1.5 to $\frac{0.5 + (0.05)5}{1 - 0.5 - (0.05)5} = 3$.

It is also clear from Equation (6) that increased employment of fixed constant capital relative to circulating (i.e., used-up) constant capital requires an increase in the relative size of Department 1. For instance, assume that $\hat{c}_1 = \hat{c}_2 = 0.5$ and that $g = 0.05$. What is the relative size of Department I that is needed in order to sustain this long-run growth rate of 5%? If no fixed capital is employed, $\hat{C}_1 = \hat{C}_2 = 0.5$ so the answer is $W^* = \frac{0.5 + (0.05)0.5}{1 - 0.5 - (0.05)0.5} = 1.1$. But if 90%

of Department I's constant capital is fixed, then $\hat{C}_1 = 5$, so $W^* = \frac{0.5 + (0.05)0.5}{1 - 0.5 - (0.05)5} = 2.1$. If 90% of Department II's constant

capital becomes fixed capital as well, then $\hat{C}_2 = 5$ and therefore

$$W^* = \frac{0.5 + (0.05)5}{1 - 0.5 - (0.05)5} = 3.$$

F. The Maximum Growth Rate

In the numerical examples above, values of g were selected arbitrarily. However, there is a maximum value of g and thus a maximum value of W^* . If Department I is "too large" relative to Department II,

the associated growth rate cannot be sustained. This is because Department II cannot produce articles of consumption in a quantity sufficient to “feed” the workers whose work is needed (given the assumption of constant technology) in order to sustain the growth rate. It is necessary that

$$W_{2t} \geq \hat{v}_1 W_{1t+1} + \hat{v}_2 W_{2t+1} \quad (7)$$

and since $W_{1t+1} = (1 + g)W_{1t}$ and W_{2t+1} approaches $(1 + g)W_{2t}$ in the long run, it is necessary that

$$W_{2t} \geq (1 + g)(\hat{v}_1 W_{1t} + \hat{v}_2 W_{2t}) \quad (7')$$

Using inequality (7') together with equation (1), the maximum values of g and W^* can be found. In the fixed-capital example in the text, the maximum long-run growth rate turns out to be 5.77% (which is also the rate of profit), and the maximum value of W^* turns out to be 3.73.