

A Critique to the So-Called Temporal Single-System Interpretation of Marx's Value Theory

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1. Introduction

The purpose of this paper is to examine the recently developed tendency in the labor theory of value dubbed as the temporal single-system interpretation (hereafter TSS interpretation) in the current literature (see, e.g. Freeman & Carchedi eds. 1996, Kliman. 1997, Ramos. 1997, Kliman & McGlone. 1999, Freeman. 1999). TSS interpretation asserts that the two conventional aggregates in the transformation procedure and the tendency of the rate of profit to fall hold in Marx's original form. It also alleges itself as an interpretation of Marx's own theory, not as a new theory or approach.

TSS interpretation classifies various interpretations of Marx's value theory according to the two criteria; (1) dual or single system according as whether value and price are regarded as two independent systems or mutually penetrating single system. (2) simultaneous or temporal system according as whether inputs and outputs are simultaneously valued in the equilibrium framework or temporally valued introducing historical time.

Whereas the main feature of dual-system is to posit value and price as mutually independent linear equation systems in the same vein of Leontief-Sraffa-Morishima, single-system interpretation argues that the values of constant and variable capital depend on the prices, not the values, of them. In so far as dual system is posited, the two aggregates in the 3rd volume of Capital can not be compatible, as shown by Bortkiewicz. However, they are compatible in single-system interpretation. Therefore, it is argued that the inconsistency critique of Marx's value theory can be refuted.

On the other hand, simultaneous valuation in the tradition of general equilibrium theory leads to the theoretical results that nullify the dynamic implications of Marx's value theory. For example, Okishio theorem, placing Marx's value concept in a static setting, could negate the law of falling tendency in the rate of profit, which can only be grasped in a dynamic perspective. Temporal system interpretation alleges that it can replicate Marx's results by defining the variables within historical time. Therefore, the redundancy critique of Marx's value concept (e.g., Samuelson. 1971, Steedman. 1977) can be imputed to the fallacy of simultaneous valuation.

The arguments of TSS interpretation are provocative in the sense that it not only vindicates Marx's theoretical results in an orthodox way (Laibman. 1996), but also aims at the dominant discourse of almost all price theories, i.e. simultaneous equilibrium concept.

First of all, can it be alleged as an interpretation loyal to Marx's value concept? However, the

question like this has tended to result in never-ending and unproductive citations and recitations of Marx's own texts including Capital. Therefore, I will try to answer the question put in a more indirect way; can the TSS interpretation refute the inconsistency critique and the redundancy critique of Marx's value theory in a more effective way than the other interpretations? I will show that it is neither a superior nor a more efficient interpretation. Furthermore, a perverse implication of the TSS interpretation will be added.

Next, I will treat the logical consistency of the concept of historical time advanced by the TSS interpretation.

2. Inconsistency critique : problems pertaining to magnitude

The TSS interpretation alleges Marx's insight that price-value difference stems only from profit-surplus value difference can only be preserved in single-system interpretation. It is true in the sense that the value concept compatible with the two conventional aggregates should be defined as the sum of prices of advances for means of production and real wages plus surplus value (see Roberts. 1997 : 486). However, the TSS interpretation defines value and price in the form of first-order difference as follows. Here ℓ , \mathbf{p} , \mathbf{A} , \mathbf{l} and \mathbf{g} denote, respectively, value vector, price vector, input coefficient matrix, labor input vector and price-value difference. And subscript denotes time defined discretely.

$$\ell(t+1) = \mathbf{p}(t)\mathbf{A} + \mathbf{l} \quad (1)$$

$$\mathbf{p}(t+1) = \mathbf{p}(t)\mathbf{A} + \mathbf{l} + \mathbf{g}(t) \quad (2)$$

Namely, the TSS interpretation rests upon abandoning one of the elementary characteristics of Marx's determination of value magnitude; it should be determined in synchronic, not diachronic way (see Flaschel. 1979 : 70). In other words, it depends on a sort of "big deal" which preserves the conventional aggregates in exchange of synchronization principle. As Kliman & McGlone(1999) admits, the temporal feature of value definition itself is not necessary for preserving the two conventional aggregates. Only single system nature is sufficient. However, the problem is more than this. Although in a more complicated and indirect way, price-value difference can also be reduced to profit-surplus value difference in dual-system interpretation. Under a certain condition, $\lim_{n \rightarrow \infty} \mathbf{A}^n = \mathbf{0}$ (footnote 1), so we can get the following result from (1) and (2).

$$\begin{aligned} \ell(t+1) &= \mathbf{p}(t)\mathbf{A} + \mathbf{l} = [\mathbf{p}(t-1)\mathbf{A} + \mathbf{l} + \mathbf{g}(t)]\mathbf{A} + \mathbf{l} \\ &= \sum \mathbf{g}(t-k)\mathbf{A}^k + \mathbf{l}(\mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \dots) + \mathbf{p}(t-k)\mathbf{A}^{k+1} \\ &= \mathbf{l}(\mathbf{I} - \mathbf{A})^{-1} + \sum \mathbf{g}(t-k)\mathbf{A}^k \quad (3) \end{aligned}$$

As $\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ is nothing other than the value definition in dual-system interpretation,(footnote 2)

$$\begin{aligned} p(t+1) - v(t+1) &= [p(t+1) - \ell(t+1)] + [\ell(t+1) - v(t+1)] \\ &= g(t) + \sum g(t-k)A^k = \sum_{k=0} g(t-k)A^k \end{aligned} \quad (4)$$

Namely, price-value difference can be expressed as a polynomial form of profit-surplus value difference in dual system interpretation. Therefore, whereas the dual-system definition of value cannot replicate Marx's text in original form, it can maintain Marx's implication that price-value difference results from redistribution of surplus value in the form of profit.

Furthermore, as (5) which can be derived from (1) shows, introducing the temporal nature into value definition necessarily leads to the conclusion that only changes in past can affect present.

$$\Delta \ell(t) = \Delta p(t-1)A \quad (5)$$

While every change in period t-1 including contingent market situation can, no sure-to-happen change in period t+1 can affect the value magnitude in period t. Marx's value concept dialectically integrates past and future into present. It must be a dynamic concept. However, its dynamic character is not automatically guaranteed by simply introducing discrete time.

3. Redundancy critique: problems pertaining to determination

The TSS interpretation argues that value concept is redundant in the simultaneous system interpretation because it can only determine relative value. For example, proportionate changes in living labor requirements will have no effect on the value rate of profit. However, TSS interpretation is alleged to avoid this problem by determining absolute value magnitude given $A, p(t), l$ in the equation (1). Therefore, to elude the redundancy critique of value theory, not only single-system nature but also temporal feature must be added.

However, TSS interpretation can no more refute the redundancy critique than the single-system interpretation can do.

Suppose the simultaneous single-system as the following.

$$\ell(t) = p(t)A + l \quad (1)' \quad p(t) = p(t)A + l + g(t) \quad (2)' \text{From (1)'}$$

and (2)', we can get the following results.

$$\begin{aligned} \Delta \ell(t) &= \Delta p(t)A \\ \Delta p(t) &= e(t)(I-A)^{-1} \text{ where } e(t) = g(t) - g(t-1) \end{aligned}$$

Therefore,

$$\Delta \ell(t) = e(t)(I-A)^{-1}A = e(t)(I+A+A^2+\dots)A = e(t)\sum A^k \quad (6) \text{On the other hand, applying}$$

the same manipulation to the TSS equations (1) and (2),

$$\begin{aligned} \Delta \ell(t) &= \Delta p(t-1)A \\ \Delta p(t) &= \Delta p(t-1)A + e(t-1) \text{ Therefore,} \\ \Delta \ell(t) &= [\Delta p(t-2)A + e(t-2)]A \end{aligned}$$

$$\begin{aligned}
&= \Delta p(t-2)A^2 + e(t-2)A = [\Delta p(t-3)A + e(t-3)]A^2 + e(t-2)A \\
&= \dots = \sum e(t-k-1)A^k \quad (6)'
\end{aligned}$$

Comparing (6) with (6)', the changes in value magnitude from period t-1 to period t can be represented by the weighted sum of physical data (A^k) in both interpretations. Only the magnitude of weight differs.

In TSS interpretation, given technical data and historical profile of past prices suffice to determine absolute value magnitude. Therefore, it cannot evade the redundancy critique in the sense that only physical data are needed to know prices and values.

4. Real vs. nominal rate of profit

Let's return to the equation (1) of TSS interpretation.

$$\ell(t+1) = p(t)A + I \quad (1)$$

The physical elements represented by matrix A were purchased at $p(t)$ in the end of period t(or in the beginning of period t+1). However, in the end of period t+1 when value magnitude is calculated, it is impossible to purchase them at $p(t)$, unless there is a lot of stocks already produced. Therefore, $p(t)$ is not a really-existing price, but only a price written in account book.

Therefore, in order to correctly represent the value of constant inputs premised upon the concept of historical time, we must calculate it as $(1+i)p(t)$ considering time factor.(footnote 3) In the equilibrium situation of $p(t+1) = (1+i)p(t)$, it is actually reduced to simultaneous system interpretation because $\ell(t+1) = (1+i)p(t)A + I = p(t+1)A + I$.

However, in a dynamic and disequilibrium setting where $p(t+1) = (1+i)p(t)$ does not hold, value definition premised upon historical time should be changed as follows.(footnote 4)

$$\ell(t+1) = (1+i)p(t)A + I \quad (7)$$

Using single commodity model, we can show what the trouble resulted from the modification of value determination as (7) is. As there is only one commodity, value is equal to price. Therefore, (7) can be changed as follows. Here a denotes input coefficient.

$$\ell(t+1) = (1+i)a\ell(t) + I \quad (8)$$

The general solution to (8) is as follows.

$$\ell(t) = \{\ell(0) - I/[1-a(1+i)]\}[a(1+i)]^t + I/[1-a(1+i)] \quad (9)$$
From (9), we can find that the value magnitude diverges to infinity as time flows, unless the specific condition between input coefficient and time factor, i.e. $a(1+i) < 1$ holds.(footnote 5) In case of decreasing living labor input with the input coefficient unchanged, we can get the same result.

For example, let us posit the situation in which 8 units of commodity are used as inputs to produce 10 units of outputs. Assuming the necessary amount of labor input diminishes by 10% in each period, value determination would be as follows.

$$10l(t+1) = (1+i)8l(t) + 10(0.9)^t \quad (10)$$

From (10), we can calculate the modified value magnitude until the period 50 as <Fig.1>. Each curve corresponds to the case in which time factor is 0.1, 0.25 and 0.3. For example, when time factor is equal to 0.3, value magnitude in period 50 would be 53.44271 while labor input is only 0.05138.6)

<Fig. 1>

Without doubt, the above result itself can not negate the TSS interpretation. As TSS interpretation rests upon the thesis that "equilibrium never happens"(Freeman. 1996 :231), it has no interest in the stability problem of long-run equilibrium. However, this problem appears in the form of the artificial distinction between real and nominal rate of profit, e.g. in Ramos(1997) and Kliman(1999). Kliman(1999) defines the profit in the following two ways. And he names (11) and (12) as nominal profit(P_n) and real profit(P_r). (footnote 7)

$$P_n = p(t+1)x - p(t)Ax - w(t)lx \quad (11)$$

$$P_r = [1/(1+i^*)]p(t+1)x - p(t)Ax - w(t)lx \quad (12)$$

In (12), i^* represents the inflation rate of the monetary expression of labor-time(τ), which is defined as $[\tau(t+1) - \tau(t)]/\tau(t)$.

Kliman(1999) notes that simultaneous system approach has difficulty in the sense that monetary expression of labor time simultaneously defined can be negative. The logical superiority of TSS interpretation is alleged to lie in the fact that τ -series are necessarily positive in so far as the initial value of τ is positive. Furthermore, he refutes Okishio's theorem by insisting that real rate of profit falls despite the rise in the nominal rate of profit. However, as can be shown from (13) derivable from the definition of i^* , $\tau(n)$ will diverge to infinity as n increases unless i^* is negative or zero. This is the same problem as the modified TSS value diverges to infinity when time factor is larger than a critical value.

$$\tau(n) = \tau(0)(1+i^*)^n \quad (13)$$

The distinction between nominal and real rate of profit is introduced to solve this difficulty. In other words, the trial of TSS interpretation to integrate value and price into single system resulted in the additional segregation of real versus nominal value. (footnote 8)

5. Discrete vs. continuous time

Equation (1) of the TSS interpretation means that the values of constant inputs should be calculated as their historical costs, not current costs. However, the context of historical versus current cost which had been implicitly presupposed came to be negated in the recent discussions as following. (footnote 9)

"...it is a complete misnomer to treat the distinction between the above and equilibrium valuations as a distinction between 'historical' and 'current' cost. The value transferred to the product is not given by the magnitude of capital when purchased ; it is given by the magnitude of this capital when it is used. This *is* its 'current' cost. The equilibrium determination substitutes a completely different notion, redefining the word 'current' to mean 'future' ; it says that the value transferred by the cotton is given by what the cotton *will* cost when it has been produced using a technology that does not exist at the time it is used" (Freeman. 1999 : 10. Italic original)

Let's examine whether the above citation is correct or not by introducing continuous time. Assume that price (value) of constant input changes at the instantaneous rate of from t_0 to t_1 . (footnote 10) The physical quantity of constant input remains unchanged as $q(0)$ and $p(t)$ denotes its price at t . Then,

$$p(t) = p(0)e^{\alpha t} \quad (14)$$

If TSS interpretation does not argue for historical cost, prices (values) of constant inputs should be calculated as follows.

$$\int_{t_0}^{t_1} p(0)q(0)e^{\alpha t} dt = p(0)q(0) \int_{t_0}^{t_1} e^{\alpha t} dt \\ = (1/\alpha)p(0)q(0)(e^{\alpha t_1} - e^{\alpha t_0}) \quad (15)$$

However, in TSS interpretation, values of constant inputs are actually calculated as follows.

$$p(t_0)q(0) = p(0)q(0)e^{\alpha t_0} \quad (16)$$

In order to find the condition which validates the calculation method of TSS interpretation, assuming (15) is equal to (16),

$$(1/\alpha)p(0)q(0)(e^{\alpha t_1} - e^{\alpha t_0}) = p(0)q(0)e^{\alpha t_0} \quad (17)$$

From (17), we can get the following result.

$$(1 + \alpha)e^{\alpha t_0} = e^{\alpha t_1} \quad (18)$$

Therefore, unless the special relation as (18) is satisfied, the calculation method of TSS interpretation is incorrect even according to its internal logic. (18) can hold only when $\alpha=0$. Namely, if TSS interpretation rests upon the concept of current cost, no technological progress

in the department producing means of production should be posited. This is a paradoxical result.

6. Conclusion

Although TSS interpretation succeeds in maintaining the two conventional aggregates and the tendential law of falling rate of profit, it cannot be regarded as a correct interpretation of Marx's value theory in the sense that it abandons one of the basic characteristics of labor value concept, i.e. synchronic determination.

Marx's insight that price-value difference stems only from profit-surplus value difference can also be preserved in dual system interpretation. In dual system interpretation, although in a more complicated and indirect way, price-value difference can ultimately be reduced to profit-surplus value difference. Furthermore, while Marx's value concept dialectically integrates past and future into present, present is affected by only past, not by future, in TSS interpretation. Therefore, TSS interpretation is not superior to the other interpretations in the problem of determining value magnitude.

On the other hand, the knowledge of technical data and historical profile of past prices suffices to determine the absolute magnitude of value. Therefore, value system is still redundant in TSS interpretation.

Finally, in order to construct the time concept in TSS interpretation consistently, time factor must be considered. In that case, however, value magnitude can diverge to infinity despite the increase in the organic composition of capital. To solve this problem, another 'unnecessary detour' such as the distinction between nominal and real rate of profit must be introduced. Furthermore, even if we introduce continuous time so as to testify the argument that the time concept of TSS interpretation is not historical but current cost, its value calculation can be shown as inconsistent.

Appendix

Footnotes

(1) If rM is the eigenvalue of A which is maximum in modulus, $|rM| < 1$ is this condition. For the proof of this property, see Pasinetti(1977), 264-5. (2) Note that $v(t+1) = v(t)$ in the dual-system interpretation which is usually accompanied by simultaneous valuation.

(3) This time factor depends upon the rate of inflation and the increase of labor productivity. (4) If not, the same problem as Samuelson(1971)'s famous metaphor of turnover tax and value-added tax will arise. Namely, while $p(t)A$ and l are simply added in (1), the rate of profit must be multiplied to both of them in (2). (5) In simultaneous system interpretation, the so-called 'productivity condition' ($\alpha < 1$) must be satisfied. If not, ℓ will be negative and so meaningless. In TSS

interpretation, however, 'productivity condition' is not needed. As $\ell(t) = \mathbf{ap}(t-1) + \mathbf{l}$, $\ell(t)$ is always positive in so far as $\mathbf{p}(t-1) > 0$. (6) See appendix.

(7) Notations are slightly changed in this paper. \mathbf{x} denotes total output vector. On the other hand, Ramos (1997) uses the concept of 'rate of profit in labor time' instead of the real rate of profit. (8) As a matter of fact, this problem is specific to the single-system interpretation, not only to TSS interpretation.

(9) This is consistent with the process of substituting the concept of production price, e.g. in Kliman & McGlone (1988), for market price in the temporal single-system interpretation. Moseley (1999) distinguishes Marx's concept of price of production from Kliman & McGlone's concept. (10) In the general case of technological progress in the department producing means of production, α will be more than 0. And we can safely assume that from t_0 to t_1 is one period without losing generality.

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Appendix

period	labor input	simultaneous valuation	TSS	TSS*(i=.1)	TSS*(i=.25)	TSS*(i=.3)
0	10	1	1	1	1	1
1	9	1.8	4.5	1.78	2	2.04
2	8.1	2.25	4.05	2.3764	2.81	2.9316
3	7.29	2.529	3.645	2.820232	3.539	3.777864
4	6.561	2.6793	3.2805	3.137904	4.1951	4.585079
5	5.9049	2.73393	2.95245	3.351846	4.78559	5.358972
6	5.31441	2.718585	2.657205	3.481065	5.317031	6.104772
7	4.782969	2.653165	2.391485	3.541634	5.795328	6.827259
8	4.304672	2.552999	2.152336	3.547105	6.225795	7.530817
9	3.874205	2.42982	1.937102	3.508873	6.613216	8.21947

10	3.486784	2.292534	1.743392	3.436487	6.961894	8.896927
11	3.138106	2.147838	1.569053	3.337919	7.275705	9.566615
12	2.824295	2.0007	1.412148	3.219798	7.558134	10.23171
13	2.541866	1.854747	1.270933	3.087609	7.812321	10.89516
14	2.287679	1.712565	1.14384	2.945864	8.041089	11.55974
15	2.058911	1.575943	1.029456	2.798251	8.24698	12.22802
16	1.85302	1.446057	0.92651	2.647763	8.432282	12.90244
17	1.667718	1.323617	0.833859	2.496803	8.599054	13.58531
18	1.500946	1.208988	0.750473	2.347282	8.749148	14.27882
19	1.350852	1.102276	0.675426	2.200693	8.884233	14.98506
20	1.215767	1.003397	0.607883	2.058187	9.00581	15.70604
21	1.09419	0.912137	0.547095	1.920623	9.115229	16.4437
22	0.984771	0.828187	0.492385	1.788625	9.213706	17.19992
23	0.886294	0.751179	0.443147	1.66262	9.302336	17.97655
24	0.797664	0.680709	0.398832	1.542872	9.382102	18.77538
25	0.717898	0.616357	0.358949	1.429517	9.453892	19.59818
26	0.646108	0.557697	0.323054	1.322586	9.518503	20.44672
27	0.581497	0.504307	0.290749	1.222025	9.576652	21.32274
28	0.523348	0.45578	0.261674	1.127717	9.628987	22.22798
29	0.471013	0.411726	0.235506	1.039492	9.676088	23.1642
30	0.423912	0.371772	0.211956	0.957144	9.71848	24.13316
31	0.38152	0.335569	0.19076	0.880439	9.756632	25.13664
32	0.343368	0.302792	0.171684	0.809123	9.790968	26.17644
33	0.309032	0.273137	0.154516	0.742932	9.821872	27.2544
34	0.278128	0.246322	0.139064	0.681593	9.849684	28.37239
35	0.250316	0.22209	0.125158	0.624833	9.874716	29.53232
36	0.225284	0.2002	0.112642	0.572381	9.897244	30.73614
37	0.202756	0.180436	0.101378	0.523971	9.91752	31.98586
38	0.18248	0.162596	0.09124	0.479343	9.935768	33.28354
39	0.164232	0.1465	0.082116	0.438245	9.952191	34.63131
40	0.147809	0.131981	0.073904	0.400436	9.966972	36.03134
41	0.133028	0.118888	0.066514	0.365687	9.980275	37.4859
42	0.119725	0.107083	0.059863	0.333777	9.992247	38.99731
43	0.107753	0.096441	0.053876	0.304499	10.00302	40.56797
44	0.096977	0.086851	0.048489	0.277657	10.01272	42.20039
45	0.08728	0.078209	0.04364	0.253066	10.02145	43.89713

46	0.078552	0.070422	0.039276	0.230553	10.0293	45.66087
47	0.070697	0.063407	0.035348	0.209956	10.03637	47.49438
48	0.063627	0.057089	0.031813	0.191124	10.04274	49.40052
49	0.057264	0.051397	0.028632	0.173916	10.04846	51.38226
50	0.051538	0.046272	0.025769	0.1582	10.05362	53.44271

