

RATE OF PROFIT AND SURPLUS IN THE CLASSICAL THEORY OF PRICES

(Working paper)

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In the contemporary classical theory, the rate of profit is usually related to the surplus product. In this paper we intend to show on the basis of the concept of the homothetic system¹ that in the classical theory of prices, the general profit rate is linked to surplus labor and profits come from the exploitation of workers. It is thus corroborated that the Ricardian idea according to which profits depend on the quantity of labor that society assigns to the maintenance of workers has a general scope, not being restricted to the particular case of systems where the value composition of capital is uniform.

1. The classical system of prices of production

The classical system of prices of production where wages are advanced by capitalists, is:

$$(1 + r) (\mathbf{A} \mathbf{p} + \mathbf{l} w) = \mathbf{p} \quad (1)$$

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¹We label homothetic system as a system in which the physical rate of surplus is the same for all commodities.

where $\mathbf{A} = [a_{ij}]$ is a nonnegative and $n \times n$ technical coefficients matrix and \mathbf{l} is a positive homogeneous labor vector. Unlike matrix \mathbf{A} , vector \mathbf{l} is not a purely technical datum because it is obtained aggregating concrete labors via their wages. We express these wages in terms of the wage mass of the economy. In this case, vector \mathbf{l} indicates the distribution of homogeneous labor among sectors and we have then:

$$\mathbf{u}'\mathbf{l} = 1 \quad (2)$$

where \mathbf{u}' is the unit vector (row). The wage rate w that appears in the classical system of prices represents then the wage mass of the economy.

This system has a degree of freedom, therefore a distribution variable must be fixed exogenously. Traditionally, the independent variable is the wage conceived as a basket of goods. In this case, the equation added to the price system is:

$$w = \mathbf{s}'\mathbf{p} \quad (3)$$

where \mathbf{s}' represents the nonnegative wage-goods vector (row). To avoid the criticism implied by the identification of these goods with the workers consumption, we propose to reinterpret them as the measure of wages. In this framework, vector \mathbf{s}' is the result of a negotiation between capitalists and workers that leads to a double social agreement concerning two distinct aspects of the wage rate: its level and the composition of the basket in terms of which it is measured. Owing to (2), vector \mathbf{s}' represents the goods which constitute the wage mass of the economy.

2. Net product, surplus product and surplus labor

In the system of production prices, three concepts of surplus are distinguished: the net product, the surplus product for profits -that from now on we will call surplus product- and the surplus labor.

The net product is a vector whose components indicates the quantities of goods produced but not utilized as means of production, that is:

$$u' [I - A]$$

The surplus product is the vector obtained discounting from the net product of the economy the goods that constitute the wage mass, that is:

$$u' [I - A] - u' L S \quad (4)$$

where L is a diagonal matrix, whose elements on the main diagonal are those of vector l , and S is a nonnegative $n \times n$ matrix, whose rows are all equal to vector s' . Since wages of the different types of labor employed in production are expressed in terms of the wage mass, we have:

$$u' L S = s'$$

The surplus labor is an aggregate that indicates the quantity of social labor -equal to the unity because of (2)-, which is not employed directly or indirectly in the production of the wage-goods, that is:

$$1 - s' [I - A]^{-1} l \quad (5)$$

where $[I - A]^{-1} l$ represents the vector of the quantities of labor employed directly and indirectly in the production of the different commodities.

While the net product is a purely technical datum, the surplus product and the surplus labor also depend, on the one hand, on the wage structure used for homogenizing the different types of labor employed in production, and on the other hand, on the negotiation between classes which defines the vector s' of goods in terms of which wages are measured.

The concepts of surplus product and surplus labor are very closely related, one being the condition of the other. In fact, if wages take in the whole net product, the surplus product does not exist and the surplus labor disappears; that is, if $s' = u' [\mathbf{I} - \mathbf{A}]$, then:

$$1 - s' [\mathbf{I} - \mathbf{A}]^{-1} l = 1 - u' [\mathbf{I} - \mathbf{A}] [\mathbf{I} - \mathbf{A}]^{-1} l = 0$$

On the other hand, surplus product can only exist if there is surplus labor, or in other words, if the production of the goods which constitute the wage mass does not take in, directly and indirectly, the whole labor employed in the economy; that is if $s' [\mathbf{I} - \mathbf{A}]^{-1} l = 1$, then:

$$u' [\mathbf{I} - \mathbf{A}] - u' \mathbf{L} \mathbf{S} = u' [\mathbf{I} - \mathbf{A}] - u' [\mathbf{I} - \mathbf{A}] = 0$$

It follows that the surplus labor and the surplus product are mutually implied. Therefore they are two logically equivalent concepts.

Unlike surplus product that, given \mathbf{A} , l y s' , also changes in face of a modification of the proportions in which commodities are produced, surplus labor does not change as a fraction of the whole labor of the society, whatever those proportions are. In this case, if the quantity of society's total labor remains constant, surplus labor always represents the same quantity of labor, whatever the structure of production is.

3. Surplus product, surplus labor and general profit rate

If prices are expressed in terms of quantities of labor commanded, we obtain from (3) the following equation whose single unknown is the general profit rate:

$$1 = s' \{ (1 + r) [\mathbf{I} - (1 + r) \mathbf{A}]^{-1} \mathbf{l} \} \quad (6)$$

The first member of (6) indicates the whole quantity of labor employed by the society - equal to the unity owing to (2)-, while the second one represents the quantity of labor commanded (in the manner of Ricardo) by the wage mass.

From equation (6) it follows that the condition for the existence of a positive general profit rate is:

$$s' [\mathbf{I} - \mathbf{A}]^{-1} \mathbf{l} < 1 \quad (7)$$

Consequently, the general profit rate will only be positive if the quantity of labor employed in the production of goods that workers receive is less than the quantity of labor that they provide. In other terms, surplus labor constitutes the condition for the existence of a positive general profit rate. But as surplus labor and surplus product are both mutually implied, the condition (7) also signifies that the general profit rate will only be positive if there exists a surplus product.

Now we are going to see how these two surplus concepts are related with the general profit rate. Lets start by showing that there exists a relationship between the magnitude of labor surplus and the level of the general profit rate. In fact, owing to $\mathbf{A} \geq 0$, $s' \geq 0$ y $\mathbf{l} > 0$, $f(r) = s' \{ (1 + r) [\mathbf{I} - (1 + r) \mathbf{A}]^{-1} \mathbf{l} \}$ is an increasing function of the general profit rate, whose second derivative is also positive. Therefore, as we can see on Figure 1, the lesser the quantity of labor employed in the production of goods which constitute the wage mass, that is, the larger the labor surplus, the greater the general profit rate.

Lets examine now if it is possible to establish a similar relationship between the general profit rate and the surplus product, which obviously supposes the evaluation of this set of commodities in terms of prices. We know that for a given technique, a given structure of wages and a given wage, the general profit rate and prices are both independent of the proportions in which commodities are produced. Hence, given the technique, the structure of wages and the wage, the same general profit rate and the same system of prices are compatible with different levels surplus product, depending on whatever the structure of production is. Therefore, it follows the absence of a relationship between, on the one hand, the surplus product and its evaluation in terms of prices, and on the other hand, the level of the general rate of profit.

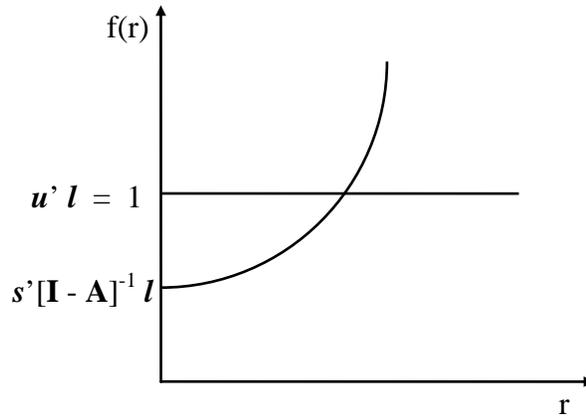


Figure 1

We can ask ourselves if it is possible to go further and determine the general profit rate on the basis of the surplus labor defined in (5), in which case this said rate should be equal to:

$$\frac{1 - s'[I - A]^{-1}l}{u'A [I - A]^{-1}l + s'[I - A]^{-1}l} = \frac{u'[I - A] [I - A]^{-1}l - s'[I - A]^{-1}l}{u'A [I - A]^{-1}l + s'[I - A]^{-1}l} \quad (8)$$

The answer is negative because, in the capitalistic society, goods are not generally exchanged in proportion to the quantity of labor employed in their production. This is owed to the capitalistic surplus distribution norm, that is: the uniformity of the profit rate. Owing to this norm, the general rate of profit is defined in terms of prices, being equal to:

$$r = \frac{u'[\mathbf{I} - \mathbf{A}]p - s'p}{u'\mathbf{A}p + s'p} \quad (9)$$

Thus, profits are not identified with surplus labor but with the evaluation of the surplus product in terms of prices. Therefore, we arrive to the following paradoxical result: surplus labor is, as we have seen, the relevant surplus concept for the general profit rate, but in the definition of this rate what appears is the surplus product evaluated in prices, whose dimension, differing from that of surplus labor, has not any relationship with the level of the general profit rate.

A second important consequence of capitalistic definition of prices is that the quantity of labor employed in the production of wage-goods constituting the wage mass does not represent what workers receive. In this case, surplus labor defined in (5) does not indicate a measure of the exploitation of workers, conceived as the difference between the quantity of labor that this class provides and the quantity that it receives in the form of wage-goods.

From all we have seen it seems that, in the classical system of prices of production, the exploitation of the workers is not the source of profits not only because profits differ from surplus labor but also because surplus labor does not constitute a measure of the exploitation of workers. This inference is opposed to the ricardian idea, exposed in the *Principles*, according to which profits depends on the proportion of annual labor assigned to the subsistence of workers, that is, on surplus labor². But it is well known that Ricardo's argument assumes the proportionality of prices and labor vectors, which is only verified in those systems where value composition of capital is uniform. This leads us to ask

²See Ricardo, D., [1821], chapters 1 and 6.

ourselves if Ricardo's conclusion can transcend the particular case in which its argumentation is based, or in other words, if there exist sufficiently general conditions which permit obtaining the Ricardian result. We intend to show that the concept of homothetic system, built on the basis of Sraffa's work³, provides a positive answer to this question.

4. Homothetic system, profit and exploitation

Lets observe first that equation (8) also can be written in the following way:

$$\frac{u'[\mathbf{I} - \mathbf{A}] [\mathbf{I} - \mathbf{A}]^{-1} l - s'[\mathbf{I} - \mathbf{A}]^{-1} l}{u' \mathbf{A} [\mathbf{I} - \mathbf{A}]^{-1} l + s'[\mathbf{I} - \mathbf{A}]^{-1} l} \quad (8')$$

Consequently, the difference between (8) and (9) resides in the distinct evaluation of the same physical quantities. That is why the source of profit can not be different in both cases. We have then to see in what conditions both evaluations can give the same result so that profit coincides with surplus labor and surplus labor constitutes a measure of exploitation.

The answer is obvious: (8') and (9) coincide, whatever the evaluation of the physical quantities would be, if the net product, the total means of production and the vector of goods in terms of which wages are measured are all composed of the same commodities in the same proportions. That is only possible in a homthetic system in which the wage is measured in terms of the net product and it represents then a quantity of the homothetic commodity. In this case we have:

$$r = \frac{1 - s_h' [\mathbf{I} - \mathbf{Q} \mathbf{A}]^{-1} \mathbf{Q} l}{u' \mathbf{Q} \mathbf{A} [\mathbf{I} - \mathbf{Q} \mathbf{A}]^{-1} l + s' [\mathbf{I} - \mathbf{Q} \mathbf{A}]^{-1} \mathbf{Q} l} = \frac{u' [\mathbf{I} - \mathbf{Q} \mathbf{A}] p - s_h' p}{u' \mathbf{Q} \mathbf{A} p + s_h' p} \quad (10)$$

³See Sraffa, P., [1960].

where Q is a diagonal matrix whose elements on the main diagonal are the multipliers that convert the actual system into a homothetic system which utilizes the whole quantity of labor employed in the economy -that from now on we will call the homothetic system-, and s_h' represents the vector of wage-goods, built from the actual vector s' , when the wage is measured in terms of the homothetic commodity.

Hence, in the homothetic system, the general rate of profit is determined before and independently of prices because (10) it is verified whatever the system of commodity evaluation would be. This rate can be defined indistinctly on the basis of the surplus product or of the surplus labor, under the condition that the wage is measured in terms of the homothetic commodity⁴. Therefore, in the homothetic system, unlike what happens in the actual system, the profit may be identified either with the surplus product evaluated in prices or with the surplus labor.

So too, in the homothetic system, the level of the general profit rate is related not only with the magnitude of the surplus labor, as in the actual system, but also with the dimension of the surplus product, which can be estimated independently of prices when the wage is measured in terms of the homothetic commodity.

Moreover, from (10) it follows that in the homothetic system, if the wage is measured in terms of the homothetic commodity, what the capitalist gain the workers lose, whatever the commodity evaluation system would be: production prices, quantities of labor or whatever price system⁵. This result can be even obtained even on the basis of the physical quantities, without any type of evaluation. In the homothetic system, the exploitation of workers in a capitalistic society is expressed in this way because, even if any type of evaluation is discarded, what both classes receive is defined according to the specific criterions of capitalism: what the workers receive is calculated on the basis of the

⁴The general rate of profit determined in this way allows the definition of the “difficulty of production” of the homothetic commodity, which is equal to $(1/1+r)$, whatever the commodity evaluation system would be.

⁵The advantage of the evaluation in terms of the quantity of labor is that it can be done without knowing the general profit rate.

wage-goods vector that results from the salary negotiation, and what capitalists obtain is profit because it is distributed according to the capitalistic distribution norm: the uniformity of the profit rate.

We can then conclude that, in the homothetic system, profit arises from workers exploitation because it is identified with surplus labor and because surplus labor represents a measure of exploitation. But, as we have seen, in the homothetic system, the surplus product and the surplus labor are at the same level, not only because both are conditions for the existence of profit, with which they can be identified, but also because there exists a direct relation between both and the level of the general profit rate. We have then to see, firstly, which of these concepts of surplus is more pertinent for the actual system and, secondly if we can explain the source of profit in the actual system on the basis of the surplus labor. With this objective, it is necessary to clarify the relationship between the homothetic and the actual system.

5. Profit and exploitation in the classical system of prices of production

Lets point out firstly that the technique is the same in the actual system and in the homothetic system, which only differs from the former in the proportions in which the commodities are produced. Consequently, given the wage measured in terms of the homothetic commodity -defined as we remember on the basis of the wage-goods vector s' that results from the salary negotiation-, both systems have the same general profit rate and the same prices, but different surplus product. On the contrary, as the surplus labor is independent of the proportions, it coincides in both systems when the wage is expressed in terms of same unit of measure. Therefore, if the wage in the actual system is measured in terms of the homothetic commodity, we have:

$$1 - s_h' [\mathbf{I} - \mathbf{Q} \mathbf{A}]^{-1} \mathbf{Q} \mathbf{l} = 1 - s_h' [\mathbf{I} - \mathbf{A}]^{-1} \mathbf{l}$$

Thus, the pertinent concept of surplus profit is surplus labor because, for a given unit of measure of the wage, a given technique and given state of distribution, the surplus labor is always the same in both systems. Furthermore, as we have seen, in the actual system, the surplus labor is related with the general profit rate.

Now, if surplus labor represents the measure of exploitation of workers in the homothetic system when the wage is measured in terms of the homothetic commodity, it must also represent its exploitation in the actual system: on the one hand, because both systems have the same surplus labor when the wage is measured in terms of the homothetic commodity and also the same general profit rate, and on the other hand, because exploitation is independent of the proportions in which goods are produced. In fact, exploitation does not vary if, *ceteris paribus*, workers produce more of one good and less of another. And, if workers are exploited in the actual system for a given vector of wage-goods, defined by the salary negotiation, when the wage is measured in terms of the homothetic commodity, we do not see why they will not be exploited, for the same given vector of wage-goods, when the wage is measured in terms of another unit of measure. Consequently, if the exploitation of workers explains the source of profit in the homothetic system, it must also explain it in the actual system. In conclusion, in the classical system of prices of production, profit arises from exploitation.

Finally, let's point out that during all our reasoning, we have maintained the classical idea of wage as a part of capital advanced and that the measure of the wage in terms of the homothetic commodity does not imply in any way the rejection of this conception of the wage. Our analysis differs then from that of Meek⁶ who assumes that the wage is paid *post factum* and intends to resolve Marx's transformation problem on the basis of the linear relation verified in Sraffa's system between the general profit rate and the *post factum* wage measured in terms of the Standard net product. We think that the homogeneous labor that appears in the classical system of prices of production, obtained aggregating concrete labors via wages, differs from Marx's abstract labor,

⁶See Meek, R. L., [1961].

which impedes the derivation from our analysis of any conclusion concerning the transformation problem.

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