A REAL PRODUCTION CRITIQUE OF CAPITAL ASSET PRICING

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I. Introduction

One of the most famous models in financial economics is the capital asset pricing model (hereafter denoted CAPM). This well-known model of asset valuation is a one-factor model which argues that the primary determinant of the required rate of return on equity (i.e. the rate of return on common stock) is a risk premium which depends upon the sensitivity of the equity security's return relative to general movements in the stock market. This model has been developed and used by, among others, Fama (1970), Lintner (1965), Mossin (1966), Rubinstein (1973), and Sharpe (1970). The model incorporates a statistical measure of the relative variability of an equity security's required return known as a beta coefficient; this statistic measures relative variability in required equity returns as the covariance between the return on the equity asset and the return in the equity market relative to the variance of the return in the equity market.

While the CAPM is probably the most well known and utilized model of asset valuation[1], there do exist several problems with it. First, the CAPM is derived only under several very strict and unrealistic assumptions (as shall be seen shortly). This makes its theoretical underpinnings, as well as its applicability, questionable at best. Second, the CAPM was originally derived as a static model of asset valuation. Hence, the model did not account for the dynamic nature of financial markets with respect to determination of rates of return.[2] Third, and in our view most damning, the CAPM was originally developed to theoretically be a link between the financial and real sectors of the economy. Unfortunately, in our judgement this model fails in this quest due to the absence of any accounting for real production effects (such as labor-management disputes). Such a failure of the model regarding these results in the CAPM not fully capturing the impact of such real production effects on required asset returns. Thus, as a potential bridge between the real and financial sectors of the economy, the model does not achieve its desired goal.

[1]The logic goes as follows: since required returns are needed to determine present, or capitalized, values of assets, models that determine such returns are referred to as "asset valuation models".

[2] As a reaction to this criticism, an "intertemporal" version of the CAPM was developed. For more on this model, see Blume and Friend (1973), Ross (1976), and Treynor (1961). This version of the CAPM will be critiqued briefly later on in this paper.

This paper explores the last point mentioned above. The organization of this study is as follows: first, a general derivation of the CAPM is presented with discussions of the weaknesses and limitations regarding the various assumptions necessary to derive the model, as well as various alternatives to the CAPM, included; second, a discussion is presented of how real production effects could alter the standard treatment of the CAPM; third, a discussion of potential reasons for the exclusion of real production effects in theories of asset valuation closes the paper.

II. Analysis and Critique of the CAPM

A. General Derivation of the CAPM

We begin by focusing on a slightly revised version of equation (20) from the paper by Diamond (1967), which deals with a general equilibrium model of the stock market under the assumption of incomplete asset markets.[3] We begin this way so as to avoid the first of many troubling assumptions often deemed necessary to derive the CAPM: that capital markets are perfect, competitive, and complete.[4] In keeping with the standard notations used in the CAPM, Diamond's equation (20) can be re-written as follows:

$$\tilde{E}i([Ui'(Ci)][\tilde{R}j - r]) = 0 \quad (1)$$

In the above formulation, $\tilde{R}j$ is the total return on firm j (in other words, it is equal to one plus the rate of return for firm j) and is a random variable, r is the risk-free return (or, alternatively, one plus the risk-free rate of interest,[5] Ci is consumption during some "state of the world" for individual i,[6] Ui'(Ci) is marginal utility of consumption for individual i, and Ei[Ui'(Ci)] is expected marginal utility for individual i. Equation (1) assumes that all beliefs are identical; this is often referred to as the assumption of homogeneous beliefs.[7] Hence, (1) can be formulated as follows:

$$\tilde{E}i([Ui'(Ci)][\tilde{R}j - r]) = 0$$

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[3] An incomplete asset market is, in general, defined as one where not all risks are insurable. In financial economic theory, this condition often is re-interpreted to mean that risk sharing is permitted within limits (i.e. that sharing of risks can occur, but only incompletely). A common way to ensure that asset markets in theory are incomplete is to assume that there are more "states of the world" relative to "payoffs". A fuller discussion of these issues can be found in Arrow (1964).

[4] Perfect capital markets preclude personal or corporate taxes, brokerage fees, underwriting costs, bankruptcy penalties, or other types of transaction costs, as well as indivisibilities of securities. Competitive capital markets imply that the same security investment opportunities are available to all investors and no investor believes he/she can influence the rate of return on any security by his/her market transactions. Finally, complete capital markets imply that all risks are insurable.

[5] This is another assumption made in any derivation of the CAPM. The principle behind this assumption is that any investor in the market can always earn a minimum rate of return by investing in riskless securities, such as U.S. Treasury securities. An aspect of this assumption is that this risk-free rate, by construction, always winds up being a positive rate of interest. More will be said regarding this later.

[6] In this context, individual consumption is also a random variable.

[7] Homogeneous beliefs allow for the existence of identical subjective probability distributions for all individuals and all firms in the economy. Since these are "used" to make decisions under uncertainty, this assumption greatly simplifies the analysis and derivation of the CAPM. However, it also poses some problems as will be explained later.

(2) $0 = E([Ui'(Ci)][Rj - r]) = E(E(Ui'(Ci)) + Ui'(Ci) - E([Ui'(Ci)][Rj - r]))$

Letting $E(Rj) = \tilde{Rj}$, (2) can be written as the following:

(3) $0 = [\tilde{Rj} - \bar{r}][E(Ui'(Ci)] + Cov(Ui'(Ci),Rj)$

At this point, another dubious assumption must be made. Ui(Ci) is assumed to be quadratic for all individuals i. Hence, Ui(Ci) and Ui'(Ci) take on the following forms:

(4a) $Ui(Ci) = Ci - aiCi^2$

(4b) $Ui'(Ci) = 1 - 2aiCi$

Due to (4a) and (4b), we have the following:

(5) $Cov(Ui'(Ci),Rj) = Cov([1 - 2aiCi],\tilde{Rj}) = -2ai[Cov(\tilde{Ci},\tilde{Rj})]$

Equation (5) holds for all individuals i and firms j. Thus, (3) now becomes the following:

(6) $0 = [\tilde{Rj} - \bar{r}][E(Ui'(Ci))] - 2ai[Cov(\tilde{Ci},\tilde{Rj})]$

Aggregating over all individuals i and rearranging yields the following:

(7) $[\tilde{Rj} - \bar{r}]\{sum_i[E(Ui'(Ci))/2ai]\} = Cov(sum_i[Ci,\tilde{Rj}])$; $sum_i[.]$ refers to the summation over all i

Equation (7) makes use of the property that $sum_i[Cov(Ci,Rj)] = Cov(sum_i[Ci,\tilde{Rj}])$.

At this point, another crucial assumption is made:

(8) $sum_i[Ci] = sum_j[Pj]$; $sum_j[.]$ refers to the summation over all j

In the above, Pj represents profits for firm j. Equation (8) states that total consumption for all individuals i must be equal to total profits for all firms j. The implication of this is the following:

(9) $\tilde{Rm} = \{sum_j[Pj/M]\}$

In equation (9), Rm represents the return on the market and M represents the total value of shares outstanding for all firms j. Due to (8), we can write the following:

(10) $Ci = [M][\tilde{Rm}]$

According to (10), we can rewrite the right-hand side of (7) as follows:

(11) $\text{Cov}(\text{sum}i[Ci,Rj]) = \text{Cov}(\{[M][Rm]\},Rj) = M[\text{Cov}(Rm,Rj)]$

If we let $v = M/\{\text{sum}i[E(Ui'(Ci))/2ai]\}$, then (7) finally can be written as follows:

(12) $Rj = r + v[\text{Cov}(Rm,Rj)]$

Equation (12) is the CAPM in general form. It states that the total return on firm j is basically composed of two parts. The first part is the risk-free return in the market. The second part is a risk premium, which compensates investors for market risk. Hence, the CAPM breaks total risk down into two components: systematic risk (the risk that is correlated with the rest of the market) and unsystematic risk (the risk that can be diversified away).[8] The term *v* in the CAPM has received much attention in financial economics, as *v* can be thought of as the market price of risk. In general, it can be shown that *v* is equal to the following:

(13) $v = \{[Rm - \bar{r}]/V(Rm)\}$

where $V(Rm)$ is the variance of the return on the market and Rm is the average return on the market.[9] Hence, *v* measures the average market return relative to the risk-free return that is required to compensate investors for variations in market returns.

B. Criticisms of the CAPM

The CAPM is a one-factor model of asset valuation, which explains the existence of a market risk premium in the required return on an asset. In the finance and financial economic literature, the CAPM is often written as follows:

(14) $\bar{Rj} = r + Bj(\bar{Rm} - r)$

In this linear framework, Rj and Rm are, respectively, the average (or expected) returns on asset j and the market, while *B*j is the beta coefficient for asset j.[10] Statistically, *B*j represents the following:

(15) $Bj = [\text{Cov}(Rm,Rj)]/V(Rm)$

In other words, the beta coefficient is simply the ratio of the covariance of market returns and asset returns to the variance of market returns. In econometric work involving the CAPM, the model is written as follows:

[8] In the financial economics literature, unsystematic risk is often referred to as idiosyncratic risk. Thus, it is the risk of the individual firm separate from the market as a whole.

[9] Full discussions of this issue, as well as derivations of (13), can be found in Fama (1970), Lintner (1965), Mossin (1966), Rubinstein (1973), and Sharpe (1970).

[10] The beta coefficient for asset j represents the sensitivity of the returns for asset j relative to the market.

(16) $\tilde{R}jt = aj + bj\tilde{R}mt + \tilde{e}jt$

where the variables $\tilde{R}jt$, $\tilde{R}mt$ and $\tilde{e}jt$ are time series data on the returns for asset j, the market, and a random error component, respectively. In this simple econometric model, *b*j should be equivalent to *B*j. In order for strict equality of these two, a strong assumption is necessary: the random error term must be independent of the market return over time. In other words, the following condition must hold:

(17) $Cov(\tilde{R}mt,\tilde{e}jt) = 0$

The CAPM is not devoid of criticism. For one thing, the assumption of quadratic utility functions implies that returns are normally distributed.[11] However, empirical evidence from the 1960s has indicated the possibility of non-normality of asset returns. This calls into question the assumption of quadratic utility functions (or utility functions at all, for that matter).

Also, the assumption that total consumption for all individuals must equal total profits for all firms (see (8) above) is most troubling. This assumption is quite crucial in the derivation of the CAPM in that it allowed for the introduction of market returns into the model. It does so by defining market return as the ratio of total firm profits to total value of shares outstanding for all firms, as well as the ratio of total consumption for all individuals to total value of shares outstanding for all firms. But why should market return depend upon total firm profits or total consumption? Market returns, in a basic sense, represent the percentage return "earned" on funds (i.e. savings) invested in the financial marketplace. Thus, they are only related to consumption in the sense that the level of total consumption determines the level of total funds or savings available. But, this is a weak relationship at best! To explicitly posit that market returns directly depend upon total consumption, it appears, implicitly assumes a predetermined relation between total consumption and total savings: that all income is spent, hence total savings is identically equal to total consumption! If this is so, then the model exists under a "pure consumption" world in which individuals "live for the moment" and do not plan for the future. In this sense, it is difficult to envision the CAPM as a model that explains required returns within a risk framework. A similar argument holds for the dependence between market returns and total firm profits.

There also exist empirical problems with the CAPM. Firstly, since $\tilde{R}mt$ is not directly observable, a proxy variable must be used when the CAPM is econometrically estimated. The most commonly used proxies are stock market indexes (i.e. the S&P 500 index). However, this leads to model misspecification in that the return on the market required for efficient estimation of the CAPM should be an ex-ante (or expected) return while the returns measured by stock market indexes are ex-post returns. Secondly, empirical results derived from the CAPM tend to be anomalous. This can be illustrated by collecting data on all the beta coefficients for all stocks traded in the stock market and running the following regression:

(18) $\tilde{R}j = a + b\tilde{B}j + uj$

[11] See Rubinstein (1973) for a good discussion of this.

The results from this regression would indicate that uj appears to be nonrandom. Thirdly, by construction the risk-free return (i.e. r) is always assumed to be a positive value. The implication of this is that at worst any financial investor could always earn this return in the financial marketplace. However, this "assumption by construction" ignores the theoretical concept in financial economics that, for any asset, total return is comprised of a real return plus a premium to compensate for expected inflation regardless of whether risk is present or not. It seems to be the case that the CAPM implicitly assumes that "typical" financial investors would be willing to substitute inflation for risk (or, put another way, treat inflation as simply a lower form of risk) provided that they could still earn a positive return on their financial investment. Thus, the model focuses on nominal returns as the driving force behind financial investing. But, suppose that the anticipated inflation is large relative to the total return. As a first approximation, subtracting the former from the latter could yield a real return which may potentially be zero or negative! Hence, one would obtain the result in the CAPM that financial investors willingly invest in securities that earn real returns that may be very low to less than zero, provided that they are riskless.[12] This ignorance of real returns is a major faux pas in the CAPM!

A final criticism of the CAPM is that it is essentially a static model of asset valuation. While the model is estimated from time series data on individual asset and market returns (see equation (16) earlier in this paper), the model itself is not intertemporal at all! Hence, the CAPM ignores the dynamic nature of financial markets with respect to rate of return determination.

Due to all these problems, alternatives to the CAPM were developed. One of the earliest alternatives was the consumption CAPM.[13] This model does not depend upon assumption (8); in other words, this model does not require that total consumption for all individuals in the economy be equal to total profits for all firms in the economy. On the surface, it appears as though the consumption CAPM is a major improvement over the original CAPM in that it does not exist under the situation that all income is spent; rather, this model allows for not only the existence of savings but also for an explicit role for savings in financial investment decisions. However, this model still needed a very strong structure (i.e. lots of assumptions) in order for it to generate results which were consistent with the original CAPM. Further, the model also suffered from many of the empirical problems that plagued its predecessor.

To incorporate a dynamic structure into the CAPM, the intertemporal CAPM was developed.[14] This model explicitly and directly derived the CAPM within an intertemporal, dynamic framework. Hence, required returns depended intertemporally upon both a time-varying risk-free return and a time-varying risk premium. However, many of the same empirical and theoretical problems existed under this model as in the original CAPM. Unfortunately, an additional dilemma cropped up in the intertemporal version of the CAPM: the time-varying beta coefficient estimates obtained from dynamic estimation of this model turned out to be inconsistent and unstable! In other words, the probability limit of these estimates did not tend to approach the "true" parameter values, nor did the expected values of these estimates tend to approach the

[12] This is analogous to either a lender receiving little or no interest from a borrower or a lender effectively paying a borrower to borrow by accepting less back than what was lent out!

[13] This model is discussed in Rubinstein (1973).

[14] See note 2 for references for this topic.

"true" mean parameter values.[15] What is intriguing with respect to all this is that rather than call into question whether or not the original framework of the CAPM was indeed valid, it was concluded that the intertemporal CAPM was seriously flawed since it did not conform to the conditions inherent in this framework. Further, due to the additional empirical problems experienced in estimation of this model, the consensus was that the intertemporal CAPM did not really add anything of benefit to the CAPM analysis and that the static, original version was good enough. An interesting way to approach this issue![16]

III. Introduction of Real Production Effects in the CAPM

Fossum (1995) discusses research which shows that financial markets react negatively to certain real production events. The example he uses to illustrate his point is a unionization effort. Assume that the workers in the company undertake such an effort. Then, even in the event that this movement proves to be unsuccessful, he claims that financial theory argues one will observe a decline in company returns.

Why should this phenomenon occur? Fossum critiques the standard argument, which states that shareholders anticipate, even in an unsuccessful campaign, that management will have to divert some additional resources away from retained equity and dividend streams to wages and benefit package costs. However, according to Fossum, this should imply greater volatility in share prices, and thus company returns.[17] Yet, he reports and shows that unionized firms actually experience relatively less volatility in share prices and returns (contrary to the prevailing wisdom). This poses a dilemma in that the theory of financial economics predicts that variation in returns for such companies should rise while Fossum demonstrates that variation in returns for such companies should be expected to fall! Hence, according to Fossum's findings, it appears as though financial markets equate unionization with less risk, leading to lower volatility and a more desirable situation for the company!

Fossum then goes on to discuss other possible factors which seem to create a result that is inconsistent with standard financial theory. He raises the idea that investors may potentially be bidding down the prices of shares of such companies out of concern that the change in operations from a nonunion environment to a unionized one will not be a transition free of problems---that it represents uncertainty. In other words, he argues that the financial markets' reactions are due to possibly either the *direction* of change (i.e. from nonunion to union) or, more simply, the *mere fact* of change. He further contends that, since change represents uncertainty and uncertainty is equated with undesirable risk, his interpretation may be more consistent with the lower volatility of both share prices and company returns for unionized firms.

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[15] In statistics terminology, the intertemporal beta coefficient estimates did not exhibit the property of "mean reversion". In the financial economic literature, it is often argued that for a financial market to be "efficient" the beta coefficients should be at least approximately normally distributed about the true mean beta coefficient for the market. As a consequence, very high betas would tend to be balanced out by very low betas.

[16] One fairly recent method developed in an attempt to address all these concerns was arbitrage pricing theory. This was formulated by Ross (1976) and refined by Huberman (1982).

[17] This is due to the shareholders interpreting such a campaign as either undesirable in their view or imposing additional risk upon the company.

What implications would this type of scenario have for the CAPM? Based upon Fossum's research, if such a situation should affect volatility of returns then the CAPM should reflect this via a change in the beta coefficient due to equation (15); in other words, the covariance between market returns and asset returns should change to account for a situation such as the one Fossum analyzed. Unfortunately, the CAPM both cannot and does not incorporate such real production effects. Thus, it would yield the unsatisfactory conclusion that such circumstances would not alter required returns even though this is apparently a contradiction to what is observed in the financial marketplace!

IV. Possible Reasons for the Ignorance of Real Production Effects

So, we arrive at the crucial question: Why does the CAPM ignore real production effects? There appear to be several reasons for this oversight.

First, it may be the case that those who developed the CAPM methodology failed to understand the role of real production effects in asset valuation. This could be seen in the general derivation of the model presented earlier in this paper. The derivation was essentially culled from various other developments of this model, utilizing common themes from all of these. In none of those was the real sector of the economy given a pronounced role in the development of the model relative to the financial sector. Yet, in all of these it was stressed how important real production was in laying the foundation for capital asset pricing.

Second, the ignorance of real production effects in the CAPM may reflect the notion that, to some, financial capital has become relatively more important than real capital in the economy. Certainly, one can see this trend of thought clearly throughout all the references cited for the CAPM in that, generally speaking, portfolio theory is the basic underlying theme around which all their respective analyses of the CAPM take place. If this were in fact so, we would argue that this is not a desirable turn of events since, in effect, the CAPM would become the financial sector's "loaded gun" with regards to the benefits of financial capitalism (a most disturbing thought, indeed). What makes this entire scenario even more troubling is that apparently the CAPM, or versions of it, continue to be used in both the financial and academic communities as though it was not simply an abstract model but rather "bible truth" with respect to how financial markets actually operate!

Finally, this ignorance could be the reflection of the idea that the bridge or link between the real and financial sectors of the economy centers around consumption as opposed to production. In all the cited references for the CAPM, production is always linked to consumption in some way; afterward, the link to the financial sector is established. Hence, the CAPM does not truly bridge production and investment directly (as it reportedly sought out to do). Rather, the bridge is indirectly developed through consumption. It may be that these references seek to create a "clear" link between partial equilibrium and general equilibrium analysis via this one linear model, using consumption as their vehicle to accomplish this. If so, we claim that they have failed to achieve this lofty goal. By failing to account for real production effects in the model, the CAPM does not succeed in being the potential link between the real and financial sectors of the economy. Further, it fails to succeed in efficiently explaining investor behavior.

What needs to be done to correct these deficiencies? Clearly, the CAPM is inadequate in providing a true picture of how the real and financial sectors of the economy are intertwined. In our view, there are two directions that financial economics can go in. On the one hand, the CAPM could be modified so as to be more correctly focused upon how these two sectors are linked. On the other hand, it may be time to leave the CAPM behind and develop a newer, more correct theory of how the real and financial sectors of the economy are interrelated. We leave these options open for vigorous discussion within both the economics and finance communities, for only through such dialogues can an answer to this puzzle have any possible hope of bearing fruition and becoming a reality.

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