1 One system or two? The transformation of values into prices of production versus the transformation problem

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1.1 INTRODUCTION

We here defend Marx's oft-refuted account of the transformation of values into prices of production in two ways. First, we argue that it was appropriate to his purpose, that of showing the transformation to be part of the process by which workers' subjectivity is transformed into an antagonistic economic 'objectivity'. To comprehend this process of transformation into opposite, we suggest, values and prices must be retained in one relation, not separated into opposed systems of calculation. Second, we show that, once values and prices are held in a single relation, Marx's account is logically coherent.

Like most contributors to this book, we contend that static equilibrium formalisations distort Marx's value theory. Our work, however, is not intended to develop an alternative, non-equilibrium political economy. Rather, we conceive our defence of Marx's account of the value-price transformation as an attempt to combat an ideological attack on his body of ideas and thus to create a place for its renewal, and as contributions to the *critique* of political economy on the foundations laid by Marx.

The difference between political economy and Marx's critique of it is, in our view, twofold. Firstly, whereas rival schools of economics primarily argue over which gives the best account of the functioning of existing society, *Capital* does not merely criticize others' *conceptions* of reality. It is a philosophical critique of economics, which critiques the existing reality of capitalism *itself*, including its thought, from the standpoint of an envisioned new, human society, the conditions for which develop through the struggles of revolutionary subjects within existing society. Because its projects and concepts – and not only Marx's own opinions – are thus inherently critical, Marx's work becomes subject to distortion when forced into the mould of economic theory.

Secondly and relatedly, whereas internal critiques within economics focus on others' inaccuracies, we believe that *Capital* is primarily a critique of the scientific, disinterested, and (largely) 'correct' political economy of the classicals. Following Dunayevskaya (1988:98-102; 1989:76-94; 1991:143-45), we regard Marx's theory of commodity fetishism as primarily not a critique of illusion and inaccuracy, but as a critique of conceptions that are topsy-turvy because they reflect accurately the inverted relations that characterize capitalism – the reification of persons and the personification of things. Thus it was precisely because the classical economists' categories did correspond to capitalist reality that Marx considered them to be, at one and the same time, both 'absurd' and 'socially valid' (Marx 1976a:169)

Although this chapter reiterates themes we have addressed earlier (Kliman and McGlone 1988), the dialogue our first paper has generated (see, especially, Naples 1993 and Kliman 1993) has convinced us of the need to sharpen and clarify our arguments. In particular, the present chapter (1) seeks to clarify further *how and why* the price of the means of production and labour power becomes the value of capital, and (2) illustrates the transformation in a slightly different and, we hope, clearer way. Also, it contains the mathematical Appendix (slightly revised) that accompanied our earlier paper but was not published due to space limitations.

Since Bortkiewicz, Marx's nonseparation of values and prices has been regarded as a logical inconsistency; Marx 'fails to keep separate rigorously enough the two principles of value- and price-calculation' (Bortkiewicz 1952:8). What has often been overlooked is that the relation between values and prices is a *methodo*logical issue. This was clear to Böhm-Bawerk. At the same time that he complained of a 'Great contradiction' between Volumes I and III of *Capital*, he argued that

> Marx has not deduced from facts the fundamental principles of his system, either by means of sound empiricism or a solid economical-psychological analysis: he founds it on no firmer ground than a formal dialectic. This is the great radical fault of the Marxian system at its birth: from it all the rest necessarily springs. (Böhm-Bawerk 1984:101)

The nondialectical 'understanding' (*Verstand*) perceives each object as isolated, uniquely itself, a whole unto itself (Hegel 1991:126-28). Thus, for instance, separate systems of value- and price-calculation are demanded, systems in which value equals value, and price of production equals price of production. Conversely, dialectical 'reason' (*Vernunft*) comprehends a judgement such as 'value is price of production' because this judgement, like every other proposition taken singly, is inadequate. It must continue to be developed until the original statement has undergone so much differentiation that we now fully comprehend *how* value becomes price of production.

To comprehend that process, we adopt neither the 'technological' nor the 'social' value paradigm (de Vroey 1982). The former confer upon technological relations a crucial role in the valuation process; the latter stress the role of money

and the market.[1] Our own approach is neither technological determinist nor market oriented, but is informed by the Marxist-Humanism of Raya Dunayevskaya, who restated the centrality of Marx's humanist philosophy of labour to his critique of political economy (see McGlone 1994). We owe to her the recognition that capitalist technological relations are themselves social relations, class relations of dead to living labour in production. '[L]abour is expressed in value' because 'the process of production has mastery over man, instead of the opposite' (Marx 1976a:174-75). Thus Dunayevskaya contended that 'it is more correct to call the Marxist theory of capital not a labor theory of value, but a value theory of labor' (Dunayevskaya 1988:138).

Neither the technological determinist conception of value, which separates value from price, nor the market centred approach, which holds that price *is* value, have been able to resolve the value-price problem. Since the early 1980s, however, a variety of authors have advanced the discussion by arguing in different ways that the total value of output equals the value added by living labour plus the *price* (rather than the *value*) of the means of production (for example Duménil (1983); Wolff, Roberts and Callari (1984a); Carchedi (1986); Glick and Ehrbar (1987); Kliman and McGlone (1988); Giussani (1991); Moseley (1993b); and several chapters in the present volume). Although we agree with this view, we wish to point out that, by itself, it does not vindicate Marx's account of the value-price transformation. Indeed, several of the authors who hold this view contend that Marx's account is incomplete or even self contradictory, precisely because the price of the means of production deviates from its value.

To defend Marx against this hoary charge, it is therefore insufficient to *assert* that the price of means of production is identical to the value of constant capital. One must show *how* – on the basis of the law of value, that is, the determination of value by labour time – the value of constant capital *comes to differ* from the value of the means of production. Our earlier paper (Kliman and McGlone 1988) demonstrated this, but failed to make explicit the conceptual basis of the demonstration. We now wish to make explicit that Marx's concept of embodied labour is the ground of that demonstration.

Marx (1976a:128) identifies *abstract* labour as the 'social substance' embodied in commodities' values. One can twist and turn a use value forever without finding any (concrete) labour lurking within. It is thus fetishistic to regard labour 'embodiment' as a suprahistorical technological reality, that is, as the expenditure of labour common to all production. The labour embodied as value is instead a '*phantom-like* objectivity' that 'arises from the *peculiar* social character of the labour that produces [commodities]' (Marx 1976a:128, 165; emphases added). As Marx (1964:122-23) wrote in 'Alienated Labour':

> The worker puts his life into the object, and his life no longer belongs to himself but to the object … The *alienation* of the worker in his product means not only that his labour becomes an object, assumes an *external* existence, but that it exists independently, *outside*

> *himself*, and alien to him, and that it stands opposed to him as an autonomous power. The
> life which he has given to the object sets itself against him as an alien and hostile force.

That which is called 'embodiment' in *Capital* is here referred to as life that 'belongs … to the object', labour that 'exists independently, *outside himself*', and life … given to the object'. It should be clear that Marx's embodied labour theory is a theory of abstract, alienated labour.

Because the embodiment of abstract, alienated labour is a peculiar social process, not a technological requirement as such, the abstract labour embodied in a commodity need not equal the amount of (concrete) labour needed to (re)produce it. Although exchange does not alter the quantum of value in existence, it does redistribute it. Because abstract labour is redistributed through exchange, some commodities *embody* more abstract labour than they would otherwise, some less. On the basis of this notion of labour embodiment, one can comprehend how the capital advanced to production does not cease to be a sum of value merely because it differs from the value of its material elements (means of production and subsistence). The illustration in Section 3 should be read with this in mind.

1.2   THE TRANSFORMATION NON-PROBLEM AND THE NON-TRANSFORMATION PROBLEM

Marx's account of the value-price transformation

It is well known that classical political economy adhered to two opposing principles which it was unable to reconcile and that, in Marx's view, this failure led to its disintegration. On the one hand, it discovered that labour is the substance of value and that the magnitude of a commodity's value is determined by the labour time needed for its production. On the other hand, it adhered to the prima facie contradictory view that profit rates tend toward equality and that a commodity's price therefore tends to be equal to the costs of its production plus an average profit. Even Ricardo failed to account for the determination of the level of the profit rate and held the disproportionality of prices and values to be an exception to the law of value.

It is also well known that Marx insisted that, rather than attempting to 'rescue' the law of value by means of a 'violent abstraction' (Marx 1976a:421) of this sort, the existence of prices of production and a general rate of profit 'have to be explained through a number of intermediate stages' (Marx 1969b:174). However, this stipulation is often interpreted as a call for successive relaxation of assumptions, for an even stricter adherence to Ricardo's method – the analytic method rooted in formal logic. In this view, the law of value is a 'first approximation' based on assumptions, such as equal compositions of capital, which do not hold in the real world and which must be dropped as the model becomes more realistic.

What Feuerbach had done in the analysis of religion,[2] Ricardo and the classicists had done in the analysis of economic life. They discovered by analysis the earthly kernel – labour – of the mystery of commodity value. The manifold phenomena of price relations were reduced abstractly, without mediation, to this undifferentiated substance, labour. Yet, the starting point in reality (prices) persisted in contradistinction to the starting point in theory (labour). The gulf between the 'real world' and the theoretical world, between appearance and essence, was not overcome.

Marx's approach was 'to do the opposite, i.e., to develop from the actual, given relations of life the forms in which these have been apotheosized' (Marx 1976a:493-94n). The difference is not only that Marx maintained a consistent starting point whereas the classicals vacillated between two inconsistent principles. Rather, instead of being a method of reconciliation, Marx's method is one of development through contradiction. His starting point thus contains within itself a duality – the dual character of labour revealed within its product, the commodity.[3] The duality between the concrete potentiality of the living workers and the abstract, value producing character of their actual activity, that is, alienated labour, is ever present in capitalist production. It is as isolated, independent individuals that the workers 'enter into relations with the capitalist … Their co-operation only begins with the labour process, but by then they have ceased to belong to themselves' (Marx 1976a:451). Their activity is not their own, but is subjected to the domination of dead labour. The social relations between persons at work have been *transformed* into thing-like relations (Marx 1976a:166).

Through a succession of 'intermediate stages', Marx traced the development of the fetishized forms in which this reification of labour manifests itself. The first of these forms is the commodity product, the materialization of the labour which is an 'objective' factor of production rather than the workers' self-expression. Each subsequent 'stage' is still another transformation, an inversion in which the worker's subjectivity takes on yet another form of a false 'objectivity', a 'social relation between things' (Marx 1976a:166). However, capitalism manifests itself not only in industrial relations, but in the market and in the categories of even 'scientific' political economy. Thus, in these realms which Marx examines in Volume III, still more transformations are revealed. As he writes in Chapter 2:

> the way that surplus-value is transformed into the form of profit, by way of the rate of profit, is only a further extension of that inversion of subject and object which already occurs in the course of the production process itself. We saw in that case how all the subjective productive forces of labour present themselves as productive forces of capital. On the one hand, value, i.e., the past labour that dominates living labour, is personified into the capitalist; on the other hand, the worker conversely appears as mere objectified labour-power, as a commodity. This inverted relationship necessarily gives rise, even in the simple relation of production itself, to a correspondingly inverted conception of the situation, a transposed consciousness, which is further developed by the transformations and modifications of the circulation process proper. (Marx 1981:136)

Thus, in Chapter 9 of Volume III, Marx argued that the consciousness of capitalists and bourgeois economists, though 'transposed', is grounded in reality's appearance. Even in the form of price of production (in which considerations of disequilibrium of supply and demand, interest, rent, and so on are excluded), price and profit for an individual capital differ quantitatively as well as qualitatively from value and surplus value. Because price appears to be determined by (not only equal to) the costs of production plus profit, and profit appears as a pure markup over costs, the law of value/surplus value seems false. Nevertheless, the alien reality of capitalist production relations remains the essential determinant of these new forms and makes its presence felt. Marx shows the determining role of the production relations by abstracting from intercapitalist competition and, via the concept 'total social capital', returning to the vantage point of capital versus labour:

> It is necessary … to avoid looking at things as if a society based on the capitalist mode of production lost its specific historical and economic character when considered *en bloc*, as a totality. This is not the case at all. What we have to deal with is the collective capitalist. (Marx 1978:509)

This was true not only of Volume II. Marx also made the total social capital the object of analysis in Chapter 9 of Volume III, viewing capital as if it 'belong[ed] to one and the same person' (Marx 1981:259). He was thereby able once again to see the capital/labour relationship through the appearance of 'many capitals'.[4] Total value and surplus value are proportional to total price and profit, respectively; the general rate of profit is the ratio of total surplus value to total capital advanced.

Throughout Volume III, rather than analysing market phenomena as self-subsistent, in their seeming independence from the sphere of production, these phenomena are developed as transformed forms of production relations. Thus, in Marx's illustration of the 'transformation *of* commodity values *into* prices of production',[5] value and price are conceived as contradictory terms in *one* relation. Value takes on a trans-formed appearance, a form of appearance that differs from itself.

The dialectical meaning of the term 'transformation' thus differs from its use as a synonym for a mathematical mapping. Many, if not most, of Marx's critics view his transformation procedure precisely as a failed attempt to map a self-contained set of values onto another, self-contained set of prices of production (or general equilibrium prices). Curiously, however, what goes unrecognized is that this transformation is but one of many transformations into opposite discussed throughout the three volumes of *Capital*, none of which are mappings. Were this fact better understood, perhaps this particular transformation would not have been singled out for criticism.

Moreover, the failure to recognize that many transformations have preceded the transformation of values into prices of production is one factor that leads critics to charge Marx with logical inconsistency. Lacking this recognition, their

misconceptions regarding the latter transformation's starting point are significant. Firstly, some critics of Marx's procedure still interpret Volume III's reference to 'value' as a reference solely to labour and labour time, and thus claim that the dimensionality of values and prices of production are inconsistent (see, for example, Abraham-Frois & Berrebi 1979:26-27). Actually, after tracing the development of the value form into the price form in Volume I, Chapter 1, Marx regularly referred to sums of money as 'values'.[6] Moreover, in a letter to Engels (27 June 1867) explaining the transformation of 'value' into price of production, Marx explicitly equates 'cost price' with the 'price of the constant part of capital + wages' and notes that this transformation 'presupposes' that various value magnitudes appear as sums of money.

The value congealed in a commodity is always expressed as a money price, a sum of money, because it is always related to the universal measure of value, money. Conversely, of course, a sum of money always represents a sum of value. As the *universal* measure of value, money is ever present, even in the absence of an exchange, since it 'serves only in an imaginary or ideal capacity' (Marx 1976a:190). Hence, the initial input 'values' in Marx's illustration of the transformation of 'value' into price of production are actually sums of money which, through the ideal presence of money, implicitly represent sums of value. Therefore, both before and after the transformation of magnitudes, inputs and outputs have the same, dual dimensionality, as will be illustrated in Section 3.

Secondly and relatedly, in Volume III 'commodities are not exchanged simply as *commodities*, but as the *products of capitals*', as results of capital's process of production (Marx 1981:275). Capital values, not the value of means of production and labour power, constitute the starting point of Marx's illustration. In circulation, capital is a sum of money which purchases means of production and labour power. The value of the *capital* is the value represented by that sum of money, not the combined value of the *means of production and labour power*. As we noted above, Marx's concept of value as embodied labour expresses the social relations of the capitalist mode of production, and is not a technological determinist conception. Thus, we reiterate that the capital advanced to production does not cease to be a sum of value merely because it differs from the values of its material elements.

At the beginning of Volume III, in discussing the transformation of value into cost price plus profit, Marx did assume that cost price equalled the combined values of the labour power and means of production used up in producing the commodity. This assumption was made in order to grasp the qualitative transformation in its 'purity', independently of any quantitative disproportionality. On the other hand, when he discussed the quantitative transformation of Chapter 9, Marx dropped this assumption, noting that '*if* the cost price of a commodity is equated with the value of the means of production used up in producing it, it is always possible to go wrong' (Marx 1981:265, emphasis added). Because they interpret his procedure as having wrongly

equated the two, his critics universally view this stipulation as an admission of error which, to be rectified, requires that values and prices be held apart in two systems.

The passage, however, continues: 'even if a commodity's cost price may diverge from the value of the means of production consumed in it, this error in the past is a matter of indifference to the capitalist. The cost price of the commodity is a given precondition, independent of his, the capitalist's, production.' Marx thereby indicated that he took the cost price as a *datum*, a given magnitude of value represented by a given price, without assuming that this magnitude equals the value of the means of production (and labour power) used up. Hence, neither his account of the transformation nor its resulting aggregate equalities depend on this assumption, as is often supposed. As we shall see in Section 3, his procedure accounts for prices of production and the aggregate equalities obtain even when inputs are purchased at their prices of production.

That the initial magnitudes of value and price are data, established in the immediate past, implies that Marx's illustration was not a system which abstracted from time. Rather, it depicted *one particular period* of capitalist production and circulation within the process of history.

The non-transformation problem

Marx's account of the transformation retains values and prices in one relation. In the transformation problem, they become separated into two opposed equational systems.

In the value system, values appear as a set of price relations ('value prices') opposed to equilibrium price relations. Rather than conceiving of price as a form of value, value becomes another form of price. The question to be answered thus becomes: in what way are these two pricing systems related? But, unlike Ricardo, Marx did not advance a labour theory of exchange ratios, that is, a theory of goods exchanging in proportion to the amounts of labour needed to produce them. The 'value price' system therefore has no basis in Marx's theory and the question of its relation to equilibrium prices is, from this standpoint, moot.

Rather, Marx asked *how* value relations assert themselves (letter to Kugelmann 11 July 1868); his account of the value-price transformation was part of the answer to this question. When value is conceived as a form of price and isolated into a separate system, this question cannot be answered. The market and the factory never come into contact; the unity of production and circulation is broken, a priori; the analysis becomes focused on different market forms alone. Moreover, since 'value prices' are abstracted from real prices, there has arisen a tendency to view value relations as abstractions from price relations, rather than as the reality of the factory.

We now turn to the price system. Solutions to the transformation problem take for granted the existence of prices and the profit rate, and seek merely to calculate

their magnitudes. This is often referred to as 'determining' prices and the profit rate. Yet not only does this differ from the real process of determination (Shaikh 1982); far more importantly, it wholly disregards the need to investigate the meaning of these variables and their qualitative relation to values.

Moreover, the conception of the rate of profit as an unknown, to be solved within the price system, differs markedly from Marx's conception. That the latter's account of the transformation leaves the rate of profit unaltered has received little notice, as if this result were a mere 'byproduct' of that procedure. However, its significance for Volume III of *Capital* is crucial. At pains to dispel the illusions which competition creates, Marx sought to demonstrate that, given a certain advance of capital, the level of the profit rate depends only on the degree to which capital succeeds in pumping out surplus labour. It is therefore determinable upon the completion of the production process, before commodities go to market. Competition merely effects the equalization of profit rates *at this previously determined level*.

The mathematical results of simultaneous solutions seem to discredit these contentions. The rate of profit appears to be determined by technology and the real wage, and established through competitively determined prices or by planning which utilizes shadow prices. Since this rate differs from the profit rate obtained through the value system, its appearance as a magnitude relatively independent of production relations is reinforced. However, inasmuch as the value system is an irrelevancy, so too is the discrepancy between its profit rate and the equilibrium profit rate. As we seek to demonstrate in the next section, when the value of the capital advanced is not confused with the value of its material elements, the logical 'existence' of the general equilibrium profit rate no longer implies its determination outside of production relations.

Even if they are of the iterative form instead of the simultaneous form, solutions to the transformation problem must employ one or another 'normalization condition' or 'invariance postulate'. Because the value and price systems are in themselves unrelated and the dimensionalities of values and relative prices are inconsistent, only the adoption of a normalization condition can create some relation between the two. It is generally recognized that, since 'there does not seem to be an objective basis for choosing any particular invariance postulate in preference to all others … the transformation problem may be said to fall short of complete determinacy' (Seton 1957:153, emphasis omitted). This indeterminacy indeed turns the transformation problem into an endless exercise. The number of possible normalization conditions (and therefore solutions) is limitless and each is, objectively, as good as any other.[7] Even in principle, then, the transformation problem cannot resolve the question of the relation of values to prices.

Even more significant is the fact that none of the 'solutions' actually *demonstrates* any relation of values to prices. Whereas Marx's procedure obtains aggregate equalities on the basis of the given data, the value-price relationships

which result from transformation problem solutions come from the theorists' heads alone. Because normalization conditions are asserted a priori and imposed externally on the otherwise unrelated value and price systems, the resulting relations are only assumed ones. That a numéraire is needed to obtain absolute prices does not justify the arbitrary imposition of a normalization condition. While the price form itself entails that a commodity find expression in some amount of money, the declaration that a specific value aggregate must remain invariant when value is redistributed is only the theorist's whim imposed on the actual data. In short, first the theorists negate the internal relation of values to prices, then they substitute whatever arbitrary relation they choose. Marx characterized this 'tendency to form arbitrary unmediated connections between things that belong together in an organic union' as '[c]rudeness and conceptual nullity'.[8]

In the transformation problem, the external mediator is the theorist, who comes from outside of the problem bearing a normalization condition that dictates how values will be reconciled with prices. The external mediator in actual life, however, must be some social force, 'independent' of both capitalists and workers, that can dictate a reconciliation of production with the market – in other words, the 'classless technical intelligentsia' responsible for planning the economy and establishing social equilibrium. Indeed, use of input-output models and equilibrium shadow pricing form the foundation of state planning. Are not solutions to the transformation problem therefore the ideological representations of a harmonious, state planned economy?

Yet, in production itself, there has been no reconciliation. To those who remain inside the factory, the plan is not classless but represents a 'social formation in which the process of production has mastery over man, instead of the opposite … production by *freely* associated men, [which] stands under their conscious and planned control' (Marx 1976a:175, 173, emphasis added). There is only the domination of labour by capital or the internal transformation of this reality, by those who live under it, into a new human society.

1.3   THE TRANSFORMATION OF INPUT PRICES: AN ILLUSTRATION

The foregoing discussion has indicated that Marx's concern was to show the transformation of values into prices of production to be only a 'further extension' of the transformation of workers' subjectivity into an antagonistic economic 'objectivity'. Comprehending this process of transformation into opposite requires that values and prices be retained in a single relationship, not separated into different systems of calculation. The charge of logical inconsistency, deriving from Marx's nonseparation of values and prices, is therefore misplaced.

While we reject this central criticism of Marx's account, often dubbed a 'failure to transform input prices', in another – quite real – sense the issue of

'input price transformation' remains. One capital's output does become the other's input and, in this interchange, the commodity's price generally *does* diverge from its value. An adequate defence of Marx's view of the transformation requires that one account for this process *without* separating values and prices into separate systems. We therefore show presently that a simple continuation of Marx's own illustration, as interpreted above, can illustrate the transformation of input prices.

To illustrate this process, some output-input relations must be assumed. For simplicity, we assume simple reproduction, but adopt *Marx's* conception of simple reproduction, not the conception formalised in the general equilibrium pricing models utilized by transformation problem solutions. Some marked differences between the two exist:

(1) 'Buying back' versus advance of capital. Were Marx to have assumed simple reproduction in his illustration of the value-price transformation, the aggregate *output* price of any component of social production (for example wage goods) generally would not have equalled the aggregate *input* price of that component (for example the total wage bill). This fact, originally noted by Bortkiewicz (1952:9), constituted his whole proof of logical inconsistency on Marx's part. Such inequalities are indeed absent from transformation problem solutions; yet they entail disruption of simple reproduction only if one accepts the implicit underconsumptionist premise that the components of output must be 'bought back'. For Marx, reproduction requires the *advance* of capital, investment. Money advanced for means of production and subsistence enables the previous period's outputs both to be sold and to serve (directly or indirectly) as inputs in the *upcoming* period. The input prices of means of production and subsistence in any period need not equal their output prices in that *same* period.

(2) Stationary prices versus prices of production. Solutions to the transformation problem look for a set of *unique*, timeless (relative) prices, perhaps in the belief that such prices alone 'support' the necessary interdependence of the various industries, or perhaps because it is thought that prices of production must be stationary by definition. In contrast, we do not regard the prices of production to which Marx refers as stationary prices. Perhaps surprisingly, support for this view has come from Garegnani (1990:51-52). He notes that 'changes in normal prices over time were ignored in traditional theory because they were considered sufficiently *small*' (Garegnani 1990:52), and not because normal prices (prices of production) were required by *definition* to be stationary. Marx's prices of production are equilibrium prices in the sense that they (a) permit each capital to achieve the average rate of profit, and (b) obtain when supplies equal demands. As we shall show, however, different sets of prices can at different times satisfy these conditions, even when technology and real wages remain unchanged.

(3) Reproduction of prices versus reproduction of use values. The relative prices obtained in transformation problem solutions are continually reproduced in

a timeless fashion. Again, these solutions assume either that material reproduction cannot occur under other prices or that stationary prices constitute an additional equilibrium condition. When Marx discussed reproduction, however, he was concerned with a prior question: in what quantities and proportions must the system produce two distinct use values, means of production and articles of consumption, to materially reproduce itself on a certain scale? Reproduction was thereby considered inseparably from the relation of dead to living labour in capitalist production, and irreducible to exchange relations among capitalists. Marx held prices fixed, not because reproduction requires fixed prices, but because changes in values and price-value deviations were irrelevant to the question at hand (Marx 1978:469-70). The simple reproduction of material relations can occur at *any* set of prices.[9] If all profit rates are uniform at the prevailing prices, whether or not they are stationary prices, it is reasonable to suppose that no further incentives for capital flows exist, and that supplies and demands should therefore actually equilibrate at these prices.

We regard Marx's illustration of the value-price transformation as entirely correct and complete, and modify it to account for simple reproduction only to defend it against the Bortkiewiczian critique. To defend it against the charge of failing to transform input prices, another modification is also made: we continue Marx's one period illustration into the next period. Whereas the value-price transformation can be depicted in a single period, the transformation of outputs into inputs, and thus the 'transformation' of output prices into input prices, takes place between *one* period of production and the *next*.[10] As Marx (1981:265) notes, the 'cost price … is a given precondition'; inputs cannot be repriced *retroactively*. Hence, to depict this second 'transformation' together with the first, without severing values and prices into self-contained systems, Marx's illustration must be continued into the next period.[11]

Table 2.1 presents a specific two department illustration. The symbols stand for:

m	capitalists' personal revenue[12]
M	money capital before production
C	cost price; commodity capital before production
MP	(price of) means of production
L	(price of) labour power
P	productive capital; process of production
s	(price expression of) surplus value
C′	commodity capital after production
M′	money capital after production
π	profit
r	general rate of profit
LL	(price expression of) hours of living labour added; generation of new value (not shown in table)
AC	articles of consumption (not shown in table)

The illustration assumes that Department I uses 160 MP and 160 LL to produce 320 MP. Department II uses 160 MP and 320 LL to produce 480 AC. At the current intensity of labour, the real wage equals 0.4 AC per LL; thus $0.4 \times 160 = 64$ AC and $0.4 \times 320 = 128$ AC are indirectly purchased by Department I and II, respectively. The remaining $480 - 64 - 128 = 288$ AC are consumed by the capitalists representing the departments. We also assume initial input prices equal to £1 per unit in both departments. (As noted above, the appearance of values as sums of money is necessarily presupposed; and Marx takes cost prices, and thus unit input prices, as *given*; see Moseley (1993b) for development of this point. Finally, in Table 2.1, Part A, we assume that each £ is the monetary expression of one hour of socially necessary labour. Every number in Part A thus signifies both a money and a labour time sum.

Beginning with money (M), the collective capitalists of the two departments each purchase two commodities (C), means of production (MP) and labour power (L), at given prices representing given values. (Any initial values could be assumed. *Solely* to facilitate comparison with transformation problem solutions, the initial values here equal the values of the means of production and labour power).

Table 2.1, Part A

Table 2.1, Part B

In production (P), the means of production become constant capital and labour power becomes labour, the labourers' activity functioning as variable capital. Upon entrance into the sphere of production, no change in material or value occurs. But production results in new outputs of greater value $(C + s)$, due to the extraction of surplus value (s) – labour for which no equivalent has been paid. These outputs are generally not priced at their values $(C + s)$; prices *tend* to

fluctuate around prices of production (C′–M′), which equal cost price (C) plus an average profit (π). Average profit differs from the surplus value each department extracts; were exchange to take place at prices of production, each capital would obtain the general rate of profit – the ratio of (1) the price expression of the total surplus value extracted in production and (2) the total cost price. The general rate of profit is determined *in production*, before circulation commences, so that its magnitude is the same whether outputs sell at their values, prices of production, or market prices differing from both. For simplicity, we assume that prices of production prevail in this period. As Marx showed, the sum of values (total C + s) equals the sum of prices (total C′–M′) and the sum of surplus value (total s) equals the sum of profit (total π).

The first circuit of money capital is now completed. For simple reproduction to occur, the collective capitalist of each department must obtain 160/320 = 0.5 of the total MP produced in Department I, requiring an outlay of 0.5 × £350 = £175. The workers in Departments I and II must obtain 64/480 = 0.133 and 128/480 = 0.267 of the total AC produced in Department II, requiring outlays by the collective capitalists of 0.133 × £450 = £60 and 0.267 × £450 = £120, respectively. The productive consumption of these means of subsistence reproduces these workers' labour power. Each department's total price (C′–M′) in period 1, minus the sum of its advances to production (M–C) in period 2, equals the revenue (m) that the collective capitalists consume unproductively on AC purchased from Department II. The sum of the revenue, though less than the sum of profit in period 1, nonetheless enables them to buy the remaining 480 – 64 – 128 = 288 AC (288/480 = 0.6, and 0.6 × £450 = £270).

Bortkiewicz (1984:212-13) alleged that, if the values in Marx's illustration permitted supplies to equal demands, the prices of production would not. Yet here the entire social product was bought and sold at its price of production, and each department's sales equalled its purchases: £175 MP were purchased by Department II and £60 + £115 = £175 AC were purchased indirectly by Department I.

The activity of production can now recommence. The workers have received their necessary means of subsistence and again perform 160 and 320 hours of labour (the sum of necessary and surplus labour, L and s) in Departments I and II. The portion of this labour for which no equivalent has been paid is greater in this period, because means of subsistence are cheaper and capital can obtain the use value of labour power at a lower price.

Since the total price and the total value of the first period are the same (£800 = 800 labour hours), the wages and capitalists' personal revenue, totalling £450 = 450 labour hours, which exit the circuit of capital between periods of production, are sums of value as well as price. The remainder, £800 – £450 = £350 = 350 labour hours, is thus *both the value and price* of the constant *capital* – even though this diverges from the value of the *means of production*. Again, while this conclusion is incomprehensible when value is conceived of as technologically

determined, when value is conceived of as a quantum of labour extracted from the living labourer that exchange can only redistribute, it makes perfect sense.

The sum of the constant capital plus the £160 + £320 = £480 = 480 labour hours of newly generated value is £830 = 830 labour hours, the total value of period 2. This total differs from the total value of the first period, but only because an additional value of £30 = 30 labour hours has been incorporated into the social capital in the interim.[13]

The sum of the surplus value in the second period, 300 labour hours, does not equal the newly added labour hours minus the *value of labour power*. As the sum of the newly added labour hours for which no equivalent has been paid it is, nevertheless, a quantum of surplus *value*.[14] And, since the value and price of the capital advanced in period 2 are equal, it follows that the general rate of profit in period 2 – the ratio of surplus value to capital advanced – is a 'price of production rate of profit', but also a 'value rate of profit'. As in period 1, addition of the resulting average profits to the cost price in period 2 yields a total price that equals total value, and subtraction of the total cost price from the total price of production yields a total profit that equals total surplus value. (This process of determination does not necessarily imply the actual attainment of prices of production or an equalised rate of profit.)

Because these *three* aggregate equalities hold in period 2, even though the values of the constant and variable *capital* diverge from the values of their *material elements*, Marx's illustration of the value-price transformation has been shown to be generally valid – his results hold even when inputs are purchased at prices deviating from values. The prices of production and general profit rate are correct in period 1, *given* its input values, and correct in period 2, *given* its different input values. In neither period did we invoke a normalization condition as an extra 'closing equation', because the known data in each period suffice to calculate prices of production (see the Appendix for the exact mathematical expressions). The difference between the total price (C′–M′) of period 2 and the total value (C + s) of period 1 is due, not to a deviation of total price from total value, but to the incorporation of additional value into the social capital in the interim, so that the total price and value *of period 2* are equal, just as in period 1. Having demonstrated these results, and having accounted for social reproduction with supplies equalling demands without severing values and prices into two systems, our defence of Marx's illustration is complete.

We now turn to two related issues. First, it may be thought that our illustration invokes a normalization condition because we have arbitrarily assumed each labour hour is expressed as £1. In 'two-system' transformation problem solutions, however, normalization conditions dictate an arbitrary equivalence between price and value aggregates even when *both* are expressed in money, or *both* in labour time (see Adolfo Rodríguez's chapter in this volume). Our illustration dictates no such equivalence independent of the data. In any case, the assumption of a constant money expression of value does not affect our conclusions. In Table 2.1,

Part B, we present monetary magnitudes corresponding to the labour time magnitudes of Part A. We assume that each labour hour is initially expressed as £1, but that, for whatever reason, the money expression of a labour hour *after* production in period 1 rises to £1.04. All output values (C + s) and prices of production (C′–M′) are thus 4 per cent greater than in Part A. Surplus value in money terms is found by subtracting cost price (M–C) from value (C + s), and profit in money terms is found by subtracting cost price (M–C) from price of production (C′–M′). Again, total value equals total price and total surplus value equals total profit.

The '*value*' and '*price of production*' rates of profit are equal, as ratios of labour hours (0.5625 in Part A), and as ratios of £s (0.6250 in Part B). The *labour time* and *money* measures of profitability are, however, unequal (1 + 0.6250 = [1.04] × [1 + 0.5625]). This discrepancy is caused only by the change in the monetary expression of value over the production period. Assuming that the money expression of value remains £1.04 per labour hour over the second period of production, the bottom row of Part B gives the money magnitudes corresponding to the labour time magnitudes of Part A's bottom row. Because no change in the monetary expression of value occurs over period 2, the monetary measure of the general rate of profit returns to equality with the labour time measure, 0.5660.

Finally, we wish to challenge the view, expounded by proponents of transformation problem solutions, that the price relations holding in general equilibrium contradict Marx's three aggregate equalities and thus invalidate his account of the value-price transformation. Were simple reproduction without technical change to continue, *ad infinitum*, and were the collective capitalists always to continue exchanging exactly at prices of production, *ad infinitum*, then the social capital would asymptotically approach the static equilibrium presented in Table 2.2 (money figures, assuming the money expression of value were to remain £1.04 per labour hour, are in the top rows; labour time figures, in italics, are in the bottom rows).

Table 2.2

Given the data of our illustration, an additional 34 labour hours and 12 labour hours would have become incorporated into constant and variable capital, respectively, the latter increase implying a 12 labour hour reduction in surplus value. Even if we imagine, for the sake of argument, that the stringent conditions needed for this static equilibrium terminus are somehow actually satisfied,

Marx's results still hold. The sums of price and profit equal the sums of value and surplus value, respectively, and the 'price of production' and 'value' profit rates are identical – and all three equalities hold both in money and in labour time terms.

1.4 CONCLUSIONS

We have not put forth the 'McGlone and Kliman solution to the transformation problem'. We have defended *Marx's own account* as internally consistent. If no self-contradiction is found within our interpretation of it, then one must reject the traditional claim that Marx's account is simply contradictory. One must at minimum concede that there is a defensible interpretation that finds it to be internally consistent (see especially, in addition to our own work, Giussani (1991), Carchedi (1991), and the chapters by Freeman, and by Carchedi and de Haan, in this volume).

We believe the issue of internal consistency must be faced squarely, irrespective of the truth value of Marx's *Capital* or the relative merits of our interpretation and the traditional approach. Marx's critics, not his defenders, are the ones who have made his alleged errors the ground upon which the economics profession has debated his work. It is a matter of simple intellectual honesty that they now either demonstrate that our defence of Marx is itself internally inconsistent, or renounce claims to have refuted him on logical grounds.

Marx's work has seemed obscure and incoherent in part because theorists have too quickly jumped to conclusions, rejecting or revising it before taking care to internalize it, learn from it, and thus work out the apparent contradictions. Precisely because such tendencies have characterized the history of post-Marx Marxism, the Marxism of Marx remains largely unexplored; much can still be learned from it. We and others have begun to find meaning and coherence in aspects of Marx's work commonly thought to be obscure or incoherent. This encourages us to continue the attempt to rediscover and learn from Marx's body of ideas.

1.5 APPENDIX

The first two sections of this Appendix present the relations depicted in Table 2.1, Part A in a general form, one applicable not only to the restrictive conditions assumed in the table (simple reproduction, two departments, no outputs used as both means of production and consumption), but to any set of single-output sectors without fixed capital. The third section shows that Marx's three aggregate equalities always hold. The proofs refer to a single period and thus do *not* require technology or input-output relations to be constant through time.

I. In period t, the physical relations are expressible by:

A = $[a_{ij}]$ $n \times n$ matrix of ith input used per unit of jth output

w $= [w_i]$ column vector of real wage components per unit of living labour extracted

l $= [l_j]$ row vector of living labour extracted per unit output

K $= [k_{ij}]$ $n \times n$ 'augmented input-output' matrix; $K = A + wl$

q $= [q_i]$ column vector of gross outputs

and the variables to be determined are:

p_{t+1} $= [p_{j;t+1}]$ row vector of unit prices of production (output prices)

v_{t+1} $= [v_{j;t+1}]$ row vector of unit values of outputs

r_t a scalar, the general rate of profit.

Initial unit input prices, p_t, are given (or determined in the prior period). ε, a scalar measured in £ per labour hour, indicating the initial monetary expression of value, is also given.

II. We now 'translate' the symbols in Table 2.1 into matrix form, assuming for simplicity that the monetary expression of value remains constant over the production period, t. The following scalar magnitudes express sums for the *total social capital*, and are amounts of money. When divided by ε, each is converted into a number of labour hours.

$$MP_t = p_t A q \tag{1}$$

$$L_t = p_t w l q \tag{2}$$

$$M_t = C_t = P_t = p_t K q \tag{3}$$

$$LL_t = \varepsilon l q \tag{4}$$

$$s_t = \varepsilon l q - p_t w l q \tag{5}$$

$$(C + s)_t = \varepsilon v_{t+1} q = p_t A q + \varepsilon l q \tag{6}$$

$$r_t = (\varepsilon l q - p_t w l q)/p_t K q \tag{7}$$

$$C'_t = M'_t = p_{t+1} q = (1 + r_t) p_t K q \tag{8}$$

$$\pi_t = (r_t) p_t K q \tag{9}$$

$$m_{t+1} = p_{t+1} q - p_{t+1} K q \tag{10}$$

Let $\hat{q}$ be an $n \times n$ diagonal matrix, with gross outputs along the main diagonal. By substituting $\hat{q}$ for q, except where q appears as an argument in the rate of profit, the corresponding *sectoral aggregates* are obtained.

By eliminating each q (except, again, where it appears as an argument in the rate of profit), the corresponding *unit magnitudes* are obtained. For example, as expressed in money the unit values of output are

$$\varepsilon v_{t+1} = p_t A + \varepsilon l \tag{6'}$$

and unit prices of production are

$$p_{t+1} = (1 + r_t) p_t K \tag{8'}$$

III. It is easy to show that Marx's three aggregate equalities hold in each period, again assuming for simplicity a constant monetary expression of value. First, the sum of prices in period t, equation (8), is

$$(1 + r_t) p_t K q$$

which (using (7)) equals

$$p_t\mathrm{A}q + \varepsilon lq$$

the money expression of the sum of values in period *t*, equation (6). Second, the sum of profit in period *t*, equation (9), is

$$(r_t)p_t\mathrm{K}q$$

which (using (7)) equals

$$\varepsilon lq - p_t wlq$$

the money expression of the sum of surplus value in period *t*, equation (5). Third, the general 'price of production rate of profit' in period *t* is

$$(p_{t+1}q - p_t\mathrm{A}q - p_t wlq)/p_t\mathrm{K}q$$

and since $p_{t+1}q$, the sum of prices, equals

$$p_t\mathrm{A}q + \varepsilon lq$$

the 'price of production' rate equals the 'value rate of profit', equation (7). (In another sense, as a ratio of money magnitudes, (7) is a 'price' rate of profit. Division of both numerator and denominator by ε yields the 'value' (labour time) rate.

NOTES

[1] See De Angelis (1994) for a well developed critique, similar to our own, of both poles of the technological/social dichotomy.

[2] Cf. Marx's fourth thesis on Feuerbach. Feuerbach 'resolv[ed] the religious world into its secular basis. But that the secular basis detaches itself from itself and establishes itself as an independent realm … can only be explained by the cleavages and self-contradictions within this secular basis [which] must then itself be destroyed in theory and in practice'.

[3] '[I]f the commodity has a double character … then labour contained in the commodity must also be of double character, while mere analysis of labour as such, as with Smith, Ricardo, etc., must everywhere come up against the inexplicable. This is indeed the whole secret of the critical conception' (Marx to Engels, 8 January 1868).

[4] For further discussion of the inner nature of capital and its relation to competition, see Andrew Kliman's chapter in this volume.

[5] This phrase (without emphases) is part of the title of *Capital*, Volume III, Chapter 9.

[6] For just one example, see Marx (1976a:417).

[7] For instance, one could let total value equal the total price of means of production and subsistence, thereby showing that profit is a pure markup on top of commodities' real value. Though this would violate the entire spirit of Marx's theory, it constitutes a 'solution' to the transformation problem which is no less legitimate than any other.

[8] Marx, quoted in Lukács (1971:9). See Nicolaus's translation in Marx (1973:88). Ironically, though Samuelson (1971) himself does not recognize the 'organic union' of values and prices, his suggestion that an eraser be used to effect the transformation indicates that he considers the use of normalization conditions to be an arbitrary way of relating values to prices.

[9] Let q_{t-1} be a column vector of gross outputs produced in period $t - 1$. In a closed, purely capitalist society, its full realization requires the recommitment to production of one portion, K_t, as means of production and subsistence, and capitalists' consumption of the rest, $\mathrm{N}_t = q_{t-1} - \mathrm{K}_t$. For *any* (row) vector of input prices, p_t (which are also the output prices of *t*–1), $p_t q_{t-1} - p_t\mathrm{K}_t = p_t\mathrm{N}_t$; that is, the revenue remaining after advances to production is sufficient to buy the remaining output. Note also that simple reproduction is just a special case of the above, so it can in principle take place at *any* prices, and not only prices that equate capitalists' consumption expenditures to total surplus value.

[10] It is clear that Marx (1976a:711, 716) regarded reproduction as a continuously renewed process taking place in real time. His verbal discussions of input price transformation (Marx 1981:265; and Marx 1972:167-68) treat it, too, as a historical process taking place in real time.

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[11]   All attempts to prove internal inconsistency in Marx's account alter the *problem* his illustration is made to address without permitting the *illustration* to be modified accordingly. We therefore consider these proofs illegitimate.

[12]   As a moment in the circuit of *productive* capital, its role in Table 1, m is used by Marx to denote 'the capitalist's revenue' (Marx 1978:149, also 152), 'the money that the capitalist spends, whether on commodities as such or on services, for his esteemed self and family' (Marx 1978:146). In the circuit of *money* capital, he uses m to denote surplus value. To avoid confusion, we use s instead.

[13]   In any period t, total value (and total price) equals $MP_t + LL_t$, so the change in total value (and total price) between periods t and $t+1$ equals $(MP_{t+1} + LL_{t+1}) - (MP_t + Ll_t)$. Capitalists' revenue between periods is $m_{t+1} = (MP_t + LL_t) - (MP_{t+1} + L_{t+1})$, so the change in total value (and total price) also equals $Ll_{t+1} - m_{t+1} - L_{t+1}$. Hence, the change in total value (and total price) between periods is due only to a difference between the quantum of new value entering the circuit of capital through extraction of living labour and the quantum of existing value exiting the circuit through capitalists' and workers' consumption.

[14]   'The workers must work for a greater or lesser amount of time in order to buy back these commodities (to replace them) and must therefore *perform more or less necessary labour* than would be needed if the prices of production of their necessary means of subsistence did coincide with their values' (Marx 1981:309, emphasis added).