1 Time, money, equilibrium: methodology and the labour theory of the profit rate

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1.1 INTRODUCTION

Marx identified the profit rate as the driving force in capitalism. The search by capitalists for an adequate return on their capital, he argued, gives capitalism its dynamic and unstable character. The profit rate is both the measure of successful accumulation and the motive for greater accumulation.

Marx insisted that the source of the profit rate was labour, labour as a process, and labour productivity as the result of that process. He (1969b:432) criticized Ricardo for limiting the determining role of labour productivity to capital and wage goods industries (that is, basics), and treating the luxury sector profit rate as passively mirroring the basics rate (see also Marx 1981:177). Yet all solutions to the transformation problem since Marx have accepted Bortkiewicz's (1984) (and Sraffa's 1960) equilibrium Ricardian model whose profit rate is the value rate of profit in basics.[1]

This chapter criticizes the methodology of neo-Ricardian models of price and profit rate determination as neoclassical[2] on three counts:

❑ its approach to time, and to how to incorporate reproduction over time;
❑ its treatment of the money-of-account;[3] and
❑ its presumption that equilibrium, and in particular a uniform profit rate, are possible in capitalism, and moreover determine the profit rate.

It emphasizes that the implicit *theory* is Ricardian rather than Marxian:

❑ production conditions in basic industries alone affect the profit rate,
❑ all technical change will lead to a higher profit rate (Marx had argued that increased capital intensity may reduce the profit rate), and
❑ money is a veil thrown over real relations.

This chapter contrasts the neo-Ricardian methodology with that of Marx and the classicals. It challenges equilibrium in terms of its method of abstraction, imposition of simultaneous time, and assertion that capitalism is rational. The causal model implicit in Marx's transformation algorithm and modern solutions

is arguably essentialist; a structuralist approach more akin to Marx's historical method and to non-equilibrium is explored. In the structural approach to reproduction, capitalist pricing causes nominal price changes as it redistributes real (surplus) value among firms. Such non-equilibrium price instability calls into question gold as the money-of-account.

The chapter then focuses on the issues of nominal price, inflation, and the money-of-account. It shows how the equilibrium assumption segregates inflation and the price level from the determination of relative price and the income distribution, a Ricardian and pre-Keynesian habit. It critically evaluates the meaning and implications of a gold money-of-account. It shows that Marx recognized that historically, the price unit changes from gold into a conventional accounting unit *before* capitalist exchange. Consequently, Marx's monetary theory is fully consistent with the qualitative differences between nominal price and real value implied by non-equilibrium.

The chapter then incorporates reproduction over time into the transformation from values to prices via a structural non-equilibrium methodology. It employs a conventional money-of-account, permits nominal price to deviate from real value, and recognizes that the profit rate *cannot* be uniform. The model determines the real rate of profit, aggregate value and surplus value. It shows that $n - 1$ prices and aggregate price (in)stability are indeterminate without analysis at a lower level of abstraction – the level of the historically contingent structure of capitalist competition (see also Naples 1989).

1.2   MARX'S METHOD OF ABSTRACTION

In Volume I of *Capital*, Marx demonstrated that when goods are produced for exchange, prices are regulated by labour productivity. Moreover, workers' productivity over and above their equivalent hourly wage creates surplus value.

In Volume III of *Capital*, Marx applied these core concepts to the case of capitalist production. He examined how surplus value is converted to a general rate of profit on capital, transforming values to prices of production. He abstracted from the actual absence of a uniform profit rate, and capitalists' need to reproduce themselves over time. Because Marx was not concerned with reproduction, he treated the following subsidiary issues as inessential:

1) the transformation of input costs from values to prices of production;
2) productivity change; and
3) the introduction of new goods tomorrow.

Marx abstracted from those real conditions he considered inessential in order to throw into relief the underlying structure of capitalism. The 'principle of appropriate abstraction' (Sweezy 1970:20) was to identify and distinguish the core elements of the problem from the inessential (see also Meek 1967:94). Marx was able to show that under capitalist pricing behaviour, goods would no longer

exchange at values, yet labour productivity would still determine price and profit. The prices of production charged would redistribute surplus value to industries with higher organic compositions of capital.

The theoretical project since Bortkiewicz has been to incorporate one aspect of reproduction, the *feedback* of today's pricing behaviour on tomorrow's costs (item 1 above). Bortkiewicz did so through a specific set of assumptions:

❑ input costs would be evaluated in prices of production;
❑ prices would be stationary, (he set input prices charged yesterday equal to output prices charged today);
❑ goods markets would clear (otherwise the markup on unit costs will not equal the profit rate, see Naples 1994).

Combined, these form an *equilibrium* specification. But equilibrium is only one way to incorporate reproduction into Marx's model; a competing formulation has been advanced (for example Naples 1989).

Bortkiewicz interpreted his equilibrium model as implying that it was not possible for both of Marx's invariance postulates to hold. His discovery of a contradiction between Marx and equilibrium is itself a major contribution. Unfortunately, the postwar generation schooled in neoclassical theory found equilibrium a more obvious point of departure than Marx's exploitation theory of the profit rate. Too few have recognized the different epistemological statuses of Marx's labour theory of value and Bortkiewicz's equilibrium assumption. Bortkiewicz allowed his simplifying equilibrium assumptions to vitiate Marx's most crucial concept: that the capitalists' ownership of the means of production permits them to extract both necessary and surplus labour from workers, and that their labour productivity explains both price and the profit rate.

But Marx would never have adopted simplifying assumptions which were inconsistent with his central theoretical discovery. The irony is, neither would any other theorist. As T. Kuhn and I. Lakatos have both argued, any scientific practice rests on an overarching 'paradigm' or 'research program'. Moreover, as Blaug quotes Lakatos,

> 'all scientific research programmes may be characterized by their "hard core", surrounded by a protective belt of auxiliary hypotheses which has to bear the brunt of tests'. The 'hard core' is irrefutable by 'the methodological decision of its protagonists' – shades of Kuhn's paradigm! (Blaug 1988:367)

The labour theory of value was Marx's 'hard core', and therefore incapable of refutation from within his own paradigm or research program (see also Dobb 1989:3-7 on vision). He would have rejected equilibrium as an 'inappropriate abstraction' inconsistent with his core principles.

Those who follow Bortkiewicz have replaced the labour theory of value with equilibrium assumptions. But they have also replaced Marx with Ricardo, where Ricardo's conception of capitalism was *not* inconsistent with Bortkiewicz's equilibrium[4] assumptions. The neo-Ricardian model treats profits as the surplus left over from nature's and technology's bounty once other class incomes are paid

(see Marx 1976a:650-651,1994:18,32 on nature). And the model treats labour and labour power as identical.

Yet Marx saw his labour/labour power distinction as his critical advance over Ricardo. The question was not only how class incomes are distributed, but the source of capitalists' income in their successful extraction of labour (more hours of work, more intensive labour, better quality performance) from employees. It is probably more than coincidence that the two theories which are consistent with an equilibrium methodology, neoclassical and Ricardian theory, treat the production process as a black box labelled 'technology', not as the conflict-ridden power struggle Marx perceived (for more on social theories of productivity, see Naples 1987).

1.3   EQUILIBRIUM AND TIME

Equilibrium in the classical tradition meant a central tendency – towards a uniform profit rate, and the exchange of goods at their natural prices or prices of production – not the achievement of a state of tranquillity (Robinson's language 1969). Since the marginalist revolution, equilibrium has been understood as a point from which there is no tendency to move. In the Marshallian (1982) long-period equilibrium, prices, wages and the rate of profit are constant over time (subject to given endowments and technology), and all markets clear.

Bortkiewicz's equilibrium methodology followed neoclassical General Equilibrium theory by employing the logical construct of simultaneous time – a moment in which all economic behaviour transpires at once. He was conscious that this formalistic approach to time and causation differed from the classical:

> Alfred Marshall said once of Ricardo: 'He does not state clearly, and in some cases he perhaps did not fully and clearly perceive how, in the problem of normal value, the various elements govern one another *mutually*, and not *successively* in a long chain of causation'. This description applies even more to Marx … [Marx] held firmly to the view that the elements concerned must be regarded as a kind of causal chain, in which each link is determined, in its composition and its magnitude, only by the preceding links … Modern economics is beginning to free itself gradually from the successivist prejudice, the chief merit being due to the mathematical school led by Léon Walras (Bortkiewicz 1952:23-24) [5]

His explicit attention to this issue highlights the incongruity between the Walrasian equilibrium *Weltanschauung* and classical thinking. His last sentence indicates how unusual simultaneous time still was even twenty-five years after the marginalist revolution.

The Surplus School, which encompasses neo-Ricardians and Post-Keynesians, eschews neoclassical equilibrium and its associated equilibration of supply and demand. They counterpose an economy in a state of tranquillity, which

develops in a smooth regular manner without internal contradictions or external shocks, so that expectations based upon past experience are … constantly fulfilled. (Robinson 1969:59)

While this is not identical to the statics of neoclassical economics, the economy is stationary in the sense of the natural sciences: despite ongoing motion, there is no substantial or structural change.

While the neoclassical and Surplus School notions of equilibria differ, they share a common equilibrium methodology. Analysts search for determinant outcomes from a set of equations representing tranquillity or equilibrium. This does *not* mean that economists from the Surplus School adopt a neoclassical *theory* of the determinants of an equilibrium position. Methods are distinct from theories, and economists of varying persuasions often employ methodologies from different traditions. For instance, some Marxian analyses of the extraction of labour from labour power have utilised an equilibrium methodology (for example Bowles 1985). The theory of profits and productivity has nothing to do with neoclassical theory, but the method is wholly an equilibrium one.

However, methods are not in themselves neutral. Thus American Keynesianism, based on an equilibrium methodology, leads to different policy prescriptions from those implicit in the Post-Keynesian representation of Keynes.

Keynes saw the decentralized nature of capitalist investment decisions in an uncertain world as a chronic source of instability and underemployment which only the socialization of investment, that is, planning, could rectify (see Keynes 1964 Chapter 24). The neoclassical interpretations of Keynes do not advocate different theories of effective demand from his. But their comparative static equilibrium models define away any determining role for uncertainty and a crisis of confidence as unfolding processes through chronological time.

Similarly, despite his classical *insight* that prices depend on production conditions, Sraffa employs an equilibrium and therefore non-classical *methodology*. This has theoretical consequences. For instance, his model has been shown to imply that every profitable change in technique will lead to a higher rate of profit or more net income to be distributed between capitalists and workers (Bortkiewicz 1952; Okishio 1961).[6] While Sraffa's model accepts the classical view of a conflict between capitalists and workers over the distribution of a given income, it implies a harmony of interest in promoting technological change. Moreover, capitalists uniformly benefit from productivity growth.

Yet Marx had insisted that labour saving technical change would hurt capitalists as a class by tending to reduce the rate of profit, which is more likely to force some firms out of business. Margaret Andrews (1981) has developed a fixed wage model where capitalists choose techniques at non-equilibrium prices. She showed how without the equilibrium assumption, labour saving technical change may reduce the average (non-uniform) profit rate.

The equilibrium methodology does not provide a neutral analytical tool, but directs economic investigations towards neoclassical results. As Bortkiewicz (1952:54) asserted:

> The mathematical method … achieves still more: by its means, the cost of production theory can, without any difficulty, be brought into harmony with the law of supply and demand … [f]ollowing the example of Walras … It is in this connection that the superiority of the mathematical method over the Marxian method appears particularly clearly.

Methodological innovations which imitate neoclassical techniques should be suspect a priori: techniques are most probably laden with theoretical priorities and perspectives, they are not paradigm-neutral.

1.4   EQUILIBRIUM AND RATIONALITY

The neo-Ricardian method posits an economy in equilibrium with an income distribution consistent with stable relative prices. This takes for granted that capitalism can be characterized in this way, as a rational, internally coherent system.[7] Interestingly the father of simultaneous equation economic models, Léon Walras, saw the demonstration of that rationality as the central goal of economic science; this would help provide the basis for rejecting a socialist alternative.

> Man [sic] is a creature endowed with reason and freedom, and possessed of a capacity for initiative and progress. In the production and distribution of wealth, and generally in all matters pertaining to social organization, man has the choice between better and worse and tends more and more to choose the better part. Thus man has progressed from a system of guilds, trade regulations and price fixing to a system of freedom of industry and trade, i.e. to a system of *laisser-faire*, *laisser-passer*; he has progressed from slavery to serfdom and from serfdom to the wage system. The superiority of the later forms of organization over the earlier forms lies not in their greater naturalness (both old and new are artificial, the newer forms more so than the old since they came into existence only by supplanting the old); but rather in their closer conformity with material well-being and justice. The proof of such conformity is the only justification for adhering to a policy of *laisser-faire, laisser-passer. Moreover, socialistic forms of organization should be rejected* if it can indeed be shown that they are inconsistent with material well-being and justice. (Walras 1984:55, emphasis added)

While Smith and Ricardo might not have balked at the neo-Ricardian effort to demonstrate the rationality of capitalism (Godelier 1972), it was Marx's enterprise to identify capitalism's contradictions, its inconsistent tendencies and counteracting tendencies. Marx sought the endogenous sources of capitalism's recurring crises. He saw capitalism as irrational, a system which a rational, self-interested working class would replace with socialism.

Bortkiewicz reframed the transformation problem in terms of a Walrasian vision of economic science.[8] He *assumed* that the outcomes of competitive individual behaviours were consistent with each other, and sought to show they

were inconsistent with Marx's theory of the profit rate. Bortkiewicz claimed that the equilibrium condition that the profit rate be uniform determines its magnitude. However, '"a static equilibrium system only expresses the *conditions* for an unspecified dynamic system to be in equilibrium"' (Dobb 1989:9, quoting R. Bentzel and B. Hansen). Bortkiewicz's model did not prove that the profit rate in capitalism must equal the Ricardian rate, but rather that a capitalist economy will only settle down into equilibrium if its rate of profit equals the Ricardian rate.

Thus Bortkiewicz and other neo-Ricardians have not demonstrated that Marx's own presentation was internally inconsistent. Rather, *they have shown that an internally consistent capitalism is irreconcilable with the exploitation theory of the profit rate*. For Marx capitalism is riddled by contradictions which render impossible the stationarity and rationality implicit in the neo-Ricardian equilibrium method.

1.5   ESSENTIALISM VERSUS STRUCTURAL CAUSATION

The neo-Ricardian equilibrium assumption is arguably not Marxian. But the model's essentialist view of causation is. In Marx's original transformation, prices were transformed quantities of labour values, but their units were not qualitatively different from values. Exchange value was reducible to its value-essence in a straightforward unmediated manner. Marx's critics, in 'correcting' his transformation, have also assumed that the price realm had to be shown to be a direct reflection of the essential underlying value realm if the labour theory of value was to be substantiated. Capitalist pricing implied that the quantity of socially necessary abstract labour time earned by a capitalist would change, but not the mechanism which allocated real income.

Yet in the field of history Marx's own analysis was much more complex, and Marxists have recognized the pitfalls of essentialist theories of causation. For instance, Louis Althusser (1970) argued that revolutionary events are never simply 'determined' by an essential cause like the contradiction between labour and capital. Rather, they are *overdetermined* by an accumulation of contradictions deriving from the structure of capitalism. This does not negate the view that that structure emerges from the fundamental conflict between labour and capital, but rather enriches it: such secondary conflicts as competition within the capitalist or working class also shape that structure and its corresponding 'laws of motion'.

Althusser cited Frederick Engels's[9] reading of the sources of social transformation as an example of essentialist thinking. He contended that Engels's attribution of a determinant role of the economy 'in the last instance' was economistic and wrong, since 'the lonely hour of the "last instance" never comes' (Althusser 1970:113). Some have interpreted this as saying that the evolution of

social formations is not determined by economic factors at all. By extension, the effort to reduce categories like 'price' to an underlying 'value', or of exchange value to production relations, is misplaced. Values and prices are mutually and reciprocally determining and determined (Kliman and McGlone 1988; Wolff, Roberts and Callari 1982), there is no *ex ante* cause after the onset of capitalist pricing.

Ironically, this echoes Bortkiewicz's view of simultaneous causation rather than the classical view. Many if not most of the authors in this collection take a similar stance, because they are critical of the dualism of the neo-Ricardian model. In the neo-Ricardian specification, values play no determining role. The value realm is orthogonal to the price realm, and the two only connect through a single invariance postulate.[10]

I too am critical of this dualism. But I believe Bortkiewicz was correct in treating Marx's sequential causation as fundamentally different from simultaneous causation. For Marx production logically preceded exchange, and value preceded price. Rather than counterposing mutual causation to no causation, (simultaneous determination to neo-Ricardian dualism), I argue that we must retain the prime causal role of values by changing our model of causation from essentialist to structuralist.

In history we are not limited to two choices: reducing everything to economics, or foregoing economic determinism altogether. Similarly, Althusser's critique of essentialism implies that instead of linear determinations, models of the transformation problem should embody a structuralist, more nuanced vision of causation. Equilibrium economic models have been likened by others to the method of Newtonian mechanics (see Bharadwaj 1978:43). A non-equilibrium model with nonessentialist causation can be exemplified by Einstein's modification of Newton's laws.

In Newtonian theory, a force applied to an object of a given mass for a period of time produces a change in speed.[11] The essential cause behind the acceleration is the force applied. Einstein recognized the contradiction between this theory and the apparent constancy of the velocity of light. He suggested that simplifying assumptions, like the constancy of an object's mass or the constancy of time's duration, should now be re-examined. The contradiction was resolved through a structural causal model.

Einstein argued that as an object approaches the speed of light, the constancy of light's velocity implies that added force may cause mass and/or time to adjust as well as speed. Because of the discovery of the constancy of light's speed, Einstein argued, other 'parameters' of the natural system had to be reconceptualized as variables. Thus changing velocity can no longer be reduced in a linear fashion to an immediate, essential cause (except for low velocities, below ninety percent of the speed of light). Nevertheless the application of force is the fundamental underlying cause of acceleration, as well as of the other resultants (changes in mass and/or time).

Bortkiewicz's discovery that equilibrium contradicts the conservation of value (socially necessary labour time) suggests that the structure of capitalism prohibits the essentialist determination of prices by values implied by an equilibrium model. A nonessentialist approach to a non-equilibrium transformation reconceptualizes a 'parameter', the price level, as a variable. Thus, a change in price can no longer be reduced in a linear fashion to an immediate essence (an equivalent change in value or surplus value realized); it could also reflect an additional purely nominal change produced by the *process* of redistributing surplus value. The relationship between values and prices of production is mediated by the phenomenon of nominal price, just as the relationship between force and velocity is mediated by such other factors as time and mass.

1.6   EQUILIBRIUM, THE MONEY-OF-ACCOUNT, AND INFLATION

In neo-Ricardian tranquillity, there can be neither changes in relative prices nor endogenous inflation. The equilibrium approach accepts a schism between the determination of relative prices and of the price level, despite the fact that the price level is only a weighted average of individual prices. The failure to allow for linkages between nominal price changes and changes in relative prices permits a Walrasian-style separation of price theory from the theory of money.

This aspect of the neo-Ricardian model follows Ricardo, but deviates from Marx's views. Ricardo is the father of the idea that changes in money will affect the price level without affecting relative price determination (see Rist 1966:160: "'I assume as a fact which is incontrovertible," says Ricardo, "that commodities would rise or fall in price, in proportion to the increase or diminution of money'"). In the equation of exchange,

$$\mathrm{M}v = \mathrm{PX} \qquad (1)$$

for a stock of paper money M, velocity of money v, row vector of nominal prices P, and column vector of goods X, Ricardo argued causation only ran from the left to the right. Changes in the stock of money, given the characteristic velocity, would change only the level of nominal prices because the output level and relative prices were given by the goods sector.

Yet Marx continually insisted (see for instance Marx 1970:193-95; 1976a:212-213; 1973:789-90, 810, 813-814, 878)[12] that causation in equation (l) ran from right to left, implying an endogenous velocity of money proper (see also Thomas Tooke's 1844 criticisms of Ricardo, and Rist 1966). For Marx inflation was not initially a monetary phenomenon, but derived from real forces.

The Marxian and Post-Keynesian macroeconomic traditions reject both an equilibrium methodology and the notion that inflation is independent of relative prices and the income distribution. A prime cause of inflation is struggles to change the distribution of income (Rowthorn 1977; Rosenberg and Weisskopf 1981). Inflation results from inconsistent nominal claims by workers and

capitalists. No excess demand or exogenous change in the money supply is required, only nominal income claims which in the aggregate exceed the value of output at historic prices. Relative price changes contribute to cyclical inflation as well: in the early expansion, prices for raw materials rise, in the late expansion, relative prices for capital goods and nominal and real wages rise, and both relative price changes cause the price level to advance (Boddy and Crotty 1975; Kalecki 1936; 1966). Inflation is inherently uneven, not 'pure', and is linked to the distribution of income and relative prices.

Nevertheless, scholars who work in the Marx-Keynes macroeconomic tradition have accepted the neo-Ricardian long period abstraction from changes in the price level. This is the more remarkable since Marx argued that prices of production involved a *redistribution* of surplus value from industries with low organic compositions of capital to those with high compositions. There is no a priori reason to assume that the redistribution mechanism which changes individual absolute prices does not generate price level changes; rather, price stability is imposed by the equilibrium methodology.

1.7   GOLD AS THE MONEY-OF-ACCOUNT

Since Marx insisted that gold was the standard of price, it is often taken for granted that his unit of account was always gold (see for instance Steedman 1981:47; critics of this view include de Brunhoff 1976:71 and Foley 1983). From this perspective it seems impossible to reconcile endogenous uneven inflation with Marx's labour theory of value. Price would always be in terms of gold, and would express real value.

This section argues that treating gold as a commodity money does not protect models against the economic forces which generate inflation. It shows that Marx's interpretation of gold as a measure of value was different from the neo-Ricardians'. Furthermore, it is not clear that Marx's unit of account in Volume III of *Capital* was gold. Marx's references to gold as measure of value do not necessarily mandate gold as the unit of account in which prices were measured. I conclude that there is nothing in Marx which either forces the accounting unit to be a commodity money, or precludes endogenous nominal inflation.

First, a commodity money accounting unit can mask inflationary forces in formal models. In 1985 I showed that there can be chronic, pure inflation in a Sraffian model despite a gold numéraire commodity if the (uniform) nominal profit rate exceeds the equilibrium rate. Since relative prices do not change, gold prices can be constant, although prices expressed in a conventional accounting-money will change over time. In 1993 I demonstrated that both Shaikh's (1977) and Kliman and McGlone's (1988) sequential models of the transformation problem masked inflation, despite their claims of a commodity money money-of-account.[13] This inflation dampens out over time, as the models tend toward

equilibrium. A commodity money accounting unit is only logical for an economy in long run equilibrium without endogenous inflation, but it does not prevent such inflation.

Equilibrium models often adopt a gold accounting unit, but treat the gold money commodity like any other commodity whose exchange value depends on the equilibrium solution. Gold has a value and a cost of production, and is produced in firms that earn the average rate of profit. Marx explicitly criticized Ricardo's similar treatment of the gold commodity money[14] as 'falsely assuming' that the 'medium of circulation, exchanges as a commodity for commodities' (Marx 1969b:200). Ricardo had suggested that a fall in wages would only reduce the prices of goods which employed a lower proportion of fixed capital than

> 'the medium in which price was estimated; all those which had more, would positively rise in price'. With regard to *money* prices this seems *wrong*. When gold rises or falls in value, from whatever causes, then it does so to the same extent for all commodities which are reckoned in gold. (Marx 1969b:200)

Is Ricardo discussing the exchange value or 'natural price' of money, while Marx discusses its value? No, rather Marx used 'value of money' and 'exchange-value of money' interchangeably because to him, gold was a non-transformed value. Alan Freeman has pointed out that in Volume III Marx (1981:50) takes the 'value of money' as 'constant throughout' despite the general transformation of values to prices.[15]

As I have suggested elsewhere (1993), Marx's language is consistent because gold is produced in mines. Thus *gold exchanges at its value* rather than price of production, since mineowners collect absolute ground rent.[16] The neo-Ricardian solution is wrong on gold because it abstracts from land, a crucial means of production in mining, and from landowners' rent. It treats gold as infinitely reproducible, like other commodities. But Marx made clear that the good which serves as commodity money must be scarce to serve as money. Just as Marx rejected Ricardo, he would reject the neo-Ricardian model where the exchange value of money is determined in the same way as other commodities' prices of production.

It is not true that Marx's unit of account was always gold. In Volume I, Marx assumed 'gold is the money commodity, for the sake of simplicity' (1976a:188). But in Volume III, where he analysed the general profit rate and prices of production, Marx abstracted from changes in 'the value of money' (1981:142) or in the 'money-expression' of given values (1981:238, 259, 266). The money form he had in mind is ambiguous. If money has value, it must be a commodity money. But at only one point did Marx explicitly refer to a commodity money. And there he made clear that

> This is so even with a purely nominal change in value, the rise or fall of mere tokens of money, as long as other factors remain the same. (Marx 1981:236)

Otherwise he spoke of 'the money-value (pp238, 280)' of capital or profits, value in 'the money form (p295)' or 'assessment of commodity values … in

money (p275)', or the 'money-expression' of value. His choice of words sets money apart from values, as if the money expression itself had no value, as would be true of a token money and a conventional money-of-account.

Moreover, in discussing 'the money-value' of capital, Marx uses gold as a measure of real value without necessarily assuming gold as the unit of account. Since he was not concerned with reproduction in Volume III, there was no need to compare yesterday's prices with today's, nor then to refer to the actual unit of account which measured prices from one period to the next.

Marx clearly did have a conventional money-of-account in mind in one section of Volume III. There Marx argued that pricing behaviour itself could cause nominal profit rates to exceed the real rate, and that this would generate endogenous inflation:

> Suppose that the general rate of profit, and hence the average profit itself is expressed in a money value that is higher than that of the actual average surplus-value. As far as the capitalists are concerned, it is all the same whether they charge one another 10 per cent profit or 15 per cent. The one percentage covers no more actual commodity value than the other does, since the inflation of the monetary expression is mutual. For the workers, however (we assume that they receive their normal wages, so that the rise in the average profit is not an actual deduction from the wage …), the increase in commodity prices resulting from this rise in the average profit must correspond to an increase in the monetary expression of the variable capital. In actual fact, a general nominal increase of this kind in the profit rate … is not possible unless it brings with it an increase in wages and similarly an increase in the price of those commodities which form the constant capital. (Marx 1981:281)

By implication the value equivalent of the money-of-account will have changed because of capitalist pricing behaviour. Gold cannot be Marx's unit of account.

1.8   THE HISTORICAL DEVELOPMENT OF A CONVENTIONAL MONEY-OF-ACCOUNT

In fact Marx showed that before capitalism, the development of exchange transformed the unit of account from gold to a conventional accounting unit whose value equivalent was affected by many factors besides the value of gold. That is, precapitalist development produced a money-of-account consistent with ongoing non-equilibrium and aggregate price instability.

Marx (1973:142, 166-67, 173, 192) saw a money-of-account as the first historical form of money (see also Keynes 1930:3). It was born of the need for a common denominator to compare qualitatively different goods or gifts in a tribal society, and came into being before a universal medium of exchange. This early money-of-account could be 'purely imaginary' (1973:167) and had no necessary relation to gold or any other real price standard.

The development of exchange led to the isolation of a general equivalent (gold) which was both the measure of value and the medium of exchange. The quantitative standard of price began as a unit of gold's weight. Then

> A general rise in the prices of commodities can result either from a rise in their values, which happens when the value of money remains constant, or from a fall in the value of money, which happens when the values of commodities remain constant. (Marx 1976a:193)

But the implicit fixed gold standard only held as long as the money-of-account and money proper remained undifferentiated.

The further development of exchange induced changes in the quantitative standard of price and therefore in nominal prices, even if there was no change in the value of goods or of gold. Circulation debased the gold money, which therefore was replaced by stamped coins. The money-of-account (£ – 'the pound' currency unit), itself the name of a coin, was thus distinguished from a unit weight of gold (lb. or pound):

> In the course of circulation, coins wear down, some to a greater extent, some to a lesser. The denomination of the gold and its substance, the nominal content and the real content, begin to move apart … The weight of gold fixed upon as the standard of prices diverges from the weight which serves as the circulating medium, and the latter thereby ceases to be a real equivalent of the commodities whose prices it realizes. (Marx 1976a:222)

These historical causes convert the separation of the money-name from the weight-name into an established habit with the community. Since the standard of money is on the one hand conventional, and must on the other hand find general acceptance, it is in the end regulated by law:

> The prices, or quantities of gold, into which the values of commodities are ideally changed are therefore now expressed in the money-names, or the legally valid names of the subdivisions of the gold standard. (Marx 1976a:194-195; see also Marx 1970:72, 107-114)

This *de facto* separation of the money-of-account from gold meant that prices were no longer denominated directly in gold. When prices are assessed in the money-of-account, the standard of price is not of necessity fixed as £1 = 1lb. of gold. What then determines the exact relationship between the money-of-account and gold, or put in a manner which Marx would abhor, the price of gold? (Marx (1970:75) argued that the 'price' of gold money is a misnomer: since gold is the standard for all other prices it cannot itself have a price.)

Marx suggested that both material factors and historical conventions determine the quantitative standard of price. He argued against the Nominalists' (Sir James Steuart, Bishop Berkeley, M. Proudhon) claim that the price standard is only in our heads and represents nothing but a subjective convention. Rather, gold is the universal equivalent because of a real historical process (Marx 1976a:197). Economic forces will determine how much gold a currency unit represents, whether the state officially honours convertibility to that standard or not (see Marx 1970:82-83).

The historical break between the money-of-account and gold means that prices are denominated in pounds (£'s), while gold remains the measure of value for both goods and the pound (£). Moreover,

> If we compare prices in England in e.g. the fifteenth century with those of the eighteenth, then we may find that two commodities had e.g. entirely the same nominal money value, e.g. 1 pound sterling. In this case the pound sterling is the standard, but expresses four or five times as much value in the first case as in the second, and we could say that, if the value of this commodity is = 1 ounce in the fifteenth century, then it was = ¼ ounce of gold in the eighteenth; because in the eighteenth, 1 ounce of gold expresses the same labour time as ¼ ounce in the fifteenth. It could be said, therefore, that the measure, the pound, had remained the same, but in one case = four times as much gold as in the other. This is the *ideal standard.* (Marx 1973:796-7)

Here the nominal prices of goods are no longer determined by the value of gold. Rather, a change in the value of gold changes the equivalence between the pound (£) and gold, instead of changing all £-prices, as a gold commodity money accounting unit would warrant.

This entire discussion refers to Marx's analysis of precapitalist exchange, abstracting from credit money as well as an equalised profit rate (see Marx 1970:116, 143, 169). Marx also identified the endogenous tendency towards replacing gold as money proper with alternative moneys that were cheaper to produce – metal tokens and paper money. He observed that the costs of circulation are paid out of surplus value (Marx 1973:548, 625), which provides a material basis for efforts to find a money proper which is less costly to produce, such as paper money.

With the emergence of paper fiat money, Marx recognized that new forces would affect the value represented by the currency unit: the quantity of paper,[17] and confidence in the monetary authority. Marx argued that a change in the quantitative standard of price, that is, gold equivalent of the £, will result from the injection of more paper tokens. A currency depreciation would change

> nothing but the nomenclature of the standard of prices, which is of course purely conventional, quite irrespective of whether it was brought about directly by a change in the monetary standard or indirectly by an increase in the number of paper notes issued in accordance with a new lower standard.[18] As the name pound sterling would now indicate one-fifteenth of the previous quantity of gold, all commodity-prices would be fifteen times higher. (Marx 1970:120)

Here the quantitative standard of price is endogenously determined by the quantity of paper money relative to the value of all goods.

Marx also recognized the role of uncertainty and expectations:

> If confidence in the government were to be thoroughly shaken … the paper thaler would in practice cease to be equal to the silver thaler and would be depreciated because it had fallen beneath the value proclaimed on its face. (Marx 1973:132)

Where Ricardo argued that trade, and therefore purchasing power parity, was the only factor determining exchange rates, Marx joined Tooke (and anticipated Keynes) by incorporating the confidence of international financial investors.

Thus the history of the money-of-account allowed for l) a change in the standard of price for metal money despite no change in the value of gold, due to a debased coinage; and for forced paper money, a change in the standard of price resulting from 2) an undue change in the quantity of money or 3) a crisis of confidence.

Marx saw that even before capitalism, the money-of-account was converted from a money commodity to a social convention. The quantitative price standard became relatively autonomous from the value of gold, while gold remained the measure of real value as distinct from nominal price. A new category of exchange value – nominal price movements – came into being historically, and was analysed by Marx. With this background, it is clearly consistent with Marx's monetary theory for the amount of real value expressed by the currency unit (or equivalently, the price level) to be affected by capitalist pricing behaviour.

1.9   A NON-EQUILIBRIUM MODEL

The neo-Marxian model combines a non-equilibrium snapshot of the economy with Marx's labour-productivity theory of price and the profit rate. Following Bortkiewicz, assume circulating capital, constant labour productivity, no product innovation, and simple reproduction (hence goods markets clear). Because this is a non-equilibrium model, a uniform profit rate is not imposed. Hence all price variables are expressed in a money-of-account, to register nominal price as distinct from real value.[19] Industry prices are then

$$P^t = P^{t-1} + Q^t = P^{t-1}A(1 + \alpha) \tag{1}$$

where

P^t = row vector of nominal unit prices at the end of the production period;

P^{t-1} = row vector of nominal prices at beginning of the production period, that is, input prices;[20]

A = the matrix of unit input requirements, including labour-power and productivity implicitly through unit labour costs;

Q^t = nominal unit profits;

I = identity matrix;

α = diagonal matrix of nominal profit rates.

r = diagonal matrix of real profit rates.

Time is measured in production periods, where the end of one production period is the beginning of the next.

Because we now examine reproduction over time, it is necessary to consider explicitly whether capitalists set aside sufficient money capital *at current prices* to buy sufficient outputs to reproduce themselves tomorrow. Consequently, real

or constant dollar profits (Q^{tc}) differ from nominal profits (Q^t) since they must be adjusted for nominal changes in input cost:[21]

$$Q^{tc} = (P^{t-1}A)\alpha - (P^t - P^{t-1})A \tag{2}$$

where Q^{tc} = nominal unit profits at time t corrected for inflated or deflated input costs.

Also, an industry's real profit rate, r_i, as distinct from its nominal rate, α_i, must evaluate all price terms consistently. (The profit rate is calculated per production period.) Therefore costs in both the numerator and denominator are in current price:

$$r_i = \frac{\alpha_i\left[\sum_j P_j^{t-1}A_{ji}\right]X_i - \left[\sum_j (P_j^t - P_j^{t-1})A_{ji}\right]X_i}{\sum_j P_j^t A_{ji}X_i} = \frac{Q_i^{tc}X_i}{\sum_j P_j^t A_{ji}X_i}, \quad i = 1, n. \tag{3}$$

Finally, given simple reproduction, real profits are spent on luxuries, so for the vector of real profit rates r,

$$rP'AX = Q^{tc}X = P_{III}{}^t X_{III} \tag{4}$$

where subscript III means only luxury goods, those produced in Marx's department III, have nonzero elements. Simple reproduction in basics is classically independent of prices:

$$AX = X_b \tag{5}$$

where subscript b means only basic goods have nonzero elements.

This way of modelling price determination transforms the structure given by the equilibrium model, even without Marx's value theory. The input prices (on the right hand side of equations (1)) are historically given parameters, rather than endogenous variables equal to output prices. Because additionally the profit rate is not assumed to be uniform, the system is not recursive to the equations for basic industries, nor does it tend towards the neo-Ricardian rate of profit as the sequential models do. Up to this point, this model is general enough to be consistent with any theory of value. Put differently, the system of equations for prices, the profit rate, and simple reproduction (1-5) could not be solved without a theory of the profit rate and of the source and magnitude of *real* value, that is, a theory which can distinguish nominal from real.

The two invariance postulates which summarize Marx's labour-productivity theory translate prices or profits in the currency unit to values or surplus value:

$$P^t X = d^t ZX \tag{6}$$

$$Q^{tc}X = P_{III}{}^t X_{III} = d^t sLX \tag{7}$$

where
- $d^t =$ the scalar dollar expression of one unit of socially necessary labour time at time t,
- $Z =$ row vector of unit labour values,

s = the scalar share of one hour's labour going to surplus value (where $v = 1-s$ is value of labour power), assumed uniform across the economy,

L = row vector of unit labour requirements.[22]

Taking prices minus profits gives capital advanced, in price and value terms:

$$P^tX - Q^{tc}X = d^t(ZX - sLX) = d^t(C + V + S - S) \qquad (8)$$

or

$$P^tAX = P^t(X_I + X_{II}) = d^t(C + V) \qquad (8')$$

where

C = scalar, total constant capital advanced,
V = scalar, total variable capital advanced,
S = scalar, aggregate surplus value.

But these two versions of Marx's invariance postulates 7 and 8′, divided by each other, imply that the real average profit rate in price terms will equal the value profit rate Π:

$$\bar{r} = \frac{Q^{tc}}{P^tAX} = \frac{S}{C+V} = \Pi \qquad (9)$$

where $\bar{r}$ = the real rate of profit. Note that d^t cancels out.

Setting the level of effective demand (that is, an output level), $n-1$ degrees of freedom can be shown to remain.[23]

This model allows prices and the price level to vary over time by adding one endogenous variable (d^t). Whether there is inflation or deflation depends on whether basics (and luxury) prices rise or fall on average. Note that the model implies that the rate of inflation in basics will be identical to that in luxuries. This is only because simple reproduction means that all of surplus value is spent on luxuries, and the money expression of surplus value must equal that for basics (and for total value).

The model's structure mandates a unique real average rate of profit, determined by the surplus value produced by labour relative to the value embodied in capital advanced. The model remains underdetermined or open ended with respect to prices, since no one unique set of relative prices is implicit. (Similarly see Alan Freeman's final chapter, which makes the same discovery on the basis of a non-uniform profit rate alone; he does not investigate the possibility of endogenous inflation.) We must be given $n-1$ markups or nominal profit rates to find absolute and relative prices, and the rate of inflation or deflation implied by d^t/d^{t-1}.

In other words, more information on the historical structure of capitalist competition is necessary before relative prices can be known (for example the degree of monopoly, extent of new industries, the structure of state regulation, and so on). In the absence of these markups, only the real value rate of profit and other real value aggregates are determinant.

As Bortkiewicz showed for a simultaneous-time model, if a uniform profit rate is superimposed on this model, the system of equations becomes

overdetermined.[24] But Marx would argue that it is not he but capitalism which is internally inconsistent. Its tendency towards a uniform profit rate cannot be realized. He would agree with Walras that the promise of socialism hinges on demonstrating the irrationality of capitalism. And Marx would applaud the success of his labour theory of value in demonstrating precisely that.

1.10   CONCLUSION

The equilibrium method of neo-Ricardian models is neither Marxian nor neutral, since it frames the Marxian transformation problem in a format which prevents its solution. The neo-Ricardian method has been shown to involve simultaneous time, mutual causation, an arbitrary commodity money-of-account, a uniform profit rate and aggregate price stability. Marx's own methodology has been counterposed as embodying historical time, sequential causation, tendential equalisation of the profit rate, and a conventional money-of-account. When this non-equilibrium method is combined with reproduction over time, the result revises Marx's view of how values cause price from an essentialist to a structural causal framework. It distinguishes nominal price from real value, and allows for endogenous aggregate price instability.

The non-equilibrium method implies that the tendency to equalise the profit rate may generate endogenous changes in the value represented by the money-of-account. In physical mechanics, the constancy of the speed of light causes time and mass to adjust when a force is applied to an object approaching that speed. In non-equilibrium Marxian price theory, the conservation of value *and* surplus value causes the currency expression of value to adjust when surplus labour extracted is allocated in proportion to capital advanced. Nominal price is determined by value, without *being* value, just as velocity is determined by force, without being reducible to force alone. Equilibrium and nominal price stability are only possible when the organic composition of capital in luxuries equals that in basics (for example a Standard Economy).

This result means that even abstracting from productivity change and from crisis tendencies, capitalism most likely exhibits chronic price instability. This implies that a stable price level would require conscious intervention on the part of the state's monetary authorities. The formation of central banks and development of monetary and regulatory policies are in part directed to such price level stabilization.

By incorporating simple reproduction into the transformation problem, this model exposes capitalism's *non-equilibrium*, internally contradictory structure. To consider how a firm is reproduced we have to distinguish nominal from real, and trace the implications of today's pricing behaviour *by other capitalists* for tomorrow's costs. This illustrates how each firm's reproduction is contingent on what every other firm is doing, which the individual capitalist cannot know in an

unplanned economy. As Marx argued, the reproduction of capitalism is always contingent, not effortless as implied by the equilibrium model.

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NOTES

[1] This includes the New Solution. Such sequential solutions as Kliman and McGlone (1988, and McGlone and Kliman's chapter in this volume) and Shaikh (1977) follow Bortkiewicz in combining simple reproduction with a uniform profit rate, although they start with non-equilibrium prices. Their models too stabilize at Ricardo's profit rate. In this paper, equilibrium as an *outcome* refers to Robinsonian tranquillity, not Walrasian equilibrium (Robinson 1969; see note 4 below).

[2] It will be argued that equilibrium as a *methodology* is the same in either Robinson's tranquillity or Walras's equilibrium formulations.

[3] In neo-Ricardian models, gold commodity money typically serves as numéraire money-of-account. But like Keynes, Marx distinguished accounting money (in terms of which prices are expressed) from real money (gold, like Keynes's money proper) (Keynes 1930; Marx 1973:190). Most economists have been taught that 'money' serves several different functions. In fact, money-of-account serves some functions (unit of account for current transactions, unit of account for deferred payment or debts), and money proper serves others (medium of exchange, store of value, means of payment). Under the forced paper monetary system we have today, gold may be the *price standard*, and gold may *measure* real value. Yet the *money-of-account* is a conventional currency unit, paper money circulates as *medium of exchange*, bank reserves are the *means of payment*, and in an inflationary crisis, a different international currency or basket of currencies may *denominate long term debts*.

[4] This is not neoclassical equilibrium: (1) there is no labour market, but a given subsistence wage, following Ricardo; (2) nor is there a capital market, but simply an assumed uniform profit rate, reflecting profits as a residual; (3) consequently price is not the sum of imputed factor incomes, as neoclassicals contend, but prices derive from production conditions, and profits are prices minus materials costs and other classes' incomes.

[5] In William Jaffé's edition of Walras (1984), he refers to frequent correspondence between Bortkiewicz and Walras regarding the *Elements*: November 1887 (p567), May 1888 (p571), February 1889 (pp588, 596), and a 'series of unpublished letters which Walras exchanged with Bortkiewicz and Edgeworth between the dates January 9 and September 14 1889' (p539); at the time, Bortkiewicz was 19-21 years old.

[6] It is not widely known that Bortkiewicz (1952) actually proved the Okishio theorem in 1906-7! He observed that for Marx, an increase in labour productivity implied an increase in the organic composition of capital (p38), and therefore a fall in the profit rate. Using his neo-Ricardian model of price and profit rate, Bortkiewicz showed that 'What is in fact true is thus the exact contrary of Marx's theory. An increase in the productivity of labour … leads to an increase in the rate of profit, with the sole provision that this increase in productivity should take place in those lines of production which are directly or indirectly relevant for the production of real wages' (pp47-8). Bortkiewicz limits productivity change to basics because a change in luxuries would have no effect on the profit rate.

[7] 'The adoption of the metaphor of equilibrium from physics onto the problem of price determination … conjur[es] up images of natural laws bringing about a natural order' (Clark 1992b:12).

[8] Schumpeter argued that an economist's underlying vision is inevitably ideological, since 'it embodies the picture of things as we see them' (Schumpeter quoted in Dobb 1989:3).

[9] Engels to Bloch 21 September 1890.

10   Section IX shows that Marx's two invariance postulates combined imply that the real rate of profit will equal the value rate, not the Ricardian rate. Thus neo-Ricardian models which claim to include two invariance postulates (examples cited in Alfredo Saad Filho's chapter) exclude some element of Marx's two invariances.

11   That is, for F = force, *t* = time, *m* = mass, *v* = velocity, *a* = rate of acceleration,

   $$F\delta t = \delta(mv)$$

   or, for a constant mass,

   $$F = \delta(mv)/\delta t = m\delta v/\delta t = ma.$$

12   'This much is clear, that prices are not high or low because much or little money circulates, but that much or little money circulates because prices are high or low', (Marx 1973:195). '[T]he money not thrown into circulation does not exist for the commodities. Thereby there exists no fixed relation between the value of money generally and the mass of it which enters into circulation. That the mass actually in circulation, divided by the number of its turnovers, is equal to the value of money is merely a tautological circumlocution for saying that the value of the commodity expressed in money is its price (1973:869-70). Marx acknowledged that excessive state issue of forced paper money could affect the value expressed by the money-of-account, see Section VII.

13   In his response, Kliman (1993) distanced himself from this view. 'Naples is correct to criticize K&M's imprecise discussion of money. Despite our mention of the "value of … money", we intended to posit the existence only of a money of account, not commodity money' (p149).

14   Ricardo was earlier quoted as claiming that a change in the quantity of money would change all prices proportionately. His two theories of the exchange ratio between gold and commodities reflect different, and inconsistent, perspectives.

15   Bortkiewicz (1952:11) interpreted Marx as claiming here only that 'the same amount of labour is always required to produce a given quantity of gold', since Marx 'always regarded the proportion in which gold … was exchanged against goods, or *rather against other goods*, as being subject to the general laws of value and of price' (emphasis added).

16   In his chapter, Adolfo Rodríguez poses a contradiction between the value and exchange value of a gold commodity money because he believes that gold's 'price' is the Ricardian exchange rate, while its value is socially necessary abstract labour time. But there is no contradiction once we recognize that Marx meant that gold's exchange value equals its value.

17   Here I differ from Suzanne de Brunhoff, who claimed that '[i]nstead of tending towards a quantity theory of paper money, [Marx] seeks to get rid of quantity theory for all kinds of money' (1976:35). But Marx argued that 'a person who restricts his studies of monetary circulation to an analysis of the circulation of paper money *with a legal rate of exchange* must misunderstand the inherent laws of monetary circulation' (1970:122) because 'in the circulation of tokens of value all the laws governing the circulation of real money [i.e. gold] seem to be reversed and turned upside down … the value of paper tokens depends on the number of tokens in circulation' (1970:121). In fact, he gave the example of a fictional 'forced gold money' system, and argued that in this case even gold becomes a token, and would not exchange in proportion to its value. If 'Gold as a token of value … fall[s] below its real value … [t]he effect would be the same as if…all commodities were evaluated in metal of lower value than gold … Commodity-prices would therefore rise …' (Marx 1973:172, emphasis added; I am indebted to Alan Freeman for this quote).

18   Modern Keynesians would disagree. If notes are issued and inflation ensues, it is the governmental issuer of notes who has redistributed real income from others in the economy who pay higher prices, to itself as borrower.

19   For Marx, real value is socially necessary abstract labour time. Nominal changes in prices are not associated with a change in socially necessary abstract labour time produced or realized. Neoclassicals convert nominal variables to real by deflating by a price index. The GDP price index is constructed by designating a representative bundle of commodities and services which serves to assess base-year prices, and asking how much that bundle costs in the present year. All price changes are interpreted as purely nominal, despite such obvious discrepancies as price declines caused by productivity advances. Moreover, the output mix also changes from year to year, but to calculate changes in the price level the commodity basket is treated as fixed over several years. Thus neoclassicals treat 'real values' as quantities of things, of the base-year commodity basket.

[20] Input prices are historical givens. They could in principle be any magnitude given from the past: values, prices of production, neo-Ricardian prices, random prices – whatever prices were consistent with the labour theory of value in the previous period.

[21] The inflation I discerned (1993) in Kliman and McGlone (1988) becomes perceptible via this technical correction of gross profits, from nominal profits (Q^t) to real profits (Q^{tc}). Without this correction, capitalists as a class may reproduce themselves over time, but individual capitalists will not. Because relative prices change over time, there is uneven inflation in Kliman and McGlone's model. For example, if capitalists in department I, whose relative output price has dropped, should continue to buy the same amount of luxuries out of gross profits, they will not set aside enough money capital to buy the inputs Kliman and McGlone assume that they do continue to buy. Because Kliman and McGlone do assume simple reproduction of capitalists in each department, they have to trace the implications of this assumption for the distribution of surplus value among departments. Once department I capitalists set aside sufficient money capital, they are forced to buy fewer luxuries as a result of department I's lower real profits realized (while department II's capitalists will have higher relative prices, higher real profits, and be buying more luxuries).

[22] I am assuming for convenience that each labourer performs only simple labour at a uniform intensity. Thus one hour's labour maps one-for-one to one hour's use of labour power. L, which is technically one unit of socially necessary abstract labour time, is also interpreted as one hour of labour power.

[23] The variables include: n elements of P^t_i, (or $n-1$ relative prices, 1 price level), n elements each of α_i, Q^{tc}_i and r_i respectively, $m-1$ relative X_i's in basics (m is the number of basic goods), one luxury X_i relative to basic outputs, one output level, and d^t. There are $4n+m+2$ variables. The independent equations include: n equations for P^t_i in terms of α_i (1), n equations for Q^{tc}_i (2), n equations for r_i in terms of α_i (3), $m-1$ independent X_i equations in basics (5), one luxury X_i equation (4), two invariance postulates (6 and 7; equation 9 for determining the real profit rate is implicit in these). There are $3n+m+2$ independent equations. Consequently there are n degrees of freedom. Using one to set the output level leaves $n-1$.

[24] To impose a uniform profit rate means setting $n-1$ real industry rates equal to the average rate, where calculating the average rate itself takes up one degree of freedom, so in all n degrees of freedom are used. But only $n-1$ degrees of freedom are available, hence the overdetermination.