

Labor values and market prices: the Spanish case

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1. The following is a possible way of conventionally transcribing¹ Marx's ideas about values and prices of production into matrix notation (when fixed as well as circulating capital are admitted):

$$\delta = a_0 + \delta \cdot A \quad (1)$$

$$s = a_0 - \delta \cdot B \quad (2)$$

$$g = (s \cdot t) / (\delta \cdot K \cdot t) \quad (3)$$

$$\pi = \delta \cdot (A + B + g \cdot K) \quad (4),$$

where δ is the vector of values; π is the vector of prices of production; a_0 are the coefficients of direct labor; A , B and K are the three usual input-output matrices (intermediate inputs, real wages and capital stocks coefficients, respectively); s is the surplus value; g is the general rate of profit; and t is the number of units produced.

Although it is not peacefully admitted that these expressions are compatible with Marx's thought, it is easy to see that they are at least respectful of the three identities claimed by him, usually summarized by means of simplified expressions such as: "total prices = total values", "total profits = total surplus values", and "the same rate of profit prevails in both spaces (values and prices)". It is easy to see that, even if, in general, individual profits are different from individual surplus values ($s \neq g \cdot K$), and, therefore, individual values are different from individual prices ($B \neq g \cdot K$ because $s + g \cdot (A+B) \neq g \cdot K + g \cdot (A+B)$), the definition of the rate of profit given by Marx suffices to conclude in this way, since from (3) we have:

$$s \cdot t = g \cdot K \cdot t,$$

i.e., total surplus value equals total profit. And therefore, using (2) and the latter equation, we obtain:

$$a_0 \cdot t - \delta \cdot B \cdot t = g \cdot K \cdot t$$

$$a_0 \cdot t + \delta \cdot A \cdot t = \delta \cdot (A+B) \cdot t + g \cdot K \cdot t$$

¹ In terms of the linear production theory.

$$* \cdot t = B \cdot t,$$

i.e., total values = total prices. Of course, there is only one rate of profit, since the global mass of profits is the same amount as the global mass of surplus value, and the denominator is always the overall capital expressed in value terms (according to Marx).

It is well known that most critics accept only the equation of values (1), but not that of prices (4), because they impute Marx the double error of not having transformed the inputs and at the same time having defined the rate of profit incorrectly. So, they prefer to write the equation of the prices of production like:

$$B = B(A+B+r \cdot K) \quad (5)$$

They say the price of the inputs must be transformed (so that it is necessary to use B instead of *), and r has to be defined differently of g (using profits and capital expressed in prices, not in values). They argue that when both errors are corrected, then it is just possible to maintain one of the equalities of Marx ("total prices = total values", or "total profits = total surplus value"), but not the two simultaneously; likewise, they argue that the rate of profit in the value space is always different from the correct one: that defined in the price space.

In my opinion, Marx's thought is not correctly represented by equations (1) to (4) above. Instead, the following system seems to be closer to his ideas:

$$\delta = a_0 + \cdot A \quad (1')$$

$$s = a_0 - \cdot B \quad (2')$$

$$g = (s \cdot t) / (\cdot K \cdot t) \quad (3')$$

$$\pi = \cdot (A + B + g \cdot K) \quad (4'),$$

Like equations (1) to (4), equations (1') to (4') also assure the double equality plus the existence of just one rate of profit in the so-called double space, but this apparent double space comes to be a single one, as we will see below. The only difference appearing in the new set of equations is that \cdot is now the vector of *market* values, which cannot be accused of not having been transformed, since they are the product of a double transformation: from direct values to production values, and from production values to market values. This change suffices to reject at the same time the second criticism, because capital invested is now valuated also in market terms. However, for a better understanding of the argument we are using in this way of thinking, we should start from the development of the conceptual scheme which can be found in Marx himself.

2. In my opinion, most debates in the field of the 'transformation problem' suffer from not distinguishing sufficiently between the different concepts implicit in Marx's thought. Those concepts can be articulated in the following scheme (where more respect is however paid to Marx's ideas than to his own terminology):

Table 1: Marx's conceptual scheme of prices or values.

		A Absolute (in hours) Prices or Values	B Relative (in terms of other things...) Prices or Values		
			...gold;	..commodity j;	...credit money)
C Theoretical Prices or Values	1 Individual	ψ_i	$y_{i0} = \psi_i/\mu_0$	$y_{ij} = \psi_i/\psi_j$	$y_{ib} = \psi_i/\mu_b$
	2 Direct	δ_i	$d_{i0} = \delta_i/\mu_0$	$d_{ij} = \delta_i/\delta_j$	$d_{ib} = \delta_i/\mu_b$
	3 Production	π_i	$p_{i0} = \pi_i/\mu_0$	$p_{ij} = \pi_i/\pi_j$	$p_{ib} = \pi_i/\mu_b$
D Real Prices or Values	4 Market	μ_i	$m_{i0} = \mu_i/\mu_0$	$m_{ij} = \mu_i/\mu_j$	$m_{ib} = \mu_i/\mu_b$

In Marx, the idea that values determine prices seems to mean three things at the same time:

1) Absolute prices determine relative prices, what means that quantities of labor express themselves in practice in monetary terms, and are measured in the market (this sense of the process of determination can be graphically represented as a *horizontal* movement from **A** to **B**).

2) *Real* prices determine *theoretical* prices in the sense that, if it is always true that social reality determines social conscience, then it will be equally true that this fraction of social reality which are real prices (i.e., actual labor and market processes) determines the area of social conscience concerned with the theory of prices. This is the vertical movement which goes from **D** to **C**.

3) Superior (in graphical terms, in the scheme) prices determine inferior prices in the sense that the *logical exposition* of the results attained by the theory of prices makes necessary (or at least adviceable) to derive the different concepts of prices in the sequence 1Ψ2Ψ3Ψ4.

Therefore, in analysing the transformation problem we must take into account those three aspects altogether. For instance, in passing from $*_i$ to p_{ib} we should be careful in order not to confuse p_{ib} with m_{ib} , and in clearly separating the quantitative nucleus of the problem (the different magnitudes of $*_i$ and B_i even if both are measured in hours of labor) from the transition from the absolute to the relative expressions of those magnitudes.

In the latter aspect, it is of the utmost importance to notice that money is different from the other commodities. Since every piece of money does not compete with other pieces, we do not need to distinguish different magnitudes of their value because of the operation of the law of competition, so that we should use the only value (*market value*) of money which appears as relevant in reality, provided the special rôle this commodity plays in the capitalist economy. Moreover, this perfectly conforms² with the *New* (see Foley 1983, 1986) and the *TSS* (see Freeman and Carchedi 1996) Interpretations of the value of money, as the inverse of the *monetary expression of labor time* (the ratio between monetary production and total abstract labor).

3. I believe that table 1 can be used as an argument in favor of the *single* interpretation supported by the *TSS* school. But do not forget there is another crucial aspect (the *temporal* one) involved in *TSS*'s interpretation of the labor theory of value with whom I have much problem. Authors like Husson (see Pérez 1980), Giussani 1993-4, and those included in Freeman and Carchedi 1996, have insisted in the idea that we should not use the kind of equations (1) to (4), nor (1') to (4'), as a correct interpretation of Marx's thought because of the importance Marx attributes to the *dynamic approach* of values, and the (in my opinion apparent) opposition in which linear static equations seem to be in their respect. From a dynamic point of view, historical time gives place to changes in techniques, improvements in the productivity of labor and modifications of unit values, all of which prevent us to use values and prices without specifying a temporal subindex which signals the moment we are referring to. If we pay attention to the fact that inputs were produced in the past and outputs are being produced in the present, we should write something like this in the equation of values:

$$v_{t+1} = a_{0t+1} + v_t \cdot A \quad (6)$$

According to this interpretation, since the inputs represented in matrix *A* were produced in moment *t*, we should apply to them their values at moment *t*, not their present values (in moment *t+1*). This idea should prevent us to use the usual conversion of equation (1) [or (1')] into the expression of values as vertically integrated labor coefficients, since in order to write:

$$v = a_0 \cdot (I-A)^{-1} \quad (7)$$

we first of all need to suppose that $v_{t+1} = v_t$, which is in general false in a dynamic context.

But in fact we do not need to suppose that $v_{t+1} = v_t$. In my opinion, the only thing we need is to realize that present values are the sum of direct plus indirect *present* labors, i.e., we must add to the quantity of direct labor presently necessary the quantity of labor presently necessary to reproduce now the means of production (not the quantity which were necessary when those inputs were produced). This is the essence of the argument in Dumenil and Lévy 1997, and I think these authors are perfectly right in this point, whereas the attempts to reply them from a *temporalist* view have not yet had enough success. See,

² See the debate Shaikh versus Mage-Foley-Itoh.

for instance, the rejoinder of Giussani 1998, which does not address the central point in debate, I believe; or that of Kliman 1997, which offers some interesting quotations from Marx that, in my opinion, correctly show that the capitalist must compute, in case of devalorization, all values invested (and not only the present values of his devaluated assets) in order to correctly compute its true profitability, but this is not enough to demonstrate that Marx's idea was not to use present necessary quantities of (direct plus indirect) labor in order to measure present values.

4. Once we realize that the *single* and the *temporal* interpretations of the LTV can (and should, in my opinion) be separated (in clear contradistinction with what Freeman and Carchedi state in the preface of their book), there is no obstacle to accept, in the context of the interpretation I am offering here, the computation of values as vertically integrated labor coefficients, and that of the so-called *prices of production*, i.e., that of the values of production, as the left-hand eigenvector associated to the dominant eigenvalue of the input-output matrix.

Perhaps, a way of gaining some support to my own interpretation is to emphasize the idea that the fact that linear algebra shows it is possible to compute *simultaneously* the rate of profit and the vector of production values just proves the opposite (in the last resort) of what most critics of Marx believe to be able to demonstrate.

In fact, the eigenequation

$$(1/g) \cdot B = B \cdot H \quad (8),$$

where $H = K \cdot (I - A - B)^{-1}$, and K includes both fixed and circulating capitals, and A includes also D , the matrix of depreciation coefficients, is formally identical to the equation which can be used to calculate direct values as the eigenvector associated to another scalar which is now related, not to the (uniform) rate of profit, but to the (uniform) rate of surplus value:

$$(1/1+p') \cdot * = * \cdot H' \quad (9),$$

where $H' = B \cdot (I - A)^{-1}$, and p' = the rate of surplus value.

What both equations altogether show is simply that *values and surplus values are formed at the same moment*, and it is therefore possible to calculate both magnitudes of value at the same time that surplus values are computed in relative terms (in terms of both variable capital and total capital advanced by the capitalists). The fact that many critics interpret the previous results in a radical different way does not change the things. They interpret these results as meaning that it is not necessary a system of values because prices and rates of profit can be calculated directly, without the recourse to values. But they are wrong, because what they actually calculate are values as well, even if in a *quantitatively* modified form.

The critics' interpretation comes from the confusion between production and direct prices, which are in fact different magnitudes, but magnitudes of the same substance, the same property of the commodities: their labor content. All what they believe to find in the prices of production is in fact something relating to the values of production. When Samuelson, Steedman and all authors located in the currents they represent think to have found that "prices of production" are superior to "values", what they have actually found is that fully *capitalist* values are superior to simple *precapitalist* values. The sense in which this is true is quite clear in the line of thought coming from Marx to (the unknown but very interesting) Marzoua, passing through other authors like Rubin and Bródy. We will turn to that issue in point 6, but deal previously with another, connected, topic (see point 5).

5. Many critics of Marx look very happy of having been able to disconnect the theory of prices from the difficult (and for them superfluous) question of the quantities of labor-value. They believe that *physical* quantities are more objective and susceptible of scientific treatment. They think that physical conditions of production (which include the physical conditions of distribution if we assume, as above, that the wage is defined in real terms) suffice to determine the vector of relative prices and the rate of profit, and interpret this result as a disproof of the LTV.

But the meaning of the concept of *physical* is necessarily insufficiently elaborated, and this is quite easy to see. In fact, there are many, many different dimensions of the physical world, and it is not possible to measure heterogeneous things in physical terms unless we decide to select a *certain* physical property and at the same neglect *all other* physical properties. The explanation of this point requires two steps.

a) Things have many different physical properties: weight, size, height, width, volume, radioactivity, calories, etc., etc. Things *A* and *B*, for instance, can possess one of those properties in the proportion 2:1, and at the same time another property in the relation 3:1, a third property in the relation 4:1, and so forth. Therefore, one needs to specify which physical property are we speaking of. On the other hand, one of the properties we can choose is certainly one of special relevance for real acts of commerce in capitalist conditions: the labor content. We have three kinds of measures which can be used in measuring quantities of commodities: physical units like Kg., liters, etc.; monetary units like dollars or pesetas; time units, like hours or days. If we compare the latter with the two former, it is quite clear that the latter is the only one which can be defined in an invariable and universal way in space and time, whereas the physical units cannot be used universally (how to measure, for instance, the height of the service of a dentist?), and the monetary units cannot be used at different moments, because a dollar or a peseta have different meanings in 1998 or in 1999. By contrary, an hour of labor appears to be a much better principle of conservation for economic matters than the other two kinds of alternative.

b) Many authors say that physical conditions determine, without any recourse to values, the relative prices of all commodities and the general rate of profit. This is because when they write $p \cdot A$ they interpret the matrix *A* as a physical matrix, where each coefficient is measured in units like Kg/liter, dozens/Ton, units/m³, and so on. Since *p* are interpreted as relative (commodity) prices, and are expressed also as physical ratios (for instance, the

relative price of wine in terms of chicken is measured in kgs of chicken meat for liter of wine), the result, $p \cdot A$, is an a-dimensional product which can be interpreted as a simple ratio, a pure number.

However, a second possibility is to use monetary prices for p , and physical units divided by monetary units for matrix A , so that the multiplication gives us the same final result, but now by using a different way. For instance, we could say that the price of wood (10 \$/m²) and a mixed ratio like 2 m² of wood for 100 \$ of sales of the *chair* sector gives us 0.2 \$ of wood for each \$ of chair. Many authors have shown the equivalence between these two ways of considering the problem (see Ochoa 1984 or Miller and Blair 1985), which is a natural result since in fact the monetary measure is not but a different approach to the physical measure.

But what many seem to forget is that there is still a third possibility, scarcely considered in input-output literature: values as a concrete, universal and exactly determined physical quantity. In this sense, the physical matrix A is expressed in (regular) physical units of input by hour of commodity produced, and the price (or value) in hours of labor included in a unit of input, so that the product gives us hours by hour, i.e., the fraction of value contributed by that specific input (also a ratio or pure number).

Moreover, there is no possibility of having A with independence of p . All of that means that input-output relationships are not merely physical, but something which could be called *labor-physical*, i.e., another, more accurate, term for *economic*. That means that there are many different possible physical relationships between different objects of the physical world, but in the economic sphere of that world what really counts is just one of the many aspects possible. In particular, in the production of social life, it is labor relationships which select one of them as the only adequate in actual economic, or capitalistic, terms: the one which reduce the labor relationship to a minimum. Think of the nails used in making, say, tables: they might be golden nails or iron nails, and the physical matrix A would be different in each case. But what in practice determines which of the two possibilities of A is selected is the respective unit values of both kinds of nails.

Moreover, actual statistics only offer us value relationships. The monetary relationship between two different sectors hides the *actual labor relationship*: hours of input by hour of output expressed as \$ of input by \$ of output. What actual statistics give us is not A but (remember table 1) the product $(:A)$. That is the reason why actual empirical studies cannot calculate real absolute values or prices, but only *relative* values, relative in the sense that direct or production prices are compared to market values (the latter being fixed as an arbitrary index = 1, 100, etc.), like in the equation:

$$*/: = */: \cdot (:A) + a_0/:,$$

which, united to the assumption that $:=1$, for instance, gives us the usual:

$$* = * \cdot A + a_0 = a_0 \cdot (I-A)^{-1}$$

6. In my opinion, if we make the effort to go beyond a war of quotations and pay instead attention to the spirit of Marx's (unfinished) work, we should conclude that the true Marxian value is what I propose to call the *value of production* (whose monetary expression is usually called the price of production). The first critics of Marx knew perfectly well that the difference between direct and production values was just a difference in magnitude and in nothing else (see Bortkiewicz 1955). But in the seventies the influence of neoclassical economics in dealing with the transformation problem (observable not only in neoclassical authors like Samuelson but also in non-neoclassical like Steedman) made most authors get confused in the consideration of values and prices as really separated worlds (remember Samuelson's dichotomy between "Marxian values" and "competitive (real world) prices").

It is true that Marx did not develop a scheme like the represented in table 1 in a direct and explicit way. However, even if he used a different terminology, he would possibly have adopted this framework. He clearly distinguished between the horizontal and the vertical dimensions I mentioned *supra*, calling the former "the formal transmutation of the value of the commodities into price" (i.e., the step from **A** to **B**), and pointing out that the latter is rather due to "determined quantitative divergences of market prices from market values, and also from prices of production" (Marx 1894, p. 246), i.e., the different magnitude of * (that he calls "market value") *B* (his "production price") and : (the "market price" for him). The difficulty of that question lies in the fact that "commodities do not exchange simply as *commodities*, but as the *product of capitals*" (ibidem, p. 222), and that is why we should distinguish between, on the one hand, their values and "the price corresponding to their values", and, on the other hand, "their value as *product of capitals*" and the prices corresponding to them, i.e., the "value (price) as a bearer of advanced capital in its production, and therefore as an *aliquote part of the total product of capital*" (Marx 1863-64, p. 128).

Another way of looking at the same idea is, as Rubín explains it, that the labor theory of value inasmuch as it refers to direct values "just presupposes relations of production between producers of commodities", whereas "the theory of the price of production assumes, on the one hand, relations of production between capitalists and workers as well, and, on the other hand, between different groups of industrial capitalists" (Rubín 1928, p. 314).

More recently, András Bródy made an important step forward in identifying production values as the real capitalist values, breaking in this way with the dualist tradition (what does not appear to have been perceived by the defenders of *TSS*). Starting from the very index of his book, Bródy writes "prices in value" and "prices of production", and even uses the same symbol, *p*, for what he calls "the vector of value or price" (Bródy 1970, p. 11). His consistent *single* approach is specially evident in the "dimensional analysis" he deploys in his book, where it can be read that "price is thus the value of a unit of the product" (ibidem, p. 99):

$$[p_k] = [v_i a_{ik}] = [W/i] [i/k] = [W/k].$$

Equally crucial is the Spaniard Marzosa's idea that we should deal with the effects of the inter-sectoral competition on the quantitative formation of values and prices as we do with those of the intra-sectoral competition on the same aspects of the valorization process. In so doing, he has gotten further than any other author in this line: "Our interpretation of the 'price of production' rejects the idea of a 'transfer of value' (and therefore that of a 'transfer of surplus value'), since it argues that in the less efficient industry less value is formed in the same time of labor, without any necessity of transfer to somewhere else; and, equally, that in the more advanced industries, with the same real time of labor, more value is constituted, without coming from anywhere else. The acceptance of this idea when different firms of the same industry are considered is usual among the scholars of the so-called 'Marxist economics'; however, it is less usually accepted in the comparison between industries" (Marzosa 1983, pp. 74-75).

Contrarily to what some hasty readings of the latter might perhaps interpret, this line of argument does not amount to substitute circulation for production as the very field of creation of value, since, as Marzosa argues, he is clear in rejecting this interpretation as well, for he believes that "it is in the act of production, but *social production as a whole* (not of an industry or branch), where it really happens that a process is more or less productive than another, where a 'social average' is formed (...) But, in what way do those values manifest themselves to the society?; in no way other than in the market; i.e., 'in circulation'. The magnitudes of value are determined in production, but the society can only know them through the market; and this is so because, in modern society, *social production as a whole* (where values are only determined, instead of in such or such particular industry or branch) is constituted as such only through the market" (ibidem, pp. 76-77).

7. Up to now, we have seen that two widely used modern instruments (input-output framework and matrix algebra), mainly associated in practice with ideas that usually come from, and can be found in, the critics of Marx do not necessarily need to be used in this way, and can also be used in developing Marx's ideas. Input-output matrices do not need to be conceived as physical matrices as something opposed to labor processes; in fact, they should be conceived as *labor-physical*, or economic, matrices. Likewise, there is no real dichotomy between values and production prices, because there is no difference in nature between them. For Marx's labor theory of value, price and value are two terms with the same meaning: labor is their content. Absolute values or absolute prices express themselves directly, in hours of labor. Relative values or relative prices express themselves indirectly, in a number of other things (usually, units of other commodities, or gold or credit money).

Thus, we can make a step forward and ask now: if modern techniques seem compatible with Marx's ideas, at least in my own interpretation, and if it is even possible to express Marx's idea of the transformation problem in a way that makes possible to maintain both equalities ("total values = total prices" and "total profits = total surplus values") and at the same time support one single and precise definition of the rate of profit (recall equations (1') to (4')), should we conclude from all this that the critics of Marx are completely wrong, and should their developments be therefore rejected? The answer is a clear no. Let us see.

To begin with, there is no logical necessity for a Marxist to be more able to contribute to Marx's line of reasoning than for a non-Marxist. In fact, this does not happen to Marx's thought only, but to anybody's else as well. It cannot and could not be otherwise. The real matter of discussion is not this one, however. The crux of the problem is which theoretical edifice will stand as the central theoretical building, growing from inside but also by receiving contributions coming from outside its own nucleus. In my opinion, the only theoretical body able to perform such a development is Marx's, which is superior to Ricardo's (on which the modern theoreticians of the *surplus* approach want to build), and to Say's and Malthus's as well (on which would wish to build the Neoclassicals), due to a long series of historical reasons which can not been developed here.

Mathematics and other technical tools belong to all scholars. We should take the developments made in those formal fields, with independence of the rightness or wrongness of the material thought shown by their contributors in *architectural* matters (if they prefer Ricardo, Say, Marx, etc.). Where everybody should be belligerent, however, is in defending the superiority of the conceptual edifice they consider really superior, and in these matters no eclecticism is even possible.

Once we have taken into account the *single* conception of values and prices in Marx's thought, and once we have seen that his dynamic approach, his concern with real time and actual temporal evolution, leads him to think that it is the *present* (rather than past) value of the inputs which is necessary to add to direct labor in order to compute correct (in the sense defined above) values, there is no obstacle to admit that direct and production values should be computed as eigenvectors (as in equations (5) or (8) and (9)), even if:

- a) Marx computed them in another way (as in (1') to (4'), in my interpretation, would he have used linear algebra);
- b) this way of computing them have been mainly developed by authors who are critics of Marx.

Marx had no problem in using concepts already developed by authors who were sharply criticized by him. He wrote: "The price of production includes the average profit. We have called it the price of production; in fact, it is the same thing that Adam Smith calls *natural price*, Ricardo *price of production*, *cost of production*, the Physiocrats *prix nécessaire* (...)" (Marx 1894, p. 250). If Marx lived nowadays, he would have probably added: "...and the Neoclassicals *long run supply price*". Marx would not have rejected the idea that his price of production might be calculated by authors like Samuelson or Steedman in exactly the same way he would. Instead, he would have put his emphasis on showing that the price of production can only be completely understood inside the apparatus of the labor theory of value, and on the necessity of taking computations as such as a part of the reality that must be explained.

8. There is another crucial difference between Marx's and others' conceptions of the price of production. Even if in all cases it is defined as the sum of costs of production plus a profit proportional to the capital invested, there are radical differences in the answer given to the

question of the magnitude of that proportion, i.e.: which is the exact rate of profit, why must it be 20% instead of, say, 2%, 2000%, etc.? Marx took advantage on other authors in giving a clear answer to that question: the rate of profit is the ratio between the number of hours of surplus labour extracted in a period and the number of total unpaid hours of labor extracted and accumulated in the past and advanced at the beginning of the same period (so that, for instance, the 20% could come from the ratio " $2 \cdot 10^9$ hours / 10^{10} hours"). Other authors, having given different answers to the same question, to which we will turn now, do not seem conscious of the problems those answers arise.

a) The most popular answer is related to the idea that the rate of profit is a kind of monetary expression of the *physical productivity of capital*. This primitive neoclassical answer is presently rejected by more sophisticated neoclassicals, mainly as a result of the famous debates on capital (the two Cambridge and so on: see Jorland 1995). As neoclassicals reject labor, they are unable to find any substance in terms of which to homogenize the different means of production in order to compute the *physical* productivity of capital. If one needs prices to such a homogenization, it is absurd to continue with the whole procedure because it was assumed we needed the physical measure of productivity in order to explain profits and then prices, and now discover we need prices to begin with the explanation of prices. This circularity makes the argument invalid.

b) The rate of profit as the *rate of interest* or the *rate of growth*. Curiously enough, neoclassicals and non-neoclassicals (*surplus approach*, etc.) meet in this point because both schools share the most sophisticated fetishism in this respect. They oscillate (see Eatwell 1987) between a financial conception of the rate of interest³ and a real one, according to which, in the latter case, each commodity has its own rate of interest, and the whole mass of commodities which form the social product and the total capital have their global rate of interest which would be nothing else than the uniform rate of growth, or general rate of expansion, of the economy. Keynes, Sraffa and von Neumann seem to share the same kind of fetishism in relation with this point, since in all of them there seems to be one element of reality vanishing from their otherwise realistic view: labor. They seem to believe that the simple passing of time suffices to make possible the physical expansion of commodities as if it were a simple expansion of things. In reality, it is labor which fills the content of time: it is time of labor which creates value, and this is why we have neither rates of interests nor rates of growth of the physical things in the moon and the planets. Were there men living and working in capitalist conditions, and it would not be impossible to see how those rates begin to be born in the new context.

The unanimous fair recognition of von Neumann's mathematical capabilities has been dubiously associated with a glorification of the significance of *his* economic model. Neoclassicals and neoRicardians share the attempt to count the heroic inside their own ranks (see Kurz and Salvadori 1995, but see also Bródy 1970 as the proof that his mathematics is compatible with Marx's ideas). But nothing of this prevents us to see the difference between the formal essence of a model and its material content. Von Neumann proved that the rate

³ Do they really believe that actual data (think of Japan in 1999, with its 0.25% of official rate of interest and 0% of inflation) support the idea that the rate of profit is the rate of interest? If their point is not "is" but "is explained by", the question remains: how the latter explains the former?

of growth and the rate of profit were the same in his model. Modern economists show that the calculation of the rate of profit and that of the rate of growth are the dual and the primal of the same mathematical problem. But this formal duality cannot be used in itself to derive a real causality. Of course, we need a theory to do such a job.

Neoclassicals and neoRicardians seem to believe that the (physical) rate of growth explains the rate of profit. Marxians can use the labor theory of value to see that rather the opposite is true. Both rates coincide when the whole surplus value is reinvested (like in von Neumann's model), but it is the existence of the rate of profit and its magnitude (and therefore the existence of labor and surplus labor) which makes possible in the real capitalist world the investment of all profits and then the growth at a uniform rate. When net output consists of goods of investment, the rate of profit, the rate of growth of output and the rate of expansion of capital are one and the same thing.

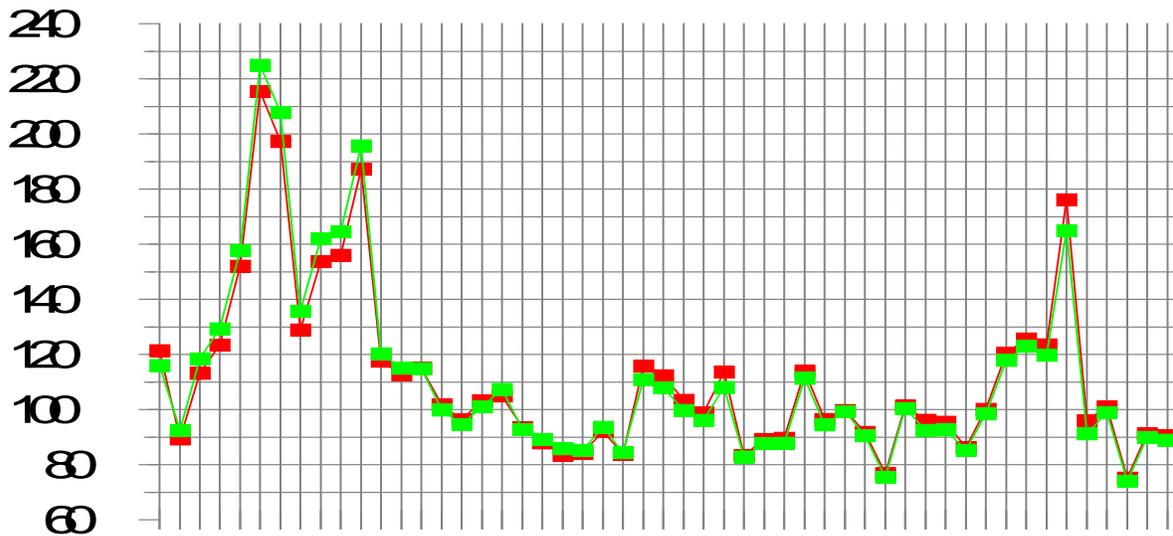
9. Empirical work is a necessary component of theoretical work. The labor theory of value deals with values which have not only a qualitative but also a quantitative dimension. The results commented in the theoretical part of this paper have been obtained partially as a consequence of the empirical study of the Spanish case which I began to comment in my paper for the 1998 Conference (see Guerrero 1998), and would like to complete now, because these results have some interest *per se* (I believe). In particular, the analysis of the Spanish case allows us to conclude: a) the labor theory of value is well supported by the results I found; b) instead, alternative value theories are discredited, at least in comparison with LTV; c) Sraffa's idea that changes in distribution should be taken into account in the determination of relative prices must be relativized due to its minimal incidence in practice; d) von Neumann model is shown to find its true sense in a Marxian context.

a. The labor theory of value.

The LTV is well supported by our empirical results, and that happens in a triple way: 1) prices of production deviate from direct prices as a result of the dispersion in the compositions of capital of the different industries; 2) the global deviation of market prices from production prices show a significantly small coefficient of variation; 3) the evolution in time of sectoral production prices is highly correlated to that of sectoral direct prices.

1) Using the methods developed by Shaikh and others⁴, it is possible to show that production prices are the result of a deviation from direct prices due to the differences in the vertically integrated value composition of capital of the industries ($vivcv_i$). For 1990 and a disaggregation of 51 sectors, we can see that putting each sectoral direct price (i.e., the monetary normalized expression of direct values defined as in equation (1) above) equal to 100, the resulting production prices oscilate above and below 100 as in the figure 1, where the first curve represents the ratio production price/direct price (p/d), and the second one the ratio $vivcv_i / vivcv^*$ (where $vivcv^*$ is the average of the $vivcv_i$).

⁴ See Shaikh 1984, 1998, Ochoa 1984, Chilcote 1997.



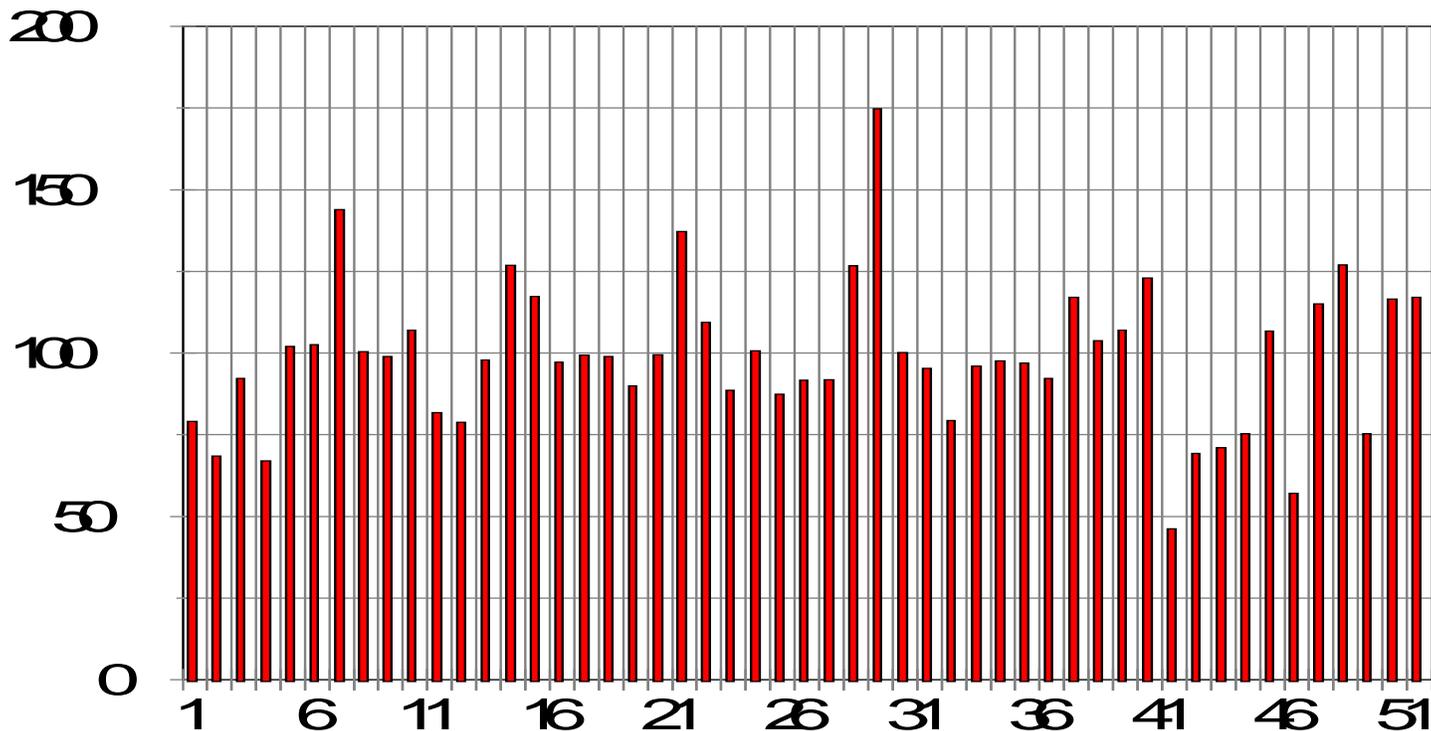
2) As for market prices (m), figure 2 shows the ratios m/p for each of the 51 industries. The coefficient of variation is 0.249, which is reduced to only 0.180 when the impact of differences in net indirect taxes (taxes less subsidies) is set aside. Steedman and Tomkins 1998 have criticized Ochoa and other authors for using the MAWD (mean absolute weighted deviations) and other statistics instead of the coefficient of variation. The importance of our data is that, taken together with the data of the relationship between direct and production prices, the global results are fully in accord with the labor theory of value: the deviations between d and p are totally explained by the differences in vivcv (vertically integrated value compositions of capital), and the deviations between m and p are partially explained by the presence of the state⁵.

3) Although the period analyzed is extremely short (since only the years 1986-1991 have been at our disposal), the evolution of d and p has been checked for the six years considered. The correlation in the evolution of both series of data is quite good in a high majority of sectors, with an average R^2 for the 51 industries of 0.757.

figure 2.

⁵ It remains to be tested the incidence of other factors like the special situation of supply and demand in the short run, etc.; but these factors cannot be tested in the absence of a conceivable objective procedure to perform it.

Preiosnercab/P.producción(p)



b. The alternative values

The computation of alternative values has been defended by some authors (see Vegara 1979 or Roemer 1981) due to formal similarities between the latter and the labor values (or, rather, between the procedures of computing both of them). The correct procedure of calculation has been used in a series of imaginary numerical examples (for instance, in Vegara 1979, following Bródy 1970), but not for actually existing economies⁶. For the Spanish economy, our procedure, inspired in Bródy 1970, is the following. Our point of departure is always the matrix **A**, i.e. the enlarged input-output matrix (which includes, as an additional row, the vector or wage costs per unit of product, and the vector of workers' consumptions as an additional column). Therefore we obtain a 52x52 matrix defined as:

$$A = \begin{pmatrix} A & cw \\ Ra / q & 0 \end{pmatrix}$$

⁶ However, Cockshott and Cottrell 1993, or Chilcote 1997, have used methods which are valid to test the correlation between prices and alternative values at the industry level. I am very grateful to all of them for helping me to understand (through private correspondence) their own procedures and the procedure I finally used. But I think the problem with their calculations is that their results are affected by the problem of spurious correlation, to whom, I believe, mine are immune, since they are obtained at the individual level.

If we are interested in computing iron-values (say, industry 8), we need first of all a matrix A_8 (51x51) which can be obtained as the result of subtracting from A both the eight row and the eight column; then we get its inverse as:

$$B_8 = (I - A_8)^{-1};$$

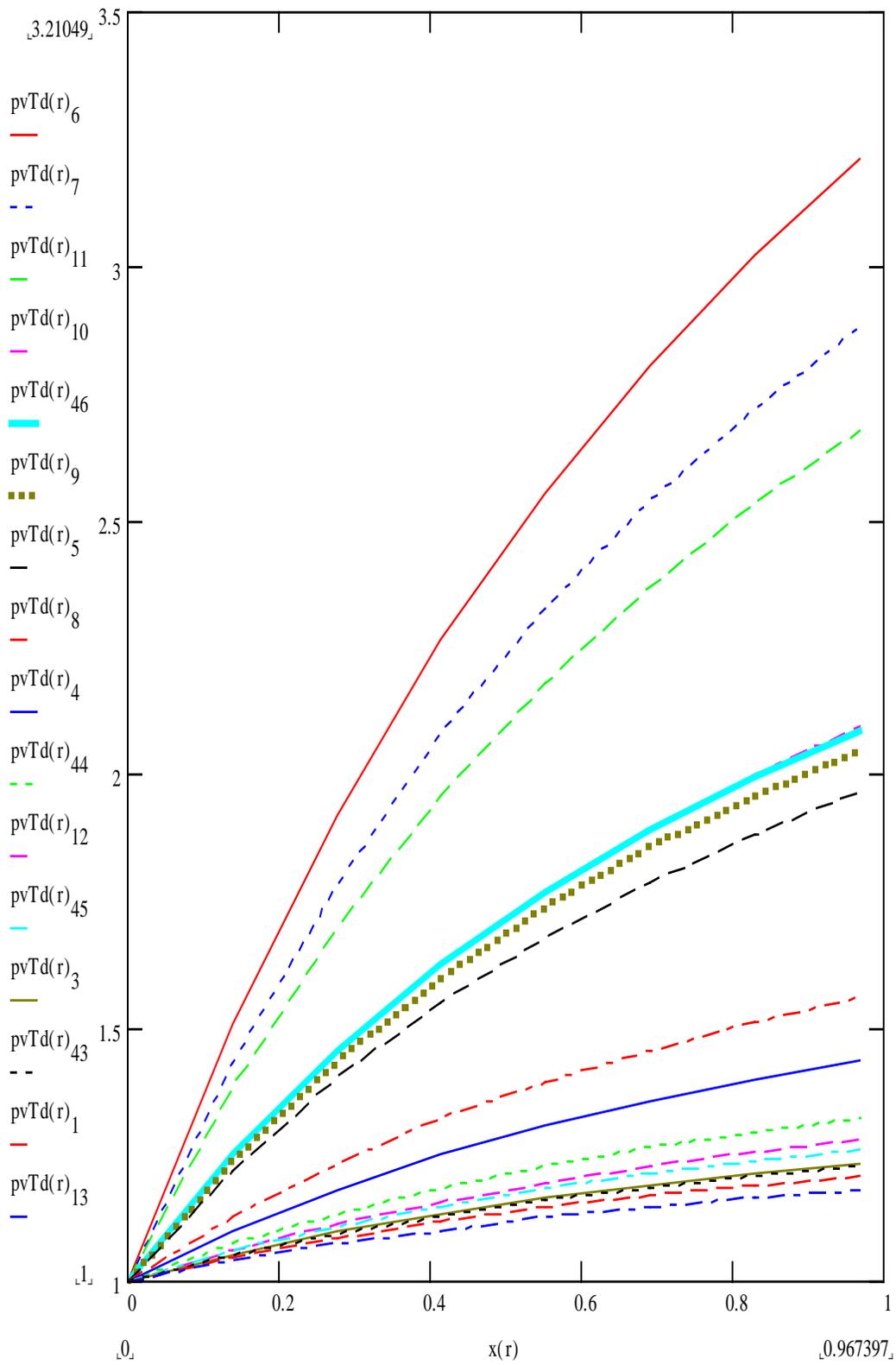
And finally we select the eight row, and after eliminating its eight element, we get the vector (say, a_8) whose multiplication by the latter matrix gives us the iron-values we are looking for:

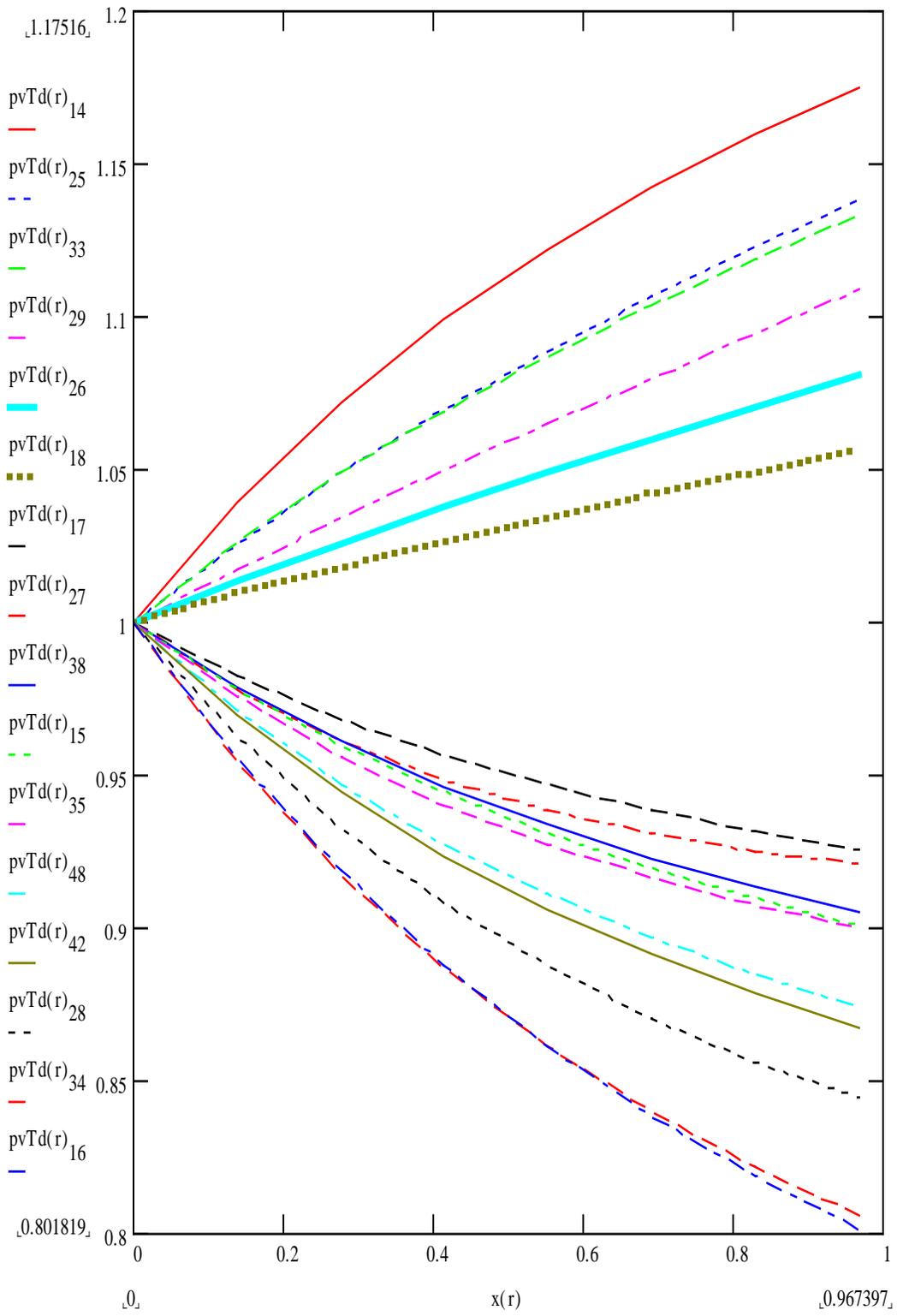
$$v_8 = a_8 \cdot B_8$$

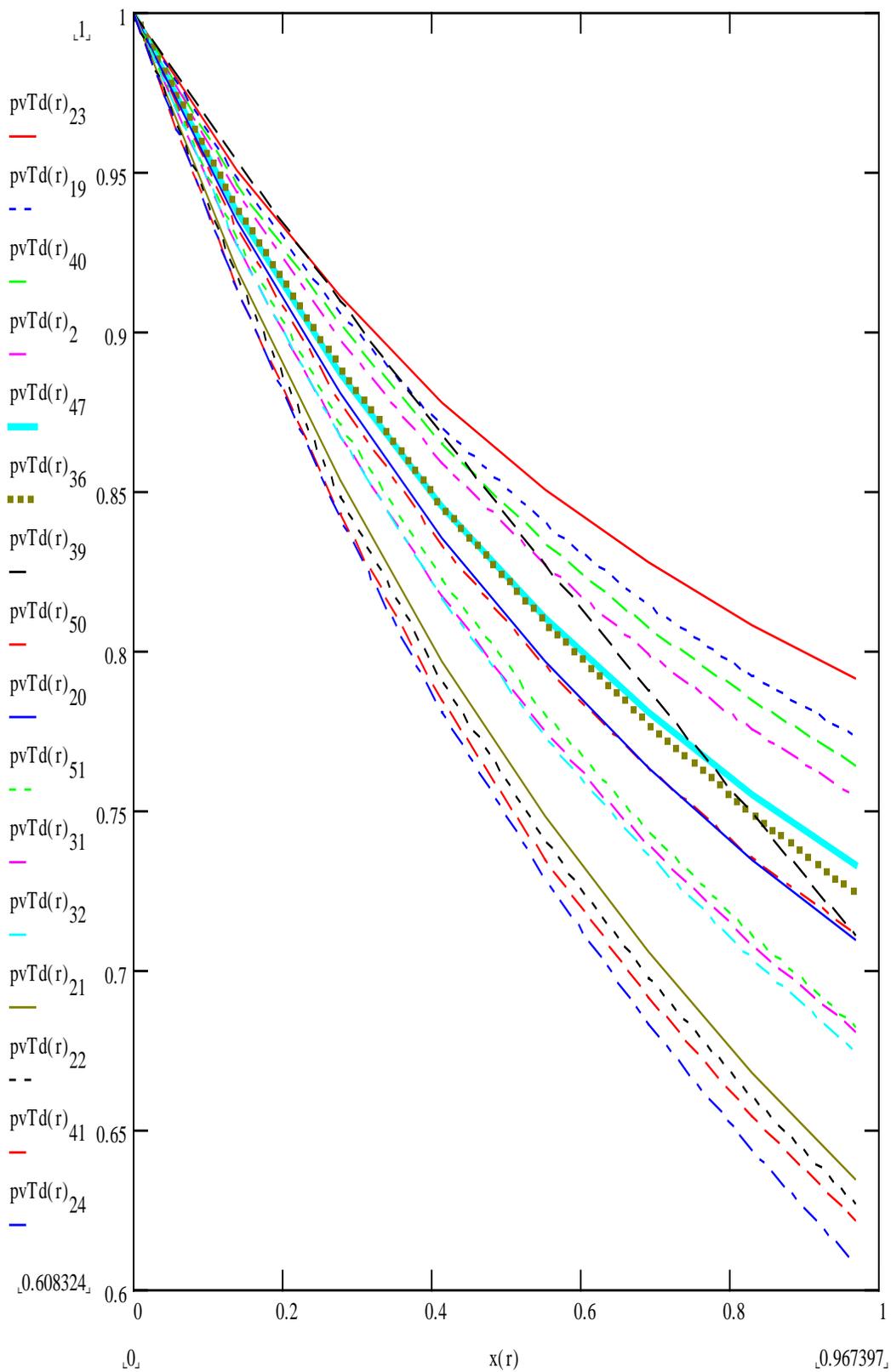
We have repeated this operation 51 times and therefore calculated the 51 possible alternative values (in terms of coal, iron, cars... and all industries defined in the Spanish I-O tables). The results of the comparison between these so-called alternative *values* and the true labor values can be summarized by saying that the correlation between the individual prices of production and individual labor values is 0.794, whereas that of the former with the average of the 51 alternative values is only 0.177.

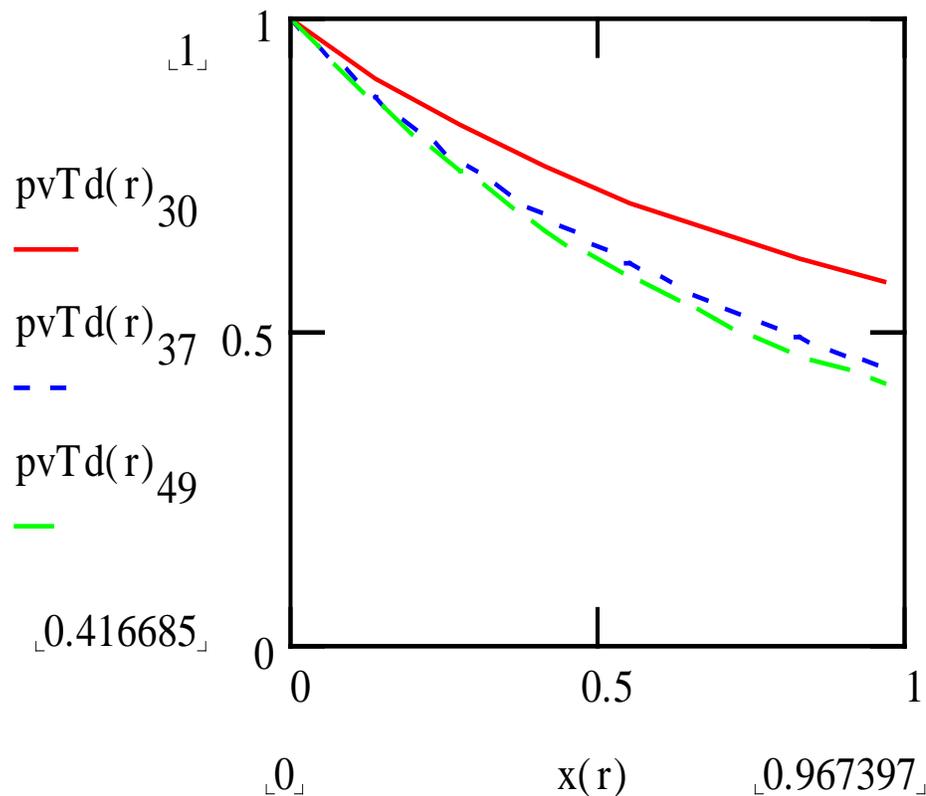
c. Distribution does not affect relatives prices

Figure 3 (in 4 sections) represents how evolve the sectoral ratios p/d as a function of the magnitude of the general rate of profit (the ratios for each industry are always 1 when $r = 0$). The evident "parallelism" shown by the 51 curves (i.e., the practical absence of crossings between them all along their course) show how constant are relative prices with independence of the value adopted by the rate of profit (r). The only effect of the changes in r seems to be a direct repercussion (an amplification) of the magnitude of r in the dispersion of the individual ratios, but it is easily seen that the structure of these ratios is the same in all cases.









d. The rate of profit explains the rate of growth, not the opposite.

Lastly, Von Neumann ray can be calculated as the dual of the problem of computation of the prices of production (5). We have in this case the right-hand (instead of the left-hand) eigenvector of the input-output matrix, and it is now associated with the dominant eigenvalue of that matrix, which amounts to be the reciprocal of the uniform rate of growth (which is therefore equal to the general rate of profit). The most interesting point here is, however, to realize that what this vector gives us is just the structure (the % in the total) of the actual capital formation (or investment) of the economy. The meaning of this result is easy to understand: the von Neumann ray shows us which should be the structure of output which would give us the maximum uniform rate of growth associated with the actual structure of the economy (given by the actual input-output matrix). This structure is necessarily the one which corresponds to the case where all net output in every industry is invested, i.e. the case when consumption is zero (above that consumption considered just as the equivalent of a physical input already included in technical coefficient matrix).

This result confirms us that it is not the rate of growth which determines the rate of profit, but exactly the opposite, as discussed in point 8 of this paper.

10. Conclusion. Marx died but his labor theory of value is well and alive. It seems to me that it is the only theory of value on which a scientific economics can be built. The fact that

Marx was the main contributor to the LTV or the fact that many of the critics of both Marx and LTV made many of their criticisms against both of them with (or without: this is an indifferent matter) the spirit of attacking them should not prevent us to use the real contributions made by those critics (usually in an unconscious way) and insert them in the body of concepts which Marx began to build. This is not eclecticism. It is the necessary appropriation of outer contributions that LTV needs, as any other theory, in their process or metabolization and growth. Ironically, whereas many believe that it is them who are being able to appropriate some elements of Marx's thought in the body of a greater current more often called the surplus approach, it has been my main intention to show that what is happening is exactly the opposite.

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