1 Money, the postulates of invariance and the transformation of Marx into Ricardo

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1.1 INTRODUCTION

Ricardo's attempt to demonstrate that labour constitutes 'the foundation of exchangeable value of all things'[1] has a series of deficiencies which he recognizes. Marx resolved these in a dialectical conceptualisation of the relationship between labour and price in which the commodity – and capitalist reproduction – is understood as a unity of opposites. To this end he had to make a theoretical break with Ricardo, particularly as regards the concept of the magnitude of value.

Participants in the controversy on the transformation of values into prices have not perceived the nature of this break. Thanks to a Ricardian concept of the magnitude of value they can separate the capitalist economy into two different spheres – price and value – a procedure completely foreign to Marx. The main consequence of this separation is the misunderstanding of the money form of value, a problem that arises in two ways. Firstly, money is a commodity like all others and there is thus an essentially contradictory relationship between its value and its exchange value, that is between its value and the expression of this value in the use value of another commodity. This contradiction is neglected in the controversy, and therefore money becomes a simple numéraire, exactly as in the Ricardian and Walrasian traditions. Second, two juxtaposed standards of price are allowed to coexist – one in the sphere of values and the other in the sphere of prices – whose relationship constitutes the only external link between the two realms. The transformation of values into prices becomes an external problem of reconciling two systems of accounting.

The abolition of the difference between the value and the exchange value of money as well as the duplication of the standard of price are presented as naïve mathematical tools needed to solve two systems of simultaneous equations (Bortkiewicz 1952, Winternitz 1948, Seton 1957) or a system of iterative equations (Shibata 1933, Bródy 1970, Morishima and Catephores 1978a, Shaikh 1984). The main goal of this chapter is to identify the conceptual mistakes

concealed behind these 'mathematical tools' and show that these 'procedures' have placed the transformation controversy outside Marx's own theoretical framework.

The first section of this chapter discusses the differences between Ricardo and Marx regarding the concept of the magnitude of value and shows that the participants in the debate separate value from price by means of a conception of value closer to Ricardo's than Marx's. The second section shows that this separation entails abolishing the contradictory nature of the money form of value and discusses what this contradictory nature is for Marx. Finally, the third section analyses the concept of 'postulate of invariance', showing that this 'postulate' conceals a spurious method of juxtaposing two standards of price.

1.2   THE SEPARATION OF PRICES FROM VALUES

The concept of value developed in the course of the transformation debate is closer to Ricardo's than Marx's. For Ricardo, the commodity's absolute value is determined by the quantity of living labour directly or indirectly required to produce it. It is an amount of labour accumulated during various production periods. For him, living labour required to produce the means of production (and the means of production involved in their production) plays the same role in determining absolute value as the living labour directly consumed in producing it. This implies that the commodity's value can be broken down into the sum of the absolute value of the used up means of production (which in turn contains the absolute value of their means of production) plus the absolute value created by fresh labour.

This is not Marx's concept of value. The difference between the authors is not that which is usually maintained. For Ricardo, as well as Marx, value is not determined by the individual value consumed in producing a particular commodity but by the *social average* labour needed to produce it.[2] Moreover for Ricardo as for Marx, value is not determined once and for all but changes with the conditions of production. Thus for both authors, value is the average labour required to *reproduce* the commodity.[3] The difference between Marx's and Ricardo's concept of value is not grounded – as is often maintained – on these two aspects of the definition of the labour needed to produce the commodity. The difference arises, rather, from the fact that, for Ricardo, the value is determined by the labour necessary to produce a commodity *as use value* while for Marx value is determined by the labour necessary to produce the commodity *as capital*. This implies that for Marx value, which is the monetary form of social labour, is determined in the process of production and circulation considered as a whole and thus not exclusively determined in the process of production.

The commodity's value, whose extrinsic measure is money,[4] has two components: the constant capital C produced during previous production periods, and the value product V+S resulting from the objectification of living labour. On

the one hand, constant capital constitutes the sum of value that the capitalist has to advance in order to replace his or her means of production. This means that it is not given by the value of these means but by their price and, specifically, by their replacement price rather than purchase price.[5] The amount of social labour transferred by the means of production is equal to that represented by their price and does not correspond to the social labour necessary to produce them as use values, unless their capital has average composition. Any variation in the price of the means of production modifies the value of constant capital C and hence the commodity's value C+V+S.

On the other hand the second component of the commodity's value, the value product, constitutes an aliquot part of the value produced by social living labour, that is, the total value product. The proportion of total value product represented by the value product contained in the commodity is given by the proportion of total social living labour represented by the living labour required to produce the commodity. This total value product is distributed between the three main classes in the form of wage, profit and rent. Its measure, like that of any value, is money. Although the amount of living labour that is objectified in the value product is given, the magnitude of the value product can change due to the contradictory character of its expression, money. This implies that the amount of money in which one hour of socially necessary labour is represented, that is the ratio between the total value product and total living labour, is not determined once for all in the sphere of production but in production and circulation as a whole. Given the total living labour, any variation in the total price of the material components of the value product (that is, in the total price of the net product) modifies the total value product and therefore the amount of money in which one hour of socially necessary labour is represented. Consequently, any variation in the price of the net product modifies the value product contained in the commodity, V+S, and hence the commodity's value C+V+S.

In this form, the two components of the commodity's value, constant capital and value product, can be modified by changes in prices, although the amounts of necessary labour needed to produce the means of production and the commodity itself remain unchanged. This is the precise meaning of the statement that the commodity's value is determined through the unity of production and circulation and that there exists an internal link between value and price.[6]

Marx's interpreters prefer to think that value is determined once for all in the sphere of production. To this end they introduce two essential modifications of Marx's concept of value. Firstly, they determine value product exclusively in the sphere of production. Bortkiewicz, who measures value in money, asserts that the amount of money in which one hour of socially necessary labour is objectified is determined as a function of the amount of money commodity produced in one labour hour.[7] This implies that the expression of social labour in the value product exclusively depends on the production conditions of the money commodity. The other contenders in the debate (except Shibata 1933) directly

measure value in labour hours. Therefore, the value product is no longer an amount of money – which may change although the living labour it expresses remains constant – but becomes the given amount of living labour consumed in producing the commodity. The second modification introduced into Marx's concept of value is to define the value transferred to the commodity from the means of production to be equal to the value, instead of the price, of these means of production. In this way both constant capital and value product are exclusively determined in production and the commodity's value is unaltered by price formation.

This different conception of the magnitude of value leads to a discrepancy between Marx and his interpreters regarding the conception of the relationship between value and price. The fact that value is unaffected by price variations allows them to conceive of values and prices as completely separated sets of exchange values. The commodity's value, says Bortkiewicz, is the amount of money received in exchange for the commodity when all commodities, money included, are exchanged according to the amounts of the labour socially necessary to produce them. The commodity's price is the amount of money received in exchange for the commodity when all commodities, money included, are exchanged at their prices of production. In this form, rather than 'conceiving of price as a form of value, value becomes another form of price'.[8] In *Capital* I and II, Marx assumes that commodities are exchanged according to their amounts of labour and calls 'value' the prevailing price vector. In *Capital* III, commodities are exchanged according to the amounts of capital advanced for their production and what Marx calls 'price of production' gives the resulting price vector. For Bortkiewicz 'value', in *Capital* III as well as in *Capital* I and II, is the price vector prevailing in *Capital* I and II:

> In what follows, *value will always be taken to mean the index of an exchange-relationship* [my emphasis – AR] … it is of the essence of that concept of value that its magnitude be determined according to the (Marxian) law of value. This in fact constitutes the difference between value and price of production … since the latter is formed not according to the Law of Value but according to the Law of the Equal Rate of Profit. (Bortkiewicz 1952:6)

Therefore the strict separation between prices and values allows them to be conceived as two juxtaposed exchange systems. This result has two important implications which will be discussed in the next section. First, it is possible to define a different standard of prices in each system in such as way that their relationship is defined as an external link between the system of values and the system of prices. The arbitrary definition of one of these standards, given the other, is known as the 'postulate of invariance'. Second, it permits an understanding of both the exchange value (or 'price') and the value of the money commodity as a relationship of exchange of money against itself which makes it impossible to perceive the contradiction between the value and the exchange value of the money commodity and, thus, the particularity of the money form of value.

Before concluding this section, it is necessary to stress that, arguing against Smith, Marx affirms that values are not modified by changes in wages.[9] In this statement he attempts to underline the fact that, if the proportion of social labour appropriated by workers as wages increases, then those appropriated by the capitalist class as profit must necessarily diminish. However, insofar as the rise of wages provokes a relative change in prices – either of the means of production or the use values in which the value product is realized – there may be a modification in the money expression of this social labour, that is, in the value of the social product. In effect, a modification of wages has a different effect on the cost price of gold and the cost price of any commodity produced by a capital whose organic composition differs from that of the capital producing gold. Given the equalisation of the profit rate, this different effect on cost prices provokes a variation in the exchange rate between gold and these commodities (that is a variation in the prices of the commodities, or, which is the same, in the exchange value of the gold). If these commodities are either the means of production or the net product, a change in wages thus implies a corresponding modification of constant capital or the value product respectively, and consequently a modification of the commodities' value, although the amount of labour socially necessary to produce the means of production, the net product and the gold remains constant.

This fact does not invalidate Marx's argument against Smith, since Marx does not require that value and value product be invariable magnitudes. The validity of the argument – and hence of the Marxist theory of value and surplus value – only requires a demonstration that prices, as aliquot parts of social value, are the objectified form of social labour. To do this, the internal relationship between labour and price has to be apprehended, which calls for an understanding of the dialectical relationship between the intrinsic measure of value – labour – and the extrinsic measure of value – money. The basis of this dialectical relationship is the contradictory nature of the money form of value; a proper understanding of this nature implies an acceptance that value (the monetary form of labour) is not given once and for all in the sphere of production but, rather, that it is quantitatively and qualitatively determined in the unity of production and circulation. Marx's theoretical results, according to which total value = total price and total surplus value = total profit can only be understood and demonstrated on the basis of this dialectical conception of the 'inner, necessary connection between value-form, value-substance and value-magnitude'.[10]

1.3   VALUE AND THE EXCHANGE VALUE OF MONEY

Rejecting the equality between total value and total price Bortkiewicz, in his second and best known article (1952, originally 1907), develops an argument which has been neglected in the literature on transformation. This reasoning is nevertheless crucial because it helps disclose the essential differences between

Marx and the other contenders in the debate regarding the concept of money. In this commentary Bortkiewicz suggests that two different standards of price coexist, one in the 'sphere of values' and another in the 'sphere of prices'.

The sum of prices is the exchange relationship between the total bulk of commodities and the money commodity. A total sum of prices of 422 *ounces of gold* means that this amount of money commodity has to be exchanged for the total output. According to Bortkiewicz's conception of value, every commodity has two exchange relationships with the gold: one corresponding with the amount of labour (called 'value' by Bortkiewicz) and other corresponding to the amount of capital, that is the 'price of production'. This implies two different exchange relationships between total output and gold, the first given by the total sum of values and the second given by the sum of prices. Only when the capital producing gold has the same organic composition as social capital would these two exchange relationships be equal. Otherwise, the exchange relationship between the gold and total output would be quantitatively different, depending on whether it is established according to the amounts of labour or to the amounts of capital, and thus, the sum of values would diverge from the sum of prices. Bortkiewicz's argumentation is developed in the following passage, where he comments Marx's first illustration in *Capital* III, Chapter 9.

> Let G be the good which serves as measure of value and price. The figures 90 and 92, which indicate the value and the price of the total output of sphere 1, would accordingly signify that this total output *is exchanged* for 90 units of G *according to the principles of the value-calculations*, and for 92 units of G *according to the principles of the price-calculation*. Such differences between price and value are due to differences in the organic composition of capital invested in the various spheres of production. These differences obviously also depend, with respect to their signs and their magnitude, on the organic composition of the capital invested in the production of G. (my emphasis – AR)

> Let us now assume that this capital has the lowest organic composition of all, i.e. that in this capital, constant capital constitutes relatively a smaller part than it does elsewhere. On this assumption, *the transition from value-calculation to price-calculation* [my emphasis – AR] should result in all goods being exchanged for more units of G than formerly, in other words, all prices should be higher than their corresponding values. The total price would consequently be greater than the total value. In the opposite case, where the capital employed to produce G has the highest organic composition, the total price would prove to be a lower figure than the expressing total value. (Bortkiewicz 1952:11, my emphasis)

Since for Bortkiewicz (1952:6) value is always 'the index of an exchange-relationship' whose magnitude is 'determined according to the (Marxist) Law of Value', he naturally assumes that the exchange relationship between gold and gold is 1 to 1 and, hence, that the value of gold is 1. He analogously reasons that the price of production of gold is equal to 1. However, these two expressions are, in themselves, wholly meaningless: What does an exchange relationship between gold and gold, or between any use value and other use value of the same kind, mean? To define the value or price of *1 ounce of gold* as *1 ounce of gold* is as irrational as asserting that the exchange value of *20 yards of linen* is *20 yards of*

linen. The reason – as Marx repeatedly says – is that the value of linen requires a different commodity to be expressed in. For instance,

> It cannot, for example express the value of linen in linen … 20 yards of linen are nothing but 20 yards of linen, a definite quantity of linen considered as an object of utility. The value of the linen can therefore only be expressed relatively, i.e. in another commodity. The relative form of the value of the linen therefore presupposes that some other commodity confronts it in the equivalent form. (Marx 1976a:140)

On exactly the same basis, it is meaningless to say that the price of *1 ounce of gold* is *1 ounce of gold*. In the words following the above passage, Marx explicitly says:

> On the other hand, this other commodity, which figures as the equivalent, cannot simultaneously be in the relative form of value. It is not the latter commodity whose value is being expressed. It only provides the material in which the value of the first commodity is expressed. (Marx 1976a:140)

Therefore, gold needs the other commodities to express its own values:

> The relative value of money is expressed in the innumerable prices of all commodities; for in each of those prices in which the exchange value of the commodity is expressed in money, the exchange value of money is expressed in the use value of the commodity.[11]

When the organic composition of a given commodity does not correspond to the average, its price diverges from its value and the profit appropriated by the producer through exchange diverges from the surplus value contained in it. Every commodity has only one exchange value, which is its price: the price vector constitutes the structure of exchange value, the only effective basis on which to calculate the cost price of commodities and hence their respective prices as well as their values. The value of a commodity is not, as Bortkiewicz thinks, an exchange relationship juxtaposed with price, a supposedly second exchange value defined according to another principle of equivalence.

The commodity's value is the *ideal* exchange relationship between the commodity and that gold which, given the structure of prices, would allow its producer to appropriate the whole surplus value produced by her or his workers. Its calculation is grounded on a certain structure of prices given for the whole commodity world, for example on a series of exchange relationships between the gold and the other commodities such that, in the case that a general rate of profit exists, all producers – including the gold producer – realize the average profit. This means that commodity values are calculated assuming that gold (as well as other commodities) is exchanged in proportions which permit its producer to pocket the average profit, and that these proportions are the unique effective exchange value between gold and commodities. This is a difficult matter due to the contradictory nature of the money form of value. An example can facilitate its understanding.

Firstly, let us assume that capital producing linen has the average composition and that the production of 20 *yards of linen* requires 1.25 *ounces of gold* as constant capital and living labour which is objectified in 1.25 *ounces of gold*, half

being appropriated as wages and half as surplus value. Linen's value can be broken down in the following form:

$$v_L = 1.25_C + 0.625_V(1 + s)$$
$$= 1.25_C + 0.625_V + 0.625_S$$
$$= 2.5 \text{ ounces of gold}$$

Let us suppose that, in the production of linen, the whole advanced capital is consumed and that its price is set at a level such that its producer appropriates as profit one third of its cost price, that is, the rate of profit is equal to $\rho = {}^1\!/_3$. In this case the price of the linen can be broken down as follows:

$$p_L = (1.25_C + 0.625_V)(1+\rho)$$
$$= 1.25_C + 0.625_V + 0.625_P$$
$$= 2.5 \text{ ounces of gold}$$

The value and price of linen correspond: its producer objectified value expressed in 2.5 ounces of gold and his commodity's price is 2.5 ounces of gold. However, the fact that the value and the price of the commodity coincide does not mean that the value contained in linen is the equivalent of the value contained in 2.5 ounces of gold. The fact that linen's value is 2.5 ounces of gold is completely independent of the value of these 2.5 ounces of gold which can be produced by a different amount of labour from that required to produce 20 yards of linen. The value of linen and gold coincides only under an additional condition: that gold is produced by an average composition capital. Yet, this is an additional condition that depends on the conditions of production and circulation of the use value gold, for which the expression of the linen's value is completely indifferent.

Thirdly, let us suppose an equal rate of surplus value in all branches and that the organic composition of the capital producing gold is different from that producing linen. What would be the expressions of value and exchange value of gold parallel to those of linen? Let us suppose that the production of one ounce of gold requires 0.25 ounces of gold as constant capital and eighty per cent of the living labour needed to produce twenty yards of linen, that is 0.5 ounces of gold, corresponding to variable capital. This allows us to calculate the value contained in the gold as well as the value appropriated by its producer in the circulation. On the one hand, assuming an equal rate of profit in all branches, the profit appropriated by the producer of gold would be 0.25 ounces of gold. If value produced and value appropriated in the production of one ounce of gold are expressed in gold, the following result is obtained:

$$v_G = 0.25_C + 0.5_V(1 + s)$$
$$= 0.25_C + 0.5_V + 0.5_S$$
$$= 1.25 \text{ ounces of gold}$$
$$p_G = (0.25_C + 0.5_V)(1 + \rho)$$
$$= 0.25_C + 0.5_V + 0.25_P$$
$$= 1 \text{ ounce of gold}$$

As stated, these expressions are in themselves meaningless because the value of gold cannot be expressed in gold: the money commodity is the only one that has neither price nor a general expression of value.[12] Money needs the use value of other commodities to express its value. So what is the correct reading of these expressions? On the one hand gold's 'price' ($p_G = 1$) is the exchange relationship that lets the gold producer appropriate the average profit.[13] It is the relationship between one ounce of gold and any other commodity whose price is one ounce of gold. Since the price of twenty yards of linen is 2.5 ounces of gold, then one ounce of gold purchases 20/2.5 = eight yards of linen. Therefore, buying linen at its price, the gold producer would appropriate the average profit, that is if she or he exchanges one ounce of gold for eight yards of linen.

On the other hand, the value of gold is the exchange relationship that, *given prices for all commodities*, permits the gold producer to appropriate the totality of surplus value produced by his or her workers. It is the relationship between one ounce of gold and any other commodity whose price is equal to 1.25 ounces of gold. Since the price of twenty yards of linen is 2.5 ounces of gold, then 1.25 ounces of gold allows the gold producer to purchase $(20/2.5) \times 1.25 =$ ten yards of linen. Therefore, if the gold producer exchanges one ounce of gold for ten yards of linen, then she or he would appropriate the whole surplus value.

It is, however, necessary to stress that this exchange relationship, ten yards of linen for one ounce of gold, is purely *ideal* and its calculation supposes that the actual exchange relationship between linen and gold is eight yards of linen for one ounce of gold (= the exchange value of gold) or 2.5 ounces of gold for twenty yards of linen (= the price of linen).

This is the fundamental contradiction of the money form of value. Money is not only the *universal form* of value but it is also a *particular commodity* and thus the relationship between its value and its exchange value is as contradictory as that of any other commodity. In a passage that, out of this context seems incomprehensible, Marx (1973:150) stresses precisely this fact in the *Grundrisse*:

> An incongruity arises not only because money, which exists only in exchange, confronts the particular exchangeability of commodities as their general exchangeability, and directly extinguishes it, while, nevertheless, the two are supposed to be always convertible into one another; but also because money comes into contradiction with itself and with its characteristic by virtue of being itself a *particular* commodity (even if only a symbol) and of being subject, therefore, to particular conditions of exchange in its exchange with other commodities, conditions which contradict its general unconditional exchangeability.

As a result of this contradiction between the value and the exchange value of gold, as far as it is the money commodity, the value contained in gold *as a specific use value* differs from the value that it represents as *universal form of value*. This contradiction is not present in the simple form of value: when the producer exchanges directly twenty yards of linen for one coat, she or he does not appropriate a *symbolic value* but rather the very *actual value* contained in the coat. But when he or she exchanges twenty yards of linen for 2.5 ounces of gold, she or he appropriates the value contained in this gold *and* the value that this gold

represents as universal form of value, that is the fraction of the total value this gold can be exchanged for. The value represented by money as the universal form of value (its symbolic value) arises from its social specific function and, therefore, it does not depend on the particular conditions of its production. These conditions determine the value contained in money as a specific use value. Conversely, the value represented by money is determined in the context of the social reproduction as a whole: If the total value product is Y thousand ounces of gold, one ounce represents $^1/_Y$ thousandths of the value produced by social living labour and, consequently, in this ounce is expressed $^1/_Y$ thousandths of the social labour objectified through this production period.

The ratio between the total value product and the total living labour gives a coefficient meaning the amount of gold in which one hour of social labour is represented, and so it constitutes the quantitatively determined relationship between the intrinsic and extrinsic measure of value. This coefficient, discussed by Foley (1982) and – I believe wrongly – termed by him the 'value of money', is not given once for all but its magnitude is determined in the process of price formation, where the value product is determined as a specific amount of money: any change in the price of the totality of commodities (and hence in the exchange value of money) is at the same time a change in the exchange relationship between money and the net product, and hence in the magnitude of the value product. Given the amount of living labour, this implies a modification of the ratio between the total value product and the total living labour, and consequently a variation in the amount of money in which one hour of social labour is represented. The basis of this fact is that labour is expressed in money, and money is not a simple unit of account but a contradictory unity of value and exchange value. To apprehend the internal and contradictory relationship between the intrinsic and extrinsic measure of value it is thus necessary to understand the contradiction between the value and the exchange value of money – in turn the basis of the money form of value – and not to treat money as a simple unit of account.

It is not guaranteed that the value *represented* by money coincides with the value *contained* in it. The possibility of this divergence, which constitutes the specificity of the money form of value, is the basis for the development of the symbol of money: long before gold is replaced by currency, gold (as money) is already a symbol whose symbolic value is divorced from its actual value. Both symbolic and actual value coincide in the specific case where the capital producing the money commodity is of average organic composition; this implies that its producer appropriates the whole surplus value created by his or her workers.

The apprehension of this specific contradiction of the money form of value constitutes the essential difference in the conception of money separating Marx from the other economists, classical as well as neoclassical. Ricardo is looking, as measure of value, for a commodity produced by an average capital, in such a

way that the value and the exchange value of this commodity coincides and their contradiction has no consequences for the value expression of the other commodities. Money would hence be 'neutral' and could be considered as a unit of account or numéraire. That is exactly what is done by the participants in the debate on the transformation problem and for the neoclassical tradition founded by Say and Walras: money is taken as a unit of account by collapsing its exchange value into its value and by considering that the 'money commodity is the one commodity which enters the circulation process with its value'. In so doing, the money form of labour becomes determined exclusively in the sphere of production (of commodities as well as the money commodity).[14] It lets values be separated from prices and opens the way for the suppression of the concept of value.

The main originality of Marx's conception of money is that money is not a simple unit of accounting but the contradictory expression of a class relationship:[15] firstly money is a commodity, and so the *possibility* exists of a contradiction between its value and its exchange value; secondly, money is a commodity which is capital, hence the contradiction between its value and its exchange value becomes a *necessity* for the reproduction of capital.[16]

By imposing the condition that the value and the exchange value of gold are equal, almost all solutions to the transformation problem leave out this specific contradiction of the money form of value. The first author to do this was Bortkiewicz, who equates the 'price' and the value of gold to 1, a method later generally followed. The next step in the dissolution of Marx's framework is to measure the value of gold (and all values) in a different unit from that which is used to measure its price (and all prices); this is supposed to make it possible to derive one of Marx's equalities. The following section is devoted to the discussion of this procedure.

1.4   THE POSTULATES OF INVARIANCE

The determination of the value of gold, as of any commodity, arises from the reproduction process as a whole and requires a definite structure of prices. To assert arbitrarily that the price and value of gold are equal to 1 – independent of the organic composition and turnover time of the capital which produces it – is only possible if values and prices are conceived as two separated and juxtaposed systems of exchange. Only if value and price are independent can the unit of measure in the sphere of prices be chosen separately from the unit of measure in the sphere of values. If we require that in the passage from one sphere to the other a certain magnitude remain 'invariant' it is enough to define two appropriate different units of measure (or 'standards of price'). As Bortkiewicz says:

> One is not at all tied to the condition that the unit of price should be the same as the unit of value. If the latter is represented by 1 ounce of gold, the former may be represented by ¾

or 1½ ounces of gold. In these circumstances, one can always, with any given model of values … select such a unit of price as will make one particular element of the price-model (e.g. the price of the total output of 1 or the variable capital invested in 3, etc.) equal to the corresponding element in the table of values. Similarly, there is nothing to prevent one making a sum of certain elements in the table of prices coincide with the sum of the analogous elements of the table of values, and thus, for instance, equating total price with total value. Such method of determination can, however, obviously be applied only to one single function of these magnitudes. It would thus not be permissible to equate total price with total value whilst simultaneously equating total profit with total surplus value. (Bortkiewicz 1952:12)

For Bortkiewicz it makes sense to express values in a different unit of measure from those used to express prices, since he thinks that both spheres are completely independent and define two alternative exchange systems. For Tugan Baranowsky the unit of measure in one sphere is thousands of man years and in the other millions of marks; for Bortkiewicz likewise, in one sphere the unit of measure could be ounces and in the other thirds of an ounce or hectograms. The relationship between the two standards of price is defined through the relationship between the value and the price of the money commodity. Since for Bortkiewicz value is an exchange relationship, the value of the money commodity (for him the exchange relationship of gold against gold) must be 1. In his equation (10)[17] he asserts that the value of one ounce of gold is one ounce of gold. When Bortkiewicz defines the standard of price in the sphere of prices, he has to ask: what is the price of one ounce of gold? If the same standard of price is defined (ounces of gold), the price of one ounce of gold is equally defined as 1. Notwithstanding this, a unit with different weight could be defined like, for example, those proposed by Bortkiewicz in the passage quoted above. If the standard of prices in the system of prices is defined as ¾ ounces of gold, the price of one ounce of gold would be $1\frac{1}{3}$ units of such standard; if it is defined as 1½ ounces, the price of one ounce of gold would be $\frac{2}{3}$ units.

The selection of two different standards is what permits at least one of the aggregates in both systems to coincide and so to define a 'linkage' between the sphere of prices and the sphere of values. Since a uniform rate of profit is compatible with any linear combination of the vector of prices of production, a level of absolute prices can be chosen imposing the equality of one aggregate in both systems; for instance the sum of values and the sum of prices. If the sum of values is 875 ounces of gold and the sum of prices is 1000 ounces of gold, then it suffices to define a unit of measure for prices equal to $\frac{8}{7}$ ounces of gold. Thanks to the selection of this standard of price, the sum of values 'coincides' with the sum of prices, that is their magnitudes are equal to 875 money units in both systems: 875 ounces in one case and 875 units of $\frac{8}{7}$ ounce in the other. Since the matter is, as Seton says, 'the selection of a definite aggregate (or other characteristic) of the value system which is to remain invariant to the transformation into prices'[18] these different alternatives have been called 'postulates of invariance'.

The selection of two juxtaposed standards of price is carried out in the form of two different normalizations, one in the system of values and the other in the system of prices. The method of carrying out such normalizations varies according to the type of systems used, namely simultaneous or iterative.

In the case of 'corrections' in simultaneous equations, the procedure sets out two different systems of equations, each of one formed from n equations and $n + 1$ unknowns. In the system of values, the unknowns are the values of the n commodities and the rate of surplus value; in the system of prices, the unknowns are the prices and the rate of profit. In contemporary notation, Bortkiewicz's system can be written

$$v = va + vwl(1 + s)$$
$$= v[a + wl(1 + s)]$$
$$p = p[a + wl](1 + r)$$

where v and p are row vectors of unit values of prices, a is a square matrix of input-output coefficients (excluding labour), w is a column vector of wage goods, l is a row vector with the amounts of living labour required to produce a unit of each use value and s is the rate of surplus value. Both systems have n equations and $n+1$ unknowns: in the first case the n values and the rate of surplus value, in the second the n prices and the rate of profit. All linear combinations of vector p corresponding to the largest characteristic root of a are compatible with the rate of profit and all linear combinations of v are compatible with the rate of surplus value that corresponds to w. So, the solution to the first system gives a vectorial hyperplane corresponding to all the linear combinations which represent the same structure of the relative values of the commodities. The selection of one single vector is carried out by posing the value of the money commodity to be 1: this constitutes one normalization of the system. In the case of Bortkiewicz, who measures value in money, this means that the value of one ounce of gold is one ounce of gold. In the case of the other solutions of simultaneous systems, which measure value in labour, this means that the value of one ounce of gold is one unit of labour.[19] On the other hand, the solution of the system of prices equally gives an vectorial hyperplane which represents relative prices; the choice of a particular vector – that is the level of absolute prices – is achieved through one particular normalization of the system. In Bortkiewicz this normalization is carried out posing the price of the money commodity as equal to 1; this implies that the standard of prices in both systems is the same: the value as well as the price of one ounce of gold (and all the other commodities) are measured in ounces of gold.[20] In the case of the other participants in the debate, the normalization is carried out by selecting a second standard of prices, which ensures the equality between some aggregate of the system of values (expressed in corresponding units of measure) and the same aggregate of the system of prices (expressed in the other unit of measure). For instance, Winternitz selects a standard which equates total price to total value, Seton selects another which

equates the price of wage goods to their value, while Duménil selects a standard of price which equates the price of 'net product' to its value.

In the case of 'corrections' in iterative systems, these usually begin from a vector corresponding to the solution of the system of values, which consequently entail a certain normalization. This normalization is generally carried out by setting the value of the money commodity equal to 1. Then an iterative sequence is defined which gives the vector of absolute prices. The normalization condition in the system of prices – that is the selection of the standard of prices corresponding to this system – is carried out implicitly through the choice of formula used to define the sequence of the rates of profit. In effect, the profit rate defines a normalization condition for the price system in such a way that, given the implicit normalization in the initial vector of values, some magnitude remains constant along the iterations. The formula of the rate of profit ensures, thus, that any modification in the normalization of values consequently implies a change in the normalization of prices and, therefore, defines a certain relationship between both standards of price. In Shibata's, Bródy's and Shaikh's iterative process the rate of profit is defined in such a way that the total price equals total value, and in Morishima and Catephores's iterative sequence the total profit equals the total surplus value.

In both traditions (simultaneous and iterative equations) values and prices are two separate systems, each of which can be solved only by adding another equation. This supplementary equation, which implies choosing one of the linear combinations belonging to the vectorial plane which constitutes the solution of the system, is their specific condition of normalization. The different 'solutions' advanced by both traditions are simple definitions of a specific relationship between the two conditions of normalization in such a way that some magnitude in one system is 'equated' to those corresponding in the other. The postulate of invariance is the selection of the normalization condition in one system which, given the normalization applied in the other, implies that some magnitude remains 'invariant' and so it implies the juxtaposition of two different standards of prices.

In *Capital* Marx mentions a series of relationships directly derived from his concepts of value and price such as, for instance, the equality between total surplus value and total profit, the equality between total value and total price, the equality between price and value in spheres of average composition, and so on. Once Bortkiewicz's concept of value replaces Marx's, all these relationships become 'mutually incompatible'. According to Bortkiewicz (1952:12), Marx neglects this incompatibility since he erroneously conceives these relationships as logical consequences of his theory, instead of correctly considering them as 'permissible, though arbitrary assumption(s)'. Many 'solutions to the transformation problem' are possible, in accordance with the selection of some 'postulate of invariance' and, therefore, they reduce the transformation to the merely external relation between units of account. Marx's interpreters are not

aware that by doing this, they have 'solved' the transformation problem by setting Marx on his head. For them, 'gold is the measure of value because its value has been established as an invariable quantity of value; it is the standard of price because it is used as variable unit of weight'.[21]

Before concluding it is necessary to ask: why do the contenders in the debate not perceive the nonsense of this procedure consisting of 'linking prices with labour values' through the arbitrary and false definition of two standards of price? They appear to think that this is a legitimate method because, according to their interpretation, Marx deploys an analogous procedure. In effect, all the contenders – excepting Bortkiewicz and Shibata[22] – argue that while Marx's values are reckoned in labour time, prices are expressed in money units: 'value is labour, its measure is labour time'.[23] According to them, when Marx 'postulates' the equality between total value and total price, he supposedly maintains that a certain number of hours or working days is equal to a certain number of pounds sterling or ounces of gold and, thus, in their systems of equations, they do exactly this. In this form, measuring value in time instead of money cancels the fact that when a 'postulate of invariance' different from Bortkiewicz's is chosen, the analysis imposes two different standards of price (for example, ounces and $^8/_7$ of an ounce). As Sweezy and Dobb[24] say, selecting the 'postulate of invariance' seems like a naïve accounting procedure. To get the equality, it suffices to choose money units in such a way that figures corresponding to the total value and price are equal. If, on the one side, there are 1000 barrels and, on the other, 2471 acres, it is enough to measure the latter quantity in hectares[25] to obtain the 'link' between the sphere of 'contents' and the sphere of 'surfaces': we can thus write 1000 barrels = 1000 hectares. Selecting the 'postulate of invariance' is deciding which figure in each sphere (in labour hours and money units) has to be equalled and 'naïvely' choosing proper units of measure.

Assuming that quantities in the schema of values are expressed in labour hours (or days) and the figures in the schema of prices in ounces of gold (or pounds sterling), how can we interpret Bortkiewicz's equation that allows him to impose the existence of one standard of price? The question is important because if the schema of values is expressed in labour hours, it is impossible to call the unit of account the 'standard of price'. Sweezy (1970:117) has the answer. Defining value and 'price' of money commodity to be equal, 'the number of units of labour necessary to produce one unit of the money commodity would provide a direct link between the two systems of accounting'. In effect, since the 'price' of one unit of the money commodity is 1,[26] the equality 'value of money = "price" of money' implies that the value of one unit of money measured in labour time is 1, and so the unit of accounting in the schema of value is defined as the amount of labour necessary to produce one unit of money. In this form, the unit of account of the scheme of values is defined as the amount of labour necessary to produce one unit of money. Thanks to this, commodity values can be expressed directly in money units or, conversely, prices in labour hours. With this, the

relationship between values and prices achieves the form of the banal functional relationship that is commonly accepted: 'the "transformation problem" in the formal sense of linking value and price of production is seen to be practically trivial mathematically'.[27]

1.5   CONCLUSIONS

The critiques advanced throughout the transformation debate are based on a conception of value different from Marx's. Thanks to this conception, values and prices are considered as two independent systems whose unique relationship consists of 'a direct link between the two systems of accounting'[28] called the 'postulate of invariance'. In this form, the relationship between value and price is presented as an external link – the arbitrary choice of two different conditions of normalization – which does not arise from the concept of value. Both systems could be determined without imposing any relation between the two conditions of normalization and so there would be no 'link' at all between the 'sphere of values' and the 'sphere of prices'. In this manner, the transformation has been reduced to the substitution of one vector of relative prices for another, as depicted by Samuelson in his famous 'theorem of rubber',[29] and the road is clear for the Sraffians to speak of the 'redundancy of value'.[30]

This conception of the relationship between value and prices has blocked any perception of the contradiction between the value and the exchange value of money. Consequently, the contradictory form in which labour is expressed by money appears incomprehensible. This implies that the transformation debate has only reproduced exactly the same deficiencies that provoked the dissolution of the Ricardian School. This chapter has attempted to clarify at least two pillars where these deficiencies are grounded, which, as the last decade testifies, have contributed to the dissolution of the new version of the Ricardian School.

NOTES

1   Ricardo (1990:13)
2   '[T]he general prices of the commodities, which are the produce of such work, will not be governed by the peculiar facilities afforded to these workmen, but by the common, usual and natural difficulties which every other manufacturer will have to encounter' (Ricardo 1990:73).
3   In the final example of his first chapter, Ricardo assumes that, given the amount of labour, the production of all commodities is doubled. His commentary clearly infers that this situation implies a 50 per cent reduction in the absolute value of each commodity including those produced under the new conditions, and those already produced.
4   See for example, the *Grundrisse* (Marx 1973:141) 'As value, [the commodity] is *money*'. Or *Capital* I (Marx 1976a:142) 'human labour creates value but is not itself value'.
5   Thus see *Capital* III (Marx 1981:238): 'the value of any commodity – and thus also of the commodities which capital consists of – is not determined by the necessary labour-time that it itself contains, but by the socially necessary labour-time required for its reproduction'. In chapter 6 of this

volume, where he discusses the effect of price changes, Marx is clear enough that the commodity's cost-price is determined by the replacement price of the material elements involved in its production.

6   To Wolff, Roberts and Callari (1982,1984a) belongs the great merit of stressing, for the first time, that the commodity's constant capital component is determined through the process where prices are formed and, thus, that value is not a pre-determined variable. However, for these authors the measure of value is not money but time; this prevents them perceiving that the second commodity's component, value-product, is also determined through price formation. For a discussion on this issue see Rodríguez (1994).

7   This idea has recently been expounded again by Moseley (1993a:170): 'The quantity of money that represents one hour of abstract labour is equal to the inverse of the labour value of a unit of money.'

8   Kliman and McGlone (1988:65).

9   See, for example, *Capital* II (Marx 1978:462).

10   Marx (1976b:34). This quotation is from chapter I of *Capital* I in the first German edition but was revised by Marx for the fourth German edition which is the basis of the standard English translation of *Capital*.

11   See *Theories of Surplus Value* II (Marx 1969b:201). See also the *Grundrisse* (1973:207) 'In order to express the price of money, the whole sphere of commodities would have to be listed, each in the quantity which equals 1 ounce of gold.' Ricardo (1817:105) presents this idea: 'To say that commodities are raised in price, is the same thing as to say that money is lowered in relative value; for it is by commodities that the relative value of gold is estimated'.

12   *Theories of Surplus Value* II (Marx 1969b:201): 'The price of the commodity which serves as the measure of value, and hence as money, does not exist at all, because otherwise, apart from the commodity which serves as money I would need a second commodity to serve as money – a double measure of values'.

13   Rigorously, the amount of any commodity given in exchange for gold is not the gold's price, but one of the many exchange values of gold. The price is the general exchange value of the commodities, the expression of their value in the use value of the commodity functioning as money. Money does not have price because no commodity can express its value in its own use value; but money has as many exchange values a there are commodities expressing their value in money.

14   de Brunhoff (1976:70).

15   See the *Grundrisse* (Marx 1973:218): 'The money relation is itself a relation of production if production is looked at in its totality'.

16   To Suzanne de Brunhoff (1976:70-71) belongs the great merit of stressing, for the first time since Marx, this contradictory character of the money-form of value: 'If money is treated as a unit of account possessing a price, it loses its specificity, and if its price is equal to its labour value, it can be considered as neutral. The confusion of the problem of prices and that of the conditions of reproduction, and the introduction of a money-commodity unit of account, wreck the bases of Marx's theory of money.'

17   Bortkiewicz (1952:14).

18   Seton (1957:152).

19   This is the case, for example, with Morishima and Catephores (1978a:158) who define the value system as $v = va + L$. As the 'distribution variables' (the vector of wage-goods and the rate of surplus value) are not specified, this system has n equations and n unknowns. Its unit of measure is not defined through a normalization (as in the above system) but it is directly given by the unit of measure of labour (hours, days, years...).

20   Bortkiewicz's system presents three productive spheres where only the third produces luxury commodities. According to his conception of value, surplus-value is given by the value of these articles and profit by their price. When he defines that this sphere produces the money-commodity and that its price equals its value, Bortkiewicz obtains the additional result according to which total surplus-value and total profit are equal. If, in Bortkiewicz's system, there were various branches producing luxury commodities, his normalization would not imply that total surplus-value equals total profit.

21   Marx (1970:71): 'Gold is the measure of value because its value is variable; it is the standard of price because it has been established as an invariable unit of weight'.

22   These exceptions are frequently neglected and, in many presentations of Bortkiewicz's system, values are measured in labour time and prices in money units. Sweezy (1970:117) who translated

Bortkiewicz's pieces into English, does not notice this either: 'in our value schemes … one hour of labour has been the unit of account'. See also Morishima and Catephores (1978a:157).

[23] Duménil (1983:441). See, for example, Tugan Baranowsky (1905:174), Moszkowska (1979:11), Sweezy (1970:117), Medio (1972:321), Shaikh (1981:128) and (1984:59), Morishima and Catephores (1978a:157), Lipietz (1982:61, note 6) and Mandel (1984).

[24] Sweezy (1970:117). 'It is important to realize that no significant theoretical issues are involved in this divergence of total value from total price. It is simply a question of the unit of accounts.' Dobb (1955: 279) fully supports this interpretation of the so-called 'postulate of invariance': 'So far as the transformation problem is concerned, the difference of assumption is purely formal: as Sweezy says, any such assumption is significant only as a way of establishing a link between Prices of Production and Labour-Values'.

[25] 1 acre = 0.4047 hectares.

[26] See equation (27) in Bortkiewicz (1952:21).

[27] May (1948:596).

[28] Sweezy (1970:117).

[29] Samuelson (1971:400): 'For when you cut through the maze of algebra and come to understand what is going on, you discover that the "transformation algorithm" is precisely of the following form: "Contemplate two alternative and discordant systems. Write down one. Now transform by taking an eraser and rubbing it out. Then fill in the other one. You would have completed your transformation algorithm".'

[30] Steedman (1977).