The Determination of the Average Profit Rate in the Production Price Equation: A Linear Solution to the Transformation Problem

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Abstract

The aim of this paper is to demonstrate that, even if Marx's solution to the transformation problem can be modified, his basic conclusions remain valid. The proposed alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices. It will be demonstrated that the general average profit rate is entirely determined from the parameters of the linear structure of the model and the labor values alone. It will also demonstrated that its value is imposed as constraint in the determination of the production price vector. This solution diverges from the Dumenil-Foley-Lipietz solution on the assumption of the money wage rate which is assumed here as an endogeneous variable while the general profit rate is assumed exogeneous. The money wage level is determined by competition on the labor market. It is no more assumed to be fixed by the value of a subsistence basket, although it is still the case in the labor space. This is also quite different from the Morishima or the neo-ricardian solution, which has accepted the samuelsonian postulate of independence between the two spaces. The assumption of a real wage rate common to both spaces is replaced by the assumption of a common general profit rate already calculated in the labor value space. This solution is an alternative to Marx's solution because it fully transforms the cost of production and maintains the two macro constraints and a general profit rate between the monetary and the social spaces.

Key words: Transformation, value, price, surplus value, profit, wage, capital, labor, equilibrium

JEL classification: B-14, B-24, D-33, D-46, D-57, E-11, P-16.

Introduction

The debate around the transformation problem is more than a century old and it continues to fuel passionate discussions among marxian and neo-ricardian economists and, to a lesser extent, mainstream economists, especially since Samuelson (1971) outlined the independence between the two value systems [1]. The latter views it as a good illustration of the redundancy of the old ricardian and marxian labor value theory while radical economists, who still believe in the validity of an alternative approach to marginal utility theory, periodically attempt to give an adequate answer to objections formulated against Marx's way of establishing the correspondence between labor values and prices. Moreover, the « new » solutions proposed by Dumenil-Foley-Lipietz (1982) or by the neo-ricardians like Morishima (1973) do not offer an adequate answer to Samuelson's critique. Some radical economists pursue the debate either by looking for a monetary solution in a static framework (Wolff-Roberts-Callari, 1982), or by proposing a dynamic approach (Shaikh 1977, Naples 1989, Freeman 1995, Freeman and Carchedi 1995). As was pointed out accurately by L. Gill (1996, p.532) in quoting M. Desai in Eatwell-Milgate and Newman... "the debate does not seem to end and is viewed as a rare example of a problem which continues to produce new solutions or reformulation of old solutions in a new mathematical language.... [perhaps] there is more than a simple technical question." Indeed, since the first critique by Bohm-Bawerk (1896) and the first algebraic solution by Bortkiewicz (1907), neo-classical economists have attempted to demonstrate the irrelevance of Marx's theory of value, since th aggregate profit rate and production prices can be determined in the monetary space independently fron the labor value theory.

Marx's solution to the transformation problem[2] is based on the hypothesis of the non- transformation of the cost of production (constant and variable costs). It is a simplifying assumption which allows him to concentrate on the distribution of the surplus value and the profit rate in the labor value space and the monetary space. His main interest was to show that, although the profit rate can be different from one sector to another in the value space, the monetary profit rate in a competitive world must be the same in all sectors. The reason why the profit rate varies from one sector to another in the value space is the differences in organic composition of capital due to unequal development between industries. It is also

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[1] This question of independence was outlined right from the start by the first critiques on Marx's transformation method, but Samuelson is the mainstream economist who did the most cogent criticism. For a review of this debate, see in particular Dostaler (1978), Beaud and Dostaler (1993), Eatwell, Milgate and Newman (1987-90), Gill (1996), Laibman (1973), Mandel and Freeman (1984), Samuelson (1971), Steedman (1981), Sweezy (1949). The authors who have contributed with new solutions to the debate are Dumenil (1980,1983), Freeman and Carchadi (1995), Foley (1982,1986), Lipietz (1982,1983), Morishima and Seton (1961), Morishima (1973), Moseley (1993), Naples (1989), Seton (1957), Shaikh (1977), Sraffa (1960), Wolff, Roberts and Callari (1982).

[2] See in particular Marx (1967) Book III, part 1 and 2.

because some sectors do not produce any surplus value, since, according to Marx, value is produced in the productive sectors and is distributed in all sectors, including those in the circulation sphere, such as the financial sector. Marx was well aware that the non transformation of production costs was a simplifying assumption and did not consider that his conclusion would be substantially changed if that assumption were modified[3].

His main conclusion is twofold: on a micro basis, prices differ from values -and this is why a labor value theory is relevant as an explanation- but on a macro basis, there are two macro constraints which ought to be satisfied if the principle of value conservation is to be preserved. The first constraint is that the sum of values must be equal to the sum of prices, i.e. the aggregate gross production must be equal in both monetary and abstract labor spaces. The second constraint is that the sum of profits must be equal to the sum of surplus values.

Since the transformation problem is a static analysis similar to a general calculable equilibrium approach[4], the same amount of value must be preserved in both spaces when one chooses a general equivalent form for measuring prices. There is more than one way of choosing that price standard:

i) Choose a level of expenditure common to both spaces and one macro constraint. This is the Marx solution with the hypothesis of no-cost transformation, but it is also the solution of equal costs as outiled by Wolff-Roberts-Callari (1982) and Moseley (1993).

ii) Choose a level of wage common to both spaces and one macro constraint. This is the Dumenil-Foley-Lipietz solution (1980, 1982, 1982)- which will be called the DFL solution- of the money wage determined exogeneously and fixed to the nominal wage in the value space.

iii) Choose a level of **real wage** common to both spaces and one macro constraint. This is the Morishima solution (1973) of money wages determined simultaneously with prices, under the constraint of equal real wage in both spaces. It is also the typical neo-ricardian solution the first formalizations of which were given by Seton (1957), Sraffa (1960), Morishima-Seton (1961).

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3 Marx (1967), in Book III, chapter 11, p. 165, says ... Since the price of production may differ from the value of a commodity, it follows that the cost- price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it.... Our present analysis does not necessitate a closer examination of this point.

4 The dynamic approach put forward by M. Naples (1989) is an interesting approach which sould be pursued. After all, even neoclassical economists have developed a dynamic general equilibrium approach; but I still consider that Marx's static equilibrium approach has to be addressed adequately in this debate.

Only the Marx solution preserves the same general profit rate between the two spaces and satisfies the two macro constraints, although only one constraint is effective. The other two solutions generate a monetary profit rate which is not equal to the general profit rate in the abstract space. Worse still, the other two solutions cannot satisfy the two macro constraints so dear to Marx and others interested by that question. This implies that, even if the macro constraint of value conservation is preserved (the role of which is to measure the money value of one hour of abstract labor), some monetary surplus value would be created or lost during the transformation process. This would constitute an awkward situation, since the core of the labor theory of exploitation is to explain how the monetary surplus value is created in the value space. Samuelson's sharp criticism of Marx's theory of labor value would still remain valid since he claims that there is no logical connection between the two spaces: the monetary values would be independent of the labor values and, hence, there is no point in having two value spaces to explain what is going on in the real world. This argument will also be one of Steedman's key reasons to reject Marx's labor theory of value in favor of the neo-ricardian approach.

Theoretical foundations of the new approach

The aim of this paper is to demonstrate that, even if Marx's solution can be modified, his basic conclusions remain valid. It will be shown in particular by the three following points:

i) Marx's solution requires the specification of one macro constraint despite the fact that his results satisfy the two macro equalities and a common general profit rate. This conclusion is already well known in the literature, but needs to be restated since it is a very crucial point, as already clearly outlined by Moseley (1993).

ii) The Dumenil-Foley-Lipietz solution (or the DFL solution) requires one macro constraint and satisfies the two macro equalities if the value added is chosen, although the general profit rate is different between the two spaces. The result is not valid any more if gross value is chosen as the macro constraint. The Morishima solution yields a similar result, even with the value added.

iii) The new alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices and will produce results which satisfy the two macro constraints.

The justification for this particular choice of hypotheses is based on the return to Marx's approach instead of the neo-ricardian approach to the transformation problem. The neo-ricardian approach leads to the independence between the two value spaces and, ultimately, to the rejection of the labor value theory, which is considered as redundant to the monetary value of the production prices and the average profit rate. Marxian theorists oftenly complained and still complaint about the use of the linear theory of production as an abuse of Marx's thinking.[5] The argument can be summarized into eight points.

i) Marx's aim was to demonstrate that the distribution of surplus value (under the form of profit, interest and rent) is dependent of the average general profit rate, which is determined in the first instance in the production space, since production is logically prior to distribution.

ii) There cannot exist two different general profit rates between the two value spaces.

iii) The determination of the average profit rate in the labor value space by the ratio of the surplus value to total capital in general (or capital owned by the capitalist class) is logically prior to the money profit rate and it can be calculated without any reference to monetary magnitudes.

iv) The solution to the transformation problem must therefore be constructed on the interdependence between the two value spaces and hence reflect the prior determination of the average profit rate in the labor value space which is used as a constraint in the determination of the monetary prices and wages.

v) Although the wage rate is determined in the labor value space by its relation to a subsistence basket, the income distributed in the monetary space is not according to a subsistence basket but a certain sum of money which can be used to satisfy a flexible demand. To that extent, the DFL contribution is a step in the right direction. However, the money wage can differ from its labor value as any other price, and there is no necessity of maintaining the hypothesis of a common real wage between the two value spaces, as it is assumed by the neo-ricardians.

vi) This last hypothesis concerning the relative autonomy of the money wage is the core of the new solution presented in this paper and represents a total reversal of position with respect to the neo-ricardian solution, which has always assumed the existence of a rigid subsistence basket as part of the augmented matrix of technical coefficients in the production price equation.

vii) The substitution of the average profit rate by the wage rate as an endogeneous variable in the production price equation allows for solving adequately the transformation problem since, with this new solution, all production costs are transformed (and not only the constant cost) and Marx's two macro equalities are preserved.

viii) Since the two macro constraints are satisfied, a third equality can be derived: total costs of production between the two spaces are equal! This last result is in accordance with the monetary approach of Mosely (1993) and Wolff-Callari-Roberts (1982). Moreover, this solution shows that there is no more inconsitency between the linear production approach or the general equilibrium approach and the Marx approach. Indeed, since the neo-ricardian critique, many economists of all tendencies had abandonned the hope of finding an adequate solution (that is without rejecting the labor theory of value) through the general equilibrium approach.

The linear production model

Let the following linear production model

(1) $x = Ax + C$

(2) $x_0 = b'x$

x is an output vector nx1, A is an nxn matrix of technical coefficients of type a_{ij} which measures the quantity of input i per unit of output j, C is an nx1 vector of value added, b is an nx1 vector of technical labor coefficients where each element b_j measures the quantity of living labor necessary per unit of output and x_0 is a scalar which measures the total quantity of living labor necessary for the production of x.[6] The solution of this system is

(3) $x = (I-A)^{-1} C$.

The dual in the labor value space (or abstract labor) is

(4) $\theta = A'\theta + b$

where θ is an nx1 vector of commodity values measured in abstract labor units. Given the exploitation rate e, the wage rate μ (or the necessary labor in one hour of labor) is also known [7], i.e. $\mu = (1+e)^{-1}$. Hence,

(5) $\theta = A'\theta + \mu b + (1-\mu)b$.

The usual hypothesis accepted by marxians and neo-ricardians is

(6) $\mu = c'\theta$

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[5] See in particular Moseley (1993) and Moseley-Campbell (1997).

[6] Knowing the distribution rate of the net product s, the C vector could be split into two components: sC = saved commodities and (1-s)C = consummed commodities.

[7] By definition, for each hour worked, there is e = (non necessary time)/(necessary time) = $(1-\mu)/\mu$. Hence, $\mu = (1+e)^{-1}$.

where c is an nx1 vector of coefficients associated with the consupmtion goods of a subsistence basket. $c'\theta$ is therefore the value of the subsistence basket contained in one hour of abstract labor.

The solution to this dual is

(7) $\theta = (I - A')^{-1} b.$

The definition of an average general profit rate in the labor value space is the ratio of total surplus value over capital as a whole. By transposing (4) and postmultiplying by x, the following global equation is obtained

(8a) $\theta' x = \theta' Ax + b' x$

or

(8b) $\theta' x = \theta' Ax + \mu b' x + (1-\mu) b' x.$

The average general profit rate is

(9) $r = (1-\mu) b' x / (\theta' A + \mu b') x = (1-\mu) b' x / b' [(I-A)^{-1} A + \mu I] x.$

As it can be seen, this general average profit rate can be calculated from the technical parameters A, b, μ and from the output vector x. There is no need to refer to production prices or to other monetary values. Since the labor value space must be linked to the monetary value space, there is no rationale to determine it again in the monetary space**. It must be imposed to the monetary space.** If redundancy exists between the two value spaces, it is the profit rate determined in the monetary space which is redundant: there are not two different average profit rates. This is the main error of interpretation in the neo-ricardian solution and it is also the weakness of the « new solution » DFL.

The production price equation

The production price equation with an average profit rate is

(10) $p = (1+r)(A' p + wb).$

a) Marx's solution

Marx's solution is based on the two following hypotheses:

i) value is preserved, i.e. $\theta' x = p' x$

ii) the general profit rate is the same between the two spaces and preserves the equality between the sum of profits and the sum of surplus value and the equality of the total costs of production between the two spaces.

Marx's production price equation is

(11) $p'x = d(p'A + wb')x$, $d = (1+r)$

Since costs are equal and not transformed from the labor value space to the monetary space, costs can be replaced by

(11a) $p'x = d(\theta'A + \mu b')x$.

After eliminating x on both sides, the equation is

(12) $p' = d(\theta'A + \mu b')$.

By substituting (7) into (12), the price equation becomes

(12a) $p' = db' ((I - A)^{-1} A + \mu I)$.

Hence, Marx's production prices are a function of (r, A, b, μ). But since r is itself a function of (A, b, μ, x), the price vector is therefore a function of the same parameters and the quantity vector x.

b) The neo-ricardian solution

The usual assumption made by the neo-ricardians is the wage rate equal to the monetary value of the subsistence basket, i.e. $w = p'c$. Substituting this hypothesis into the production price equation, the following system is derived

(13) $p = (1+r)(A'p + bc'p) = (1+r)Mp$

(14) $p'x = \theta'x =$ constant.

Posing $\nu = (1+r)^{-1}$, under the hypothesis that p and M are semi-positive, the Frobenius theorem asserts the existence of one latent vector corresponding to the maximun value of the latent root ν^*. Given the additionnal macro constraint (14) which determines one of the prices (by fixing the monetary equivalent of one hour of labor), the price vector is entirely determined. There is no need to go through the labor value theory in order to calculate a general profit rate and the associated production price vector. This is the neo-ricardian viewpoint, but it is not the viewpoint of Marx and of many other theorists who defend the interdependence between the two value systems. There is more than a purely technical question of the best representation: there is a fundamental ideological difference.

c) The DFL solution

The DFL solution is an improvement with respect to the neo-ricardian solution. The wage rate is assumed exogeneous with respect to the price system. The originality of the DFL approach is to fix the wage rate equal to the one already determined in the labor value space and standardize the solution with respect to the value added in the two spaces. The DFL solution is therefore built on the following system:

(10) $p = (1+r)(A'p + wb)$

(15) $p'x - p'Ax = \theta'x - \theta'Ax = b'x$.

After transposing, the last equation is

(16) $x'(I - A')p = x'b$.[8]

This solution has the great merit to preserve the equality between the sum of profits and the sum of surplus value because, with $w = \mu$ and given (15), it follows

(17) $p'x - p'Ax - wb'x = \theta'x - \theta'Ax - \mu b'x$.

The major flaw with this solution is that the general rate of profit is not preserved equal between the two spaces. Moreover, there is no specific reason to prefer the definition of the monetary unit from the value added instead of the gross value, since, in the abstract labor value space, one hour of labor incorporated in constant capital has the same value as one hour of direct or living labor. Therefore, if standardization is made from gross value (14), the sum of surplus value is not preserved any more in the monetary space. Finally, as outlined by Moseley (1993), there is no reason to consider differently constant capital and variable capital. After all, both enter the production cost, and, if it is important to preserve the same amount of value added between the two spaces, it should be also possible to preserve the same amount of value for total costs between the two spaces. The new solution which will be introduced in the next section will take into account these two objections.

The new solution (Loranger)[9]

A breakthrough has already been made with the interpretation of the neo-ricardian approach by the DFL solution showing the assumption that w is determined outside the production price system. It is an excellent idea, but there is an error in the choice of variables**: it is the profit rate and not the wage rate which is exogeneous to the price system!** Indeed, it is not necessary to preserve the value of the subsistence basket in the monetary space, because the labor force is a commodity as any other commodity, and its production price can differ from its value, the latter being always equal to the labor value of a subsistence basket. . The advantages of this new approach are numerous:

i) Prices and wage differ from values, i.e. $p_i \neq \theta_i$, $\forall$ i = 1, ...,n and $w \neq \mu$.

[8] This equation is used to isolate one particular price with respect to other (n-1) prices. The system (10) is solved by substitution by eliminating first the scalar (1+r) from each price equation and by substituting the isolated price already obtained from (16). The solution is non linear with respect to one particular price.

[9] This solution has already been presented on two occasions: International Conference on Politics and Languages of Contemporary Marxism, University of Massachusetts, Amherst, Dec. 5-8, 1996 and at the Mini Conference on Value Thory, Eastern Economic Association Convention, Washington, April 3-6 1997. Both papers are available at the Department of Economics, University of Montreal (working papers 9625 and 9703).

ii) The two macro equalities are satisfied, because the simultaneous use of the gross value constraint (14) and the equality of the general average profit rate $r = r'$ (where r' is the profit rate in the monetary space) preserves the sum of surplus values and the sum of profits between the two spaces.

iii) If gross values and surplus values (profits) are preserved between the two spaces, it follows that the sum of total costs is also maintained equal.

A) Formal solution

The production price equation is

(10) $p' = d(p'A + wb'), \quad d = 1 + r$

(14) $p'x = a, \quad a = \theta' x.$

After re-arranging (10),

(10a) $p'(I - dA) = dwb'.$

Post-multiplying by $(I - dA)^{-1}x$,

(10b) $p'x = dwb'(I - dA)^{-1}x.$

Substituting (14) in (10b), w is calculated as

(18a) $dwb'(I - dA)^{-1}x = a$

(18b) $w = a / db'(I - dA)^{-1}x.$

Substituting (18b) in (10b) after elimination of x, the price vector p' is

(19) $p' = a\, b'(I - dA)^{-1} / b'(I - dA)^{-1}x.$

Therefore, in the monetary space, prices and wage can be specified by the following function:

(20) $(p' \quad w) = f (r, \theta, A , b, x).$

Interdependence between the two spaces is well illustrated by this function.

B) Verification of the macro constraints

The first constraint -preservation of the gross value - is obvious since the equality (14) is part of the solution. The preservation of the second constraint -equality between the sum of surplus values and profits- is implicit with the constraint of a common general rate of profit between the two spaces. According to (17), the sum of surplus values and profits are written respectively as

	Labor value space	Monetary space
(17)	$\theta'x - (\theta'Ax + \mu b'x)$	$p'x - (p'Ax + wb'x)$

The profit rate is

(21a) $r = [\theta'x - (\theta'Ax + \mu b'x)] / [\theta'x -(\theta'x - \theta'Ax - \mu b'x)]$

$r' = [p'x - (p'Ax + wb'x)] / [p'x - (p'x - p'Ax - wb'x)]$

(21b) $r = [\theta'x - (\theta'Ax + \mu b'x)] \;/\; (\theta'Ax + \mu b'x) \qquad r' = [p'x - (p'Ax + wb'x)] / (p'Ax + wb'x)$

(21c) $1 + r = \theta' x / (\theta' Ax + \mu b' x)$ $\quad$ $1 + r' = p' x / (p' Ax + wb' x)$

Since$(1 + r) = (1 + r')$ and $\theta' x = p' x$ by hypothese, it follows

(22) $$(\theta' Ax + \mu b' x) = (p' Ax + wb' x)$$

that is

(total costs in abstract labor) = (total costs in money).

Substituting (22) into (17), it follows that the sum of surplus values is equal to the sum of profits.

Conclusion

The proposed new solution diverges from the DFL solution because the average general profit rate is assumed exogeneous in the monetary space while it is the wage rate which is assumed endogeneous as the other prices. The profit rate is entirely determined in the value space and is a function of all the technical coefficients and the output quantity vector. Because production is prior to distribution, the general average profit rate is logically determined prior to the production prices and is used as a constraint in the solution of the system on the monetary side, since it is impossible to have two different general average profit rates in a static general equilibrium system. In a dynamic approach, there would exist the possibility to introduce a time lag function between production and distribution as already outlined by Foley (1982) and Loranger (1991). It would then be more obvious that distribution follows production and prices are determined in the realization phase with the possibility of a crisis or distortion between production and demand. The equalization of the wage rate to the subsistence level is possible in the abstract labor value space but not in the monetary space. This contradicts the neo-ricardian solution which imposes the equalization of the real wage in both spaces to the value of a subsistence basket. The imposition of the equality of the general average profit rate coupled with the equalization of the gross value constraint in both spaces re-establishes the interdependence between the two spaces and satisfies Marx's two macro equalities while transforming all costs and not only the constant cost, as in the DFL solution. Samuelson's critique as well as the critique of other neo-ricardians are not addressing Marx's problem but an invented problem!

Appendix

Numerical example with a three sector model

In prior presentations, Loranger (1996, 1997), a numerical example was used with a two sector economy from a model that was invented with my students. Some small excentricities were introduced such as two different rates of exploitation and wage rates in the labor value space. Some critics found that more confusing than anything else. So, in this numerical example, I will revert to the very classical model first formulated by Tougan-Baranosky (1905), and re-utilized by Bortkiewciez (1907) and Samuelson (1971). The value tableau contains figures associated to the case of simple reproduction (or zero growth model).

Table 1
A three sector model of simple reproduction.
(Units are in abstract labor time)

sector or department	constant capital(c)	variable capital(v)	surplus value(s)	total
I Production goods	225	90	60	375
II Basic goods	100	120	80	300
III Luxury goods	50	90	60	200
Total	375	300	200	875

As it can be seen, the total input required for constant capital is equal to the total output of sector I. Similarly for the two other sectors or departments: the total value added or labor force required (v+s) = (300+200) is equal to the goods consumed and produced (whether basic or luxury). The exploitation rate $e = s/v = 2/3$ in each sector. Therefore, the wage rate $\mu = (1+e)^{-1} = 3/5 = .60$. Since the organic composition $\gamma_i = c_i / v_i$ is different for each sector, (i.e. 2.5, 5/6, 5/9), the profit rate earned by each department is different, (i.e. 4/21, 4/11, 3/7). The general average profit rate is r = 200/675 = .2963. In order to use the linear production model of input-output, it is necessary to formulate some hypothesis about the distribution of constant capital among the three sectors. Since each c is defined by the following equation

$c_i \;=\; a_{1i}\,\theta_1\, x_i + a_{2i}\;\theta_2\, x_i \;+ a_{3i}\,\theta_3\, x_i\;, \quad i = 1,2,3,$

it is possible to determine only three a_{ji} coefficients out of a total of nine. Therefore, to make it easier for calculation, it will be assumed that only the three elements on the diagonal matrix A are different from 0. Of course, the assumption of a different technical matrix would lead to different computation, but the basic results would remain the same. Moreover, it will be assumed that the output quantity vector is simply defined as x' = (1000 1000 1000). Therefore, the other information necessary to the illustration of the new solution is summarized in table 2.

Table 2

Value and technical coefficients[10]

sector	output vector x	abstract labor value θ	capital input a_{ii}	labor input b
I	1000	0.375	0.60	0.15
II	1000	0.300	0.33	0.20
III	1000	0.200	0.25	0.15

A) The Marx solution

According to equation (12), Marx's solution is

$p' = d(\,\theta'A + \mu b'\,)$, where $d = (\,1 + r\,) = 1.2963$ and $\mu = 0.6$.

Hence,

$$p' = 1.2963\,[(\,.375\;.30\;.20\,)\begin{vmatrix} .60 & & \\ & .33 & \\ & & .25 \end{vmatrix} + .6\,(\,.15\;.20\;.15\,)] = (\,.408\;.285\;.182\,)$$

Therefore, assuming that monetary equivalent of one unit of labor is unity, the gross value p'x = 875 is also equal to the gross value θ'x in the labor space. Moreover, the total cost is by definition (θ'A +

[10] The calculation of the elements on the diagonal matrix is a_{11} = 225/375 = .6, a_{22} = 100/300 = .33, a_{33} = 50/200 = .25. The calculation of the elements of the b vector is simply the value added in each sector / 1000.

μb')x = [(.225 + .09) + (.10 + .12) + (.05 + .09)] 1000 = 675. Hence, the sum of profits is (875 - 675) = 200 which is also the sum of surplus values.

B) The new solution (Loranger)[11]

From equations (18b) and (19), w and the price vector p are

(18b) $w = a / db'(I - dA)^{-1}x$

(19) $p' = a\, b'(I - dA)^{-1} / b'(I - dA)^{-1}x.$

Therefore, the following matrices have to be computed:

$$(I - dA)^{-1} = \begin{vmatrix} 1 - 1.2963 (.60) & & \\ & 1 - 1.2963 (.33) & \\ & & 1 - 1.2963 (.25) \end{vmatrix}^{-1}$$

$$= \begin{vmatrix} 4.5\,00 & & \\ & 1.7608 & \\ & & 1.4794 \end{vmatrix}$$

$b'(I - dA)^{-1} = (.6750 \quad .3522 \quad .2219)$

$b'(I - dA)^{-1}x = 1249$

$w = a / db'(I - dA)^{-1}x = 875 / 1.2963 (1249) = .5404$

$p' = a\, b'(I - dA)^{-1} / b'(I - dA)^{-1}x = 875 (.675 \quad .352\ .222) / 1249 = (.473 \quad .247 \quad .155)$

Hence,

$p'x = (.473 \quad .247 \quad .155)\, 1000 = 875$. Therefore, the gross value in both spaces is equal. The equality of profits and surplus value is also verified. Indeed, the total surplus value is

$(1 - \mu)b'x = 0.4(.15 \quad .20 \quad .15)1000 = 200.$

The total profit is calculated from the following matrix product: $r(p'A + wb')x$

$$p'A = (.473 \quad .247 \quad .155)\begin{vmatrix} .60 & & \\ & .33 & \\ & & .25 \end{vmatrix} = (.2838 \quad .0823 \quad .0388)$$

[11] Since the DFL solution is also identified as the new solution, it would be a little more cumbersome to compute by hand because it would be required to slove a cubic equation for the determination of the price vector and the average profit rate. This is another advantage of the new solution proposed since it can be computed by linear matrix algebra as the Marx solution.

$wb' = .54(.15\ .20\ \ \ .15) = (.081\ \ \ .108\ \ \ .081)$

$r(p'A + wb')x = .2963(.3648\ \ \ .1903\ \ \ .1198)!000 = 200.$

The equality of total costs is also verifed from equation (22):

$(\theta'Ax + \mu b'x) = \ \ (.315\ \ \ .220\ \ \ .140)1000 = 675$

$(p'Ax + wb'x) = (.365\ \ \ .190\ \ \ .120)1000 = 675.$

Bibliography

Beaud M. and Dostaler G. 1993. La pensée économique depuis Keynes, Paris, Seuil.

Bohm- Bawerk E. von 1896. « Zum Abschluss des Marxschen Systems », in P. Sweezy « Karl Marx and the Close of his System by Eugen Von Bohm-Bawerk » Clifton, A.M. Kelly, 1949

Bortkiewicz L. von 1907. «Zur Berechtigung der Grundlegenden Theoretischen Konstruction von Marx im Dritten Band das Kapital » in P. Sweezy « On the Correction of Marx's Fundamental Theoretical Construction in the Third Volume of Capital » Clifton, A.M. Kelly, 1949.

Dostaler G. 1978. Valeur et prix. Histoire d'un débat, Grenoble, Montreal, Presses de l'Université de Grenoble et Presses de l'Université du Québec.

Dostaler G. and Lagueux M. 1985. Un échiquier centenaire, Paris-Sainte-Foy, La Découverte-Presses de l'Université du Québec.

Duménil G. 1980. De la valeur aux prix de production, Paris, Economica.

Duménil G. 1983. " Beyond the transformation riddle: a labour theory of value ", Science and Society, vol. 47, no 4, p. 427-450.

Eatwell J., Milgate M. and Newman P. (dir), 1987-1990. The New Palgrave. Marxian Economics, London, Norton.

Foley D. 1982. « Realization and Accumulation in a Marxian Model of the Circuit of Capital », Journal of Economic Theory, vlo. 28, pp. 301-319.

Foley D. 1982. " The value of money, the value of labour power and the marxian transformation problem ", Review of Radical Political Economics, vol. 14, no 2, p. 27-47.

Foley D. 1986. Understanding Capital. Marx's Economic Theory, Cambridge Mass., Harvard University Press.

Freeman A. 1995. « Marx without equilibrium », Capital and Class, 56, Summer 1995, p. 49-90.

Freeman A., Carchedi G. (eds) 1995. Marx and non equilibrium economics, London, Elgar

Gill L. 1996. Fondements et limites du capitalisme, Montreal, Boréal.

Gleicher D. 1989. “ Labour specialization and the transformation problem ”, Review of Radical Political Economics, vol.21, no 1-2,p. 75-96.

Laibman D. 1973. “ Value and prices of production: the political economy of the transformation problem ”, Science and Society, vol. 4, p. 404-436.

Lipietz A. 1982. “ The so-called transformation problem revisited ”, Journal of Economic Theory, vol. 6, no 1, p. 59-88.

Lipietz A. 1983. Le monde enchanté: de la valeur à l’envol inflationiste, Paris, La Découverte/Maspéro.

Loranger J.G. 1991. « Modelling the Soft Budget Constraint: Inflation Explained by a Diequilibrium in the Circuit of Capital », Metroeconomica, vol. 42, pp. 71-92.

Loranger J.G. 1996. « The transformation problem: an alternative solution with an identical aggregate profit rate in the labor value space and the monetary space », Working paper 9625, Département de sciences économiques, Université de Montréal.

Loranger J.G. 1997. « The wage rate and the profit rate in the price of production equation: a new solution to an old problem », Working paper 9703, Departement de sciences economiques, Universite de Montreal.

Mandel E. and Freeman A. (dir), 1984. Ricardo, Marx, Sraffa, London, Verso.

Marx K. 1967. Capital, Vol.I-III, New-York, International Publishers.

Moseley F. (ed) 1993. Marx’s Method in Capital. New-Jersey, Humanities Press.

Moseley F., Campbell M (ed) 1997. New Investigation in Marx’s Method, New-Jersey, Humanities press.

Morishima M. and Seton F. 1961. “ Aggregation in Leontief matrices and the labour theory of value ”, Econometrica, vol. 29, no 2. p. 203-220.

Morishima M. 1973. Marx’s Economics, Cambridge, Cambridge University Press.

Naples M. 1989. “ A radical economic revision of the transformation problem ”, Review of Radical Political Economics, vol. 21, no 1-2, p. 137-158.

Robinson J. 1942. An Essay on Marxian Economics, London, Macmillan.

Robinson J. 1950. “ The labour theory of value ”, Economic Journal, reprinted in Collected Economic Papers, Cambridge, Mass., MIT Press, vol. 1, p. 146-151.

Samuelson P. A. 1957. “ Wages and interest: a modern dissection of marxian economic models ”, American Economic Review, vol.47, no 6, p. 884-912.

Samuelson P.A. 1971. “ Understanding the marxian notion of exploitation: a summary of the so-called transformation problem between marxian values and the competitive prices ”, Journal of Economic Literature, vol. 9, no 2, p. 399-431.

Seton F. 1957. “ the transformation problem ” Review of Economic Studies, vol. 24, p. 149-160.

Shaikh A. 1977. "Marx's theory of value and the transformation problem", in The Subtle Anatomy of Capitalism, J. Schwartz, ed., p. 106-139.

Shaihk A. 1982. “ Neo-ricardian economics: a wealth of algebra, a poverty of theory ”, Review of Radical Political Economics, vol. 14, no 2, p. 67-83.

Sraffa P. 1960. Production of Commodities by Means of Commodities, Cambridge, Cambridge University Press.

Steedman I. (ed)1981. The Value Controversy, London, Verso and New Left Books.

Sweezy P. M. (ed) 1949. Karl Marx and the Close of His System by Eugen Bohn-Bawerk and Bohn-Bawerk's Critism of Marx by Rudolf Hilferding, Clifton, A.M. Kelly.

Tougan-Baranovsky M. 1905. Theoretische Grundlagen des Marxismus, Leipzig, Dunker and Humbolt.

Wolff R., Roberts B. et Callari A. 1995. « Marx's (not Ricardo) « transformation problem »: a radical conceptualization », History of Political Economy, 14, no 4, p.564-582.