1 Price, value and profit – a continuous, general, treatment

Alan Freeman

Laβ dir von keinem Fachmann imponieren, der dir erzählt: 'Lieber Freund, das mache ich schon seit zwanzig Jahren so!' – Man kann eine Sache auch zwanzig Jahre lang falsch machen.

Kurt Tucholsky

1.1 INTRODUCTION

This chapter replaces the simultaneous equations approach of General Equilibrium theory with an economically superior and more general formalism based on Marx's analysis, removing the arbitrary and restrictive assumptions needed to obtain a simultaneous solution. Its values, prices and rate of profit are in general different from those predicted by simultaneous models. Former debates, which assume a common framework, are therefore superseded. There are now two frameworks; one confirms Marx's thought and one falsifies it; one expresses the inherent phenomena of a capitalist economy, the other assumes they do not exist.

The features of the formalism which distinguish it from equilibrium are:

- ❑ Reproduction is treated as a chronological, not a simultaneous process.
- ❑ Goods are sold at market prices instead of fictional equilibrium prices.
- ❑ Goods exchange for money, not for each other.
- ❑ Profit rates are not assumed actually to equalise.
- ❑ Technology is not assumed either constant or uniform.
- ❑ Supply and demand do not balance and unused goods accumulate as stocks.
- ❑ Variations in the price and value of existing stocks are rigorously accounted for in the calculation of prices, values, and profits.
- ❑ In a uniform treatment of fixed and circulating capital, the period of reproduction has no definite length. In the continuous case it is arbitrarily small.

A formalism is not a model. It does not yield predictions or 'solutions' from particular restrictive assumptions. It is an axiomatic system, a methodical framework for presenting concepts and their relations, in which a variety of different assumptions can be represented and in which it is possible to deduce the general laws that apply to all such special assumptions. Anyone who wants to build a model – that is, asserts that capitalism obeys more precise laws under

more specific circumstances or assumptions – can frame this mathematically in this system, and anyone who wants to study certain phenomena in abstraction from disequilibrium can do so by introducing special restrictions. Equilibrium systems are hence a special case of this more general formulation.

It is hence an alternative paradigm to the simultaneous equation method which, under the guise of simplifying, imposes a particular assumption – market clearing – and claims it as a general model. We apply the classical procedure of moving from the general to the particular.[1]

The word 'general' does not mean that every aspect of a real economy is represented, but that the construction has introduced no obstacles to representing them, at a more concrete stage of analysis. There is no scope to cover commercial capital, finance capital, landed rent, credit, unproductive labour, noncapitalist production, or skilled and complex labour. Geographical factors are not assessed. The state is not treated apart from its role in monetary regulation, nor relations between states and hence imperialism or the world economy. This shows how far we have to travel. But to travel at all we must leave the territory we are confined in: it is impossible to study finance capital rigorously in a simultaneous framework, since the assumptions of simultaneity spirit away the money relation. My aim is to remove those limits to a proper study of these questions inherent in existing treatments, above all the ideological assumptions of General Equilibrium, frozen in place by the simultaneous equation model and the elimination of time.

The use and limits of mathematics

Parts of this chapter are mathematical. The non-mathematical reader can skip them, but I hope she or he will glance at them, because the mathematics is new but not inherently difficult and one function of this chapter is to develop a complete alternative way of going about things so as to break the stranglehold of equilibrium thinking.

Mathematics suffers the same limitations as formal logic, which has to separate things conceptually that are not isolated actually. For this reason alone it is dangerous to credit it with powers greater than those invested in it. In the last analysis mathematics is a technology of mental processes, and should be taken neither for real things nor real thinking.

However, it often is. Its very power lends it the aspect of a supernatural force, capable of revealing any truth. It unites the two most powerful human mental faculties, the power to symbolize and the power to depict: magic and religion abound with mathematical lore.[2] Walras and Bortkiewicz were early worshippers at this shrine and economics has yielded itself almost entirely to the mystical power of pictures and symbols, a cosy substitute for the complexity of the real world. For this reason some Marxist writers despair of using mathematics.

The problem, however, is not mathematics as such but its worship as an independent source of truth.[3] Political economy is subject to the laws of

arithmetic, which are not abolished by refusing to express numbers in symbols. It is true that the real world imposes itself, if not through conscience then through the facts, but it is not enough just to assert 'the figures add up'; it has to be proved. This calls for a mathematical framework whose generality admits the facts, and whose simplicity displays the concepts.

As shown in Chapter 1, Walrasian mathematics imposes concepts that deny access to the facts. However these concepts exist independent of the mathematics, which merely exhibit them in pure form. Even the most seasoned casuist cannot make five from two and two: his best hope is to stop two and two coming together in the presence of four. Mathematics does not help him in this respect; the problem is not its use but its abuse, which this chapter seeks to end.

1.2   A BRIEF READER'S GUIDE

This chapter has two audiences: non-mathematical readers, and those with a background in linear production models.[4] Mathematical detail is given separately at the end of each section and can be skipped, except for the final part and the section on notation which introduces symbols used throughout.

Sequential value calculation, time, and the labour process

First, I introduce the sequential calculation of value, correcting the basic weakness of simultaneous models which assume that input values are equal to the corresponding output values at the end of production. In fact they equal the output values of the *preceding* phase of production. The notation is introduced.

Exchange, circulation, values and market prices

This introduces circulation in a money economy.[5] In contrast to the standard treatment commodities exchange against money, not each other; money functions as a hoard, not a flow; and I show that Marx's 'first equality' holds for arbitrary market prices, not just prices of production. Being derived from pure circulation, the analysis applies to any exchange of the products of labour regardless of the conditions of production.

Value transfers and the origin of profit

Any set of market prices effects a transfer of values given by a special vector, the value transfer vector. It summarises the impact of the market on the values emerging from production. Profit is shown to be the sum of surplus value and this transfer vector – Marx's 'second equality'.

Capital as such: stocks, flows and accumulation

Stocks are the form which capital – dead labour – takes in production. This central section rigorously examines their formation from commodity flows.

Equilibrium theory is deficient in two vital respects. First, it assumes that supply equals demand when in fact fluctuations in stocks, the pulse of capitalism, both express and regulate the differences between them. Second, it ignores the way old stocks enter the formation of new prices, which is why the profit rate falls.

Value, price and profit in the presence of fixed capital

I extend Marx's derivation of market values to fixed capital, based on his concept of moral depreciation. I show how to calculate profit and surplus value in the presence of fixed capital and that Marx's two equalities remain true.

This makes it possible to correct the traditional distinction between fixed and circulating capital which assumes a fixed period of reproduction, an arbitrary accounting construct. This restriction is removed so that results are independent of it. This is the basis of the passage from difference, or discrete dynamics, to continuous dynamics.

I deduce the general law of accumulation and a general account of sale at market prices. On this basis the Okishio theorem is refuted and it is shown that the value and price of society's capital rises – and its profit rate falls – unless the capitalists disinvest. Finally, the theory is restated with a variable value of money, and its role in the mechanism of the business cycle is established.

1.3   SEQUENTIAL VALUES: AN ILLUSTRATION

To fix ideas and explain the contrast with the simultaneous method, consider a simple example involving two producers P_1 and P_{II}, producing homogeneous commodities C_I and C_{II} respectively. Suppose over some period of time they and their labourers consume, produce or reproduce the following quantities of C_I and C_{II} and labour power V, measured in their natural units.

FLOWS		C_I	C_{II}	V		C_I	C_{II}	Labour Power
Producer P_I	used	35		300	and produced	50		
Producer P_{II}	used	10		200	and produced		100	
Labourers	consumed		50		and reproduced			500

Table 11.1 Quantities consumed and produced in period 1

Let λ_1, λ_2 and λ_L be the value per unit of C_I, C_{II} and V. The simultaneous approach proceeds thus: the unit values of inputs must equal the unit values of outputs. Then the following must hold:

$$50\lambda_1 = 35\lambda_1 + 300 \qquad (1)$$
$$100\lambda_2 = 10\lambda_1 + 200 \qquad (2)$$

The unique solution – the only one compatible with equilibrium[6] – is

$$\lambda_1 = 20 , \lambda_2 = 4 \qquad (3)$$

However, we have no real right to assume that input values are equal to output values. Suppose during the previous period productivity was different for

whatever reason, and the quantities consumed and produced were given by Table 11.2:

FLOWS		C_I	C_{II}	L		C_I	C_{II}	Labour Power
Producer P_I	used	40		400	and produced	50		
Producer P_{II}	used	10		300	and produced		100	
Labourers	consumed		70		and reproduced			700

Table 11.2 Quantities consumed and produced in period 0

The corresponding simultaneous equation values are given by

$$50\lambda_1 = 40\lambda_1 + 400 \tag{4}$$
$$100\lambda_2 = 10\lambda_1 + 300 \tag{5}$$

and are
$$\lambda_1 = 40, \lambda_2 = 7 \tag{6}$$

The simultaneous calculation faces an insuperable difficulty. If C_I was worth 40 per unit at the end of period 0, then it must also be 40 at the beginning of period 1, since these are one and the same time. But according to equation (1) this cannot be: λ_1 must be 20. The two simultaneous solutions are incompatible.[7]

This reverses the charge made by Marx's critics. The input values of period 1 are not equal to output values of period 1 but of period 0; the same applies to prices *mutatis mutandem*. A perfectly rational alternative is thus available. Suppose, for example, the values given by equation (6) are valid at the beginning of period 1. A perfectly determinate calculation gives new, and different values at a later time – the end of period 1. To distinguish them, we use a time suffix: λ^t represents value at the beginning of period t, that is the end of period t–1. Hence

$$\lambda_1^0 = \text{unit value of } C_I \text{ at the start of period 0}$$

and
$$\lambda_1^1 = \text{unit value of } C_I \text{ at the start of period 1}$$

and so on. The problem then becomes to write down an equation giving the *relation between* λ^1 and λ^0 for each commodity; This is technically a small step, but conceptually a giant one; it removes us once for all from a mathematical framework which logically imposes constant prices and values.

The equations follow naturally enough from Marx's account of the labour process. For each commodity the value of outputs is the sum of two quantities; the value transferred from consumed constant capital, and the labour product. That is, in this instance, the value of consumed inputs, plus the hours worked. Hence

$$50\lambda_1^1 = 35\lambda_1^0 + 300 \tag{7}$$
$$100\lambda_2^1 = 10\lambda_1^0 + 200 \tag{8}$$

that is

$$50\lambda_1^1 = 35 \times 40 + 300 \tag{9}$$
$$100\lambda_2^1 = 10 \times 40 + 200 \tag{10}$$

Giving

$$\lambda_1^1 = 34, \lambda_2^1 = 6$$

These are lower than the values of the previous period, because labour productivity has risen, but not as high as the hypothetical simultaneous values of the current period, whose inputs were produced less efficiently than its outputs because of the inherited more costly inputs. Now that λ^1 are known, the same method can be used with the data of the next period (which in general will be different) to define λ^2, λ^3, and so on. There is no single unique value but a time sequence of different values, none of them in general equal to the simultaneous solution. Provided the data about consumption and production are available for each period, this is a determinate definition of values in every period. A few relevant points should now be noted:

❑ No technological assumptions (such as constant returns to scale) were made; we calculated values from the observed consumption and production of each good, on the scale at which the economy actually performed.

❑ Hence a linear production model is not assumed. The only linear assumption is that the value of a composite is equal to the sum of its parts. This is intrinsic to the nature of value and involves no assumptions about production.

❑ The straitjacket of a fixed technology has vanished. No matter how technology changes from period to period, the value calculation remains valid.

❑ The calculation depends on initial values as for all dynamic analysis. We shall show that starting data are given by the economy itself – observed input prices. The only truly unknown initial quantity is the value of money, dealt with later.
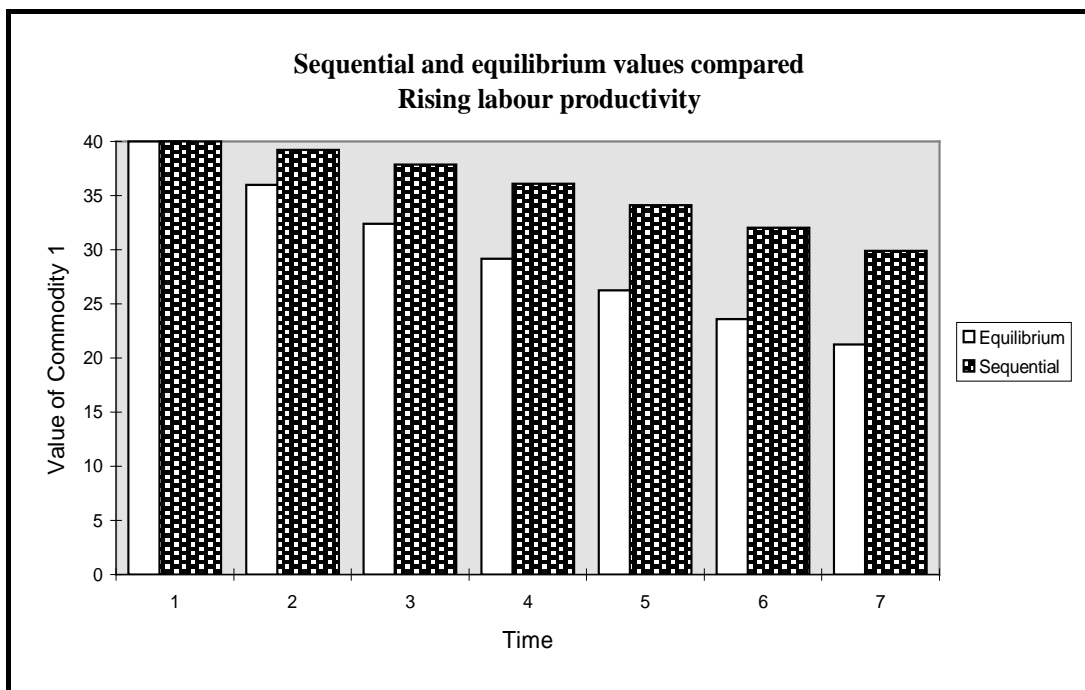

Figure 11.1 Sequential and equilibrium values compared

It cannot be stressed enough that sequential and equilibrium values are *different.* This is so even when technology is fixed but becomes even clearer when it is changing. If consumption and production levels do not change, the sequence converges to equilibrium for any economy producing a surplus, whatever the starting point, which appears to justify treating the fluctuations as an ignorable disturbance. But production and consumption levels will in fact change on a time scale similar to the period of convergence: equilibrium never happens.

Suppose labour productivity steadily improves and 10 per cent less labour is required in each period. Then as Figure 11.1 shows, even though the sequential and simultaneous calculations start with the same values, from then on the two diverge. The reason is that inputs in each cycle come from a *previous* period and embody past labour time. The sequential calculation recognizes these historically-inherited production conditions, the equilibrium calculation cancels them.

In this respect it should be remembered that for the whole of *Capital* except the section of Volume II which deals with simple reproduction, Marx assumes *relative surplus value*.

1.4   A MATHEMATICAL REPRESENTATION

This section may be skipped by the non-mathematical reader although it may be useful to read the first three paragraphs of the section on 'notation' which introduce simple standards used throughout this piece.

Difference equations and the sequential method

The sequential approach can be understood without vectors or matrices. Consider a single commodity serving as its own input such as corn. Suppose 10 person-weeks of labour transform 5 tons of seed corn into 10 tons of new corn. At time $t = 0$ suppose the seed corn's unit value is λ^0 weeks per ton. New corn is produced at $t = 1$ with a new value, λ^1. Basic value theory tells us that

$$10\lambda^1 = 5\lambda^0 + 10 \tag{11}$$

Suppose now that at $t = 0$ the value λ^0 of the seed corn is known to be 1. Then

$$10\lambda^1 = 5 + 10 = 15$$

hence $$\lambda^1 = 1.5$$

If corn and labour continue to be produced and consumed at the same rate, we can define a relation between values at any successive times in the same way:

$$10\lambda^{t+1} = 5\lambda^t + 10 \tag{12}$$

By successively substituting we can get $\lambda^2 = 1.75$ from λ^1, $\lambda^3 = 1.825$ from λ^2, and so on. These values are defined at all subsequent times, that is, they can be calculated from observed data. Equation (12) is a difference equation which given the initial values can be solved for these values. More generally it reads

$$\lambda^{t+1}X = \lambda^{t}C + L \tag{13}$$

where

    C represents consumed inputs

    X represents outputs.

    L represents the value-product of labour

More generally, if technical relations are changing (as they are), the equation has to reflect this by adding a time parameter to all magnitudes:

$$\lambda^{t+1}X^{t+1} = \lambda^{t}C^{t} + L^{t}$$

In this general case, sequential and simultaneous values have no necessary relation to each other: If we suppose for example that labour inputs shrink by 10% per year we get the equation that produced Figure 11.1:

$$10\lambda^{t+1} = 5\lambda^{t} + 10 \times (0.9^{t}) \tag{14}$$

A sequence of magnitudes for all data of the economy, at all times, is a *trajectory*. A 'model' of this sector of production is an attempted prediction of its trajectory from past data. (The Sraffa model, for example, assumes that X, C, and L are constants.) This is in general impossible.[8] However certain general laws hold for *all* models – for example, the rate of profit falls unless the capitalists disaccumulate in value terms. The function of mathematical analysis is to establish such general laws and the conditions in which they hold.

Notation

Mathematical notation is not neutral. The unorthodox notation used here is chosen to reflect and encourage the conceptual structure needed to analyse a commodity economy. It is designed to make the logic clearer and the argument easier to follow. The principle is that the same symbol always stands for the same commodity in the same capital, while value is distinguished from use value, and stocks from flows, by varying the type or additional symbols. This emphasises the unity of the commodity form. It also makes it easier to use the same letters as Marx, who tends to use C for everything and V for everything else.

    Every commodity has two aspects: use value and value. The value of a commodity will be represented with a £ sign in front unless the context is unambiguous. Thus equation (13) can be written

$$\pounds X^{t+1} = \pounds C^{t} + \pounds L$$

or $$\lambda^{t+1}X^{t+1} = \lambda^{t}C^{t} + \pounds L$$

    This leads to a pedantic but important distinction: One pound's worth of value will be represented as £1 but one pound coin or note itself will be represented 1£.

    Every commodity exists both as a flow, or turnover and as a stock. The notation to distinguish these will be introduced in section 10 on accumulation. The basic symbols are matrices C, W, X and B, and vectors V, L, λ and *p*:

    C_j^i constant capital employed: quantity of commodity *i* in capital *j*

    V_j variable capital (labour power) employed by capital *j*, in hours

    $\pounds L_j$ value-creating capacity of V_j, (value-product) in pounds[9]

X_j^i produced output of commodity i in capital j
W_j^i quantity of commodity i in the wage-fund of workers in capital j
B_j^i quantity of commodity i owned by capitalists in sector j
λ_j value of a unit of commodity j measured in pounds
p_j price of a unit of commodity j measured in pounds

Columns represents capitals and rows represent commodities. This is slightly confusing since Marx's tables show capitals or producers as rows and commodities in columns. Modern usage is too well rooted to change it.[10]

There may be more than one producer of the same commodity so C_j^i may not be square. In this chapter we use a *reduced form* (Freeman 1991) of C in which each column aggregates all capitals in a sector and activities corresponding to joint production are allocated to distinct commodities, so each sector makes one distinct good. X is therefore a diagonal matrix. Workers' consumption is represented by a matrix (W) rather than a vector, so wages may differ from sector to sector though of course they may be the same.

To distinguish rows from columns we use the convention that superscripts vary over columns and subscripts over rows.[11] Hence:

£C^m is a row vector in which £C_j^m represents the value of money held in sector j,

£C$_{farmers}$ gives the farmers' constant capital, and so on

Column and row totals and correspondence with Marx's notation

We often refer to column or row sums of C, V, X and their derived matrices, for example $\Sigma^i£C^i$, the constant capital employed in each sector (note that this is the same as λC). Thus

$\Sigma^i£C^i$ is the total value of constant capital employed in each sector.

$\Sigma_j£C_j$ is the total value of each commodity employed in production

This lets us use most symbols as Marx does: £C is what he terms constant capital and £V is variable capital.

Sign convention and the problem of the stock-flow relation

The important matrix K gives the distribution of the total stocks of all commodities in the economy except labour power. A problem of signs then arises as follows. It is conventional, and any other usage would be obscurantist, to represent the *consumption* of C, W and B as a positive quantity. Consumption actually diminishes a stock and, strictly speaking, should be represented as a negative number. It seems a rather strong illustration of the scant attention economics has paid to the stock-flow relation that this is not often recognized.[12] We find that the stock of a commodity is *minus* the sum (or integral) of consumption flows. Therefore the stock of C is represented by –C, not by C, just as assets on a balance sheet appear as a debit, something owing to the owner. In

writing down the relation between K and the other stocks in society this problem cannot be avoided and we have to recognize that

$$K = X - C - W - B$$

Thus $\Sigma_j \, K_j^i$ gives the amount of commodity i in the economy. The diagonal matrix formed from this is called $\hat{K}$, so that $\hat{K}_j^i = 0$ when $i \neq j$ and $\hat{K}_i^i$ is the quantity of commodity i in existence. $\hat{C}, \hat{W}$ and so on are similarly defined. Note that $\hat{X} = X$.

The *n*-sector value equation

The simple difference equation for one good carries over to the *n*-sector case provided we assume (which Marx did not) that all goods are turned over exactly once, in which case $X = K$.[13] Then

$$\lambda^{t+1}X = \lambda^t C + \pounds \, L \tag{15}$$

or more simply

$$\pounds X^{t+1} = \pounds C^t + \pounds L \tag{16}$$

This can be read off as it appears: value at time $t+1$ is equal to consumed constant capital plus the value product. It provides a solution for λ at all times:

$$\lambda^{t+1} = \lambda^t C X^{-1} + \pounds L X^{-1} \tag{17}$$

which is positive and determinate provided consumption of inputs and hours worked are positive. It is difficult to conceive how this could be violated.

This concludes the first mathematical section.

1.5   CIRCULATION AND MARKET PRICES

Whether or not goods sell in proportion to their values, prices appear with circulation. Commodities do not exchange for each other but a third commodity, money. This, like all others, is neither consumed nor produced by exchange. It functions as a hoard which grows when people sell, and shrinks when they buy.

By the very fact that prices differ from values, the intrinsic value of the commodity serving as money does not fix the ratios in which it exchanges. If I buy clothes produced in nine hours with metal produced in ten, then just as if I barter meat or drink for them instead of money, the value of money measured in clothes has fallen. In this respect it is like any other commodity.[14]

Ricardo's famous unanswerable question – whether the high price of clothes is 'due to' a rise in their value or a fall in the value of the metal – arises only because he never really accepted that nothing exchanges in proportion to its value, not even money. The issue has to be posed differently: how does money regulate the transfer of values in circulation between all goods, including itself?

As universal equivalent, only money functions both as measure of value and standard of price. If clothes previously worth ten pounds now sell for nine then society does not say money has fallen by one sock, but clothes have risen by one

pound. If Ricardo adopted any other practice in his daily life he would at best be recorded as mildly eccentric. If, in exchange, ten pounds come to represent less value then *all* money prices rise, whereas a fall in sock prices does not change every ticket in the shop. Both changes transfer value between capitals; but the change is expressed differently. This is the essence of the price-value relation.

Circulation as such

Circulation is a distinct phase of reproduction. Everyone enters with stocks of commodities and money derived from previous times, whose values also derive from previous times. In general, they exchange at prices different from values. Like Marx, we distinguish two moments of this process; sale and then purchase.

The analytical reason for this separation is not that all commodities are sold at once, but that the sale of any one commodity does not depend on the purchase of another.[15] All commodities are exchanged with money, none with each other. The gains and losses of each capitalist are therefore the net result of two magnitudes, their sales and their purchases. The determinants of each are quite distinct.

Virtually all equilibrium models translate a particular theory of demand into a universal theory of economics. Usually they *derive* sales from purchases which are already past, as if capitalists were under a compulsion to replace their inputs in kind and quantity. Actually this never happens.[16] Demand and supply are concretely and separately determined differently for every society at every stage. A general framework has to translate *any* given pattern of demand into symbols and relations and deduce what is necessarily common to all of them.

We assume only that at the 'end' of circulation, everyone possesses different stocks from the beginning, and that the changes were effected by money exchange at a definite set of prices. Our aim is to express the underlying transfer of value resulting from an arbitrary exchange of commodities in a market economy.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I	£200[25]		£300[300£]	£500
Capital II		£80[20]	£300[300£]	£380

Table 11.3a Two-party exchange, starting position with prices equal to values λ_1=£8, λ_2=£4

Suppose two capitals constitute a society and exchange with the endowments of Table 11.3a. Assume £1 represents one hour of socially necessary labour time or as Marx puts it, one hour of labour time is expressed in £1. Assume all stocks possess initial values given by λ_1=£8, λ_2=£4, so that, for example, the value of 30 units of C_{II} is £5, representing 5 hours. Suppose proprietors exchange at prices equal to values (p=λ) as shown in Table 11.3b. The following propositions then hold:

I.   The sum of values in society is the same before as after
II. The sum of values in society is equal to the sum of prices in society
III. Each capital has the same value before as after exchange

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I		£80[20]	£300[300£]	£500
Capital II	£200[25]		£300[300£]	£380

Table 11.3b Two-party exchange after all goods are sold at prices equal to values (p_1 = £8, p_2 = £4)

This representation cannot be assimilated to barter. There is no necessary correspondence between the C_I sold and C_{II} purchased. Capital I, if it wanted, could have bought the C_{II} without selling anything leading to Table 11.3c.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I	£200[25]	£80[20]	£220[300£]	£500
Capital II			£380[300£]	£380

Table 11.3c Two-party exchange after C_{II} is sold at prices equal to values (p_1 = £8, p_2 = £4)

Walras, who writes as if C_I exchanges directly against C_{II}, thinks the demand for C_I is matched by the demand for C_{II} in a ratio given by their relative prices.[17]

Thus these propositions apply regardless of how much is actually traded. A change of ownership transfers use values, including money, from one place to another; therefore if the two capitals do actually trade as shown, there must be a net transfer of money. But *value* can only change hands as a result of price variations, and as we shall see this is independent of the volume of trade.

Exchange at market prices

Consider the results of the same exchange at prices different from values. Suppose the unit price of C_I falls to £4 and that of C_{II} rises to £9. This represents no qualitative change: commodities are assessed in the same units. But the commodities have now lost or gained value. The 30 units of C_I whose value is £5.00, for example, are now priced at £4.50, so that £0.50 of their value has been transferred elsewhere. The result is shown in Table 11.3d.

Value has been transferred between capitals as well as commodities, so proposition III no longer holds, but the first two propositions remain true. Social wealth is the same but its distribution has changed. Capital I, which owned the commodity whose price has fallen, has lost £0.50 (representing ½ hour) and capital II, whose goods rose in price, has gained the same amount.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I		£180[20]	£220[220£]	£400
Capital II	£100[25]		£380[380£]	£480

Table 11.3d Two-party exchange, all goods sold at prices p_1 = £4, p_2 = £9.

In no sense have commodity values been 'wiped out' and replaced with prices. Table 11.3b did not purport to give values before exchange, but after exchange at prices equal to values. Now we have same table with a new assumption: exchange at prices different from values.

Figure 11.2 shows how value is redistributed between capitals, and between the different stocks of commodities in society. This shows that the effect of pricing the commodities at any given market price is a transfer of value, both between the stocks of these commodities and the capitals which are composed of

them. To foreshadow a later discussion, it adds a *transfer vector* to society's commodity stocks and another vector, induced by the first, to capitals.
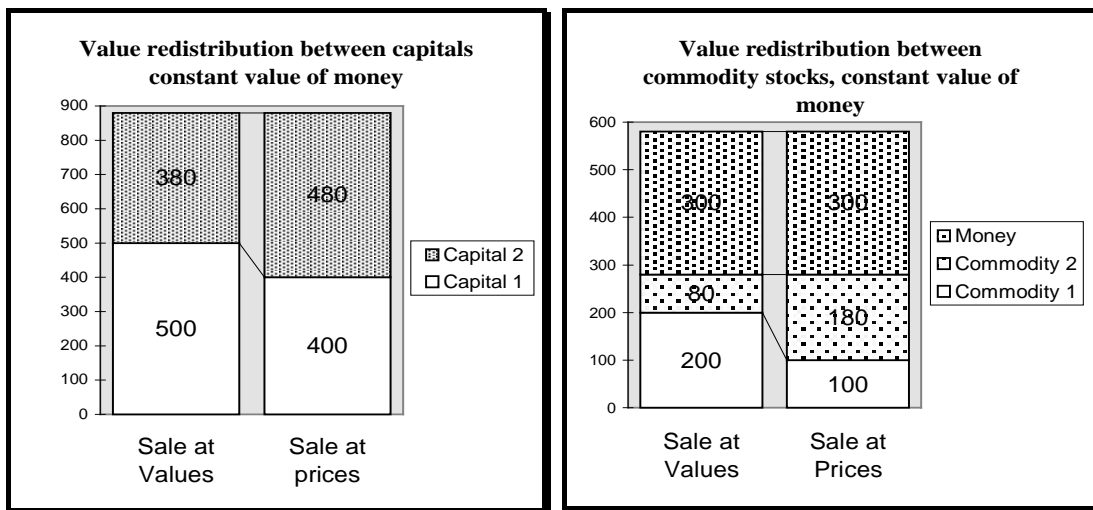

Figure 11.2 Value redistribution with a constant value of money

Two points are not immediately obvious. First, these losses and gains are not concealed. There are no secret transactions in hours hidden by public ones in pounds. If I sell books for £50 that cost me £100 my loss is tangible and concrete. It is not fetishized, dialectically complex, abstract, contradictory, metaphysical or even subtle. I am £50 down, period. The disguise effected by capitalism does not lie in deceit as to the role of exchange or arbitrage. It consists in disguising the source of *profit*, in making the prices of things appear as their real social cost. It disguises only the effects of selling the commodity labour power.

Second, as previously stated the value transfers are effected not by trade but by the change in prices. They remain independent of the volume of trade. In Marx's words, the commodities 'are assessed in gold before it circulates them' (1969b:200). Value is transferred between all goods in circulation, that is, all commodities in society. It is not confined to what is sold. If I speculate in palm oil and £50mn is wiped off its value, then I lose £50mn. I cannot fob off creditors by saying I haven't sold it yet. I may be able to conceal it, but it has happened. The price system hurls accumulated labour across the globe like the mediæval wheel of fortune tumbling crowns and enthroning pretenders with sublime indifference.

This led Proudhon to say exploitation lay in exchange other than at values. Marx said it lay in the nature of the commodity labour power, a result not of purchase at prices other than values, but the purchase of this commodity *at* its value. To demonstrate this he had to abstract, in Volume I, from price-value differences. Twentieth century 'Marxism' has erroneously taken this to mean that exploitation involves hours and exchange concerns money. In fact exploitation can be expressed directly in money terms, as Marx did throughout his work.

Simple exchange with a variable value of money

We now turn to prices different from those discussed by Marx in the first ten chapters of Volume III dealing with prices of production. Marx states throughout that he assumes the value of money to be constant.[18] This is hardly discussed in the literature but has produced indescribable confusion.

The rate at which any good exchanges for money results from the general interaction of supply and demand. The basic difference between Marx's theory of money and Ricardo's is that Ricardo, like Hume before him and many after him, assumed the price level was determined by the relation between the 'supply and demand' for money.[19] Marx held that it was determined, in effect, by the supply and demand for everything else. The quotation given in chapter 1 is instructive:

> The most common and conspicuous phenomenon accompanying commercial crises is a sudden fall in the general level of commodity-prices occurring after a prolonged general rise in prices. A general fall of commodity-prices may be expressed as a rise in the value of money relative to all other commodities, and, on the other hand, a general rise in prices may be defined as a fall in the relative value of money. (Marx 1970:183)

During a boom when goods are in short supply, all prices rise relative to money, and in a slump they all fall. While an inflationary paper issue of course raises prices, these fluctuations happen regardless of the money supply. This is inverted in Ricardian and monetarist formulations so that a slump is presented as a shortage of money and a boom as a surplus.

Fluctuations in the value of money over the boom-slump cycle are in fact an integral component of the process of that cycle itself, though not its primary or sole causal factor. They reflect the very conditions of general shortage or general glut which Say's law forbids. Exchange rate relations between different monies are also a central mechanism of the world operation of the law of value.

Suppose as a result of a general fluctuation in global demand, all money prices rise. For simplicity suppose C_I and C_{II} exchange 'at values' – that is in the proportion $\lambda_1:\lambda_2$ – but at twice the money price. Prices are now $p_1 = £16$, $p_2 = £8$. What final disposition of money and value corresponds to any given disposition of products? No special knowledge of value theory is needed, just solid bookkeeping. We price the commodities at the new rates, charge capitalists with their purchases and credit them with their sales.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I		£160[20]	[300£−160£+400£=540£]	£700
Capital II	£400[25]		[300£−400£+160£=60£]	£460

Table 11.4 Two-party exchange, all goods sold at $p_1 = £16$, $p_2 = £8$

Now a pound no longer purchases an hour of socially necessary labour time. To express this, Table 11.4 shows the value of each commodity estimated in money at the rate used to effect the exchanges, that is, at the given prices; and then as before, the use value of the commodity estimated in its natural units.

At first sight this table appears to violate the carefully-specified conditions which hold with a constant value of money. Although price *ratios* have not altered, their money measure has. Everything has inflated in the proportion $^{880}/_{1160} = ^{11}/_{14}$. Note, incidentally, that no change in the quantity of money was needed for this price revolution.

Something, however, is clearly amiss with the picture as both a neoclassical and a surplus approach advocate would surely recognize. The same pattern of commodity exchanges has taken place as before. No new commodities have made their appearance. How can wealth have been created out of nothing? Our 'society' has clearly not created an extra £260; it has moved the goalposts, changed the scale of the reckoning so that it appears so.

Why do 'total prices equal total values'?

This illustrates the most misunderstood issue in the literature on transformation. Total prices equal total values because of exchange, not production. We have isolated this so the matter can be studied in its pure form; moreover we used an example where goods exchange, as any true Ricardian would prefer, in proportion to values. Yet the problem persists. It would be a rash economist indeed who would claim that doubling prices creates a profit of £260 with no new products.

How can we represent this? The perceptive reader may notice something missing from the third column. In Tables 11.3a and b, money stocks were given as for all other commodities in both exchange value and use value terms. Capital I was given 300 units of value expressed in 300 pounds of use value, written £300[300£]. But things are no longer so simple. Although the relative prices of C_I and C_{II} have not changed, money no longer possesses the same purchasing power. We thus have two money measures of value: the money in which commodities were estimated before exchange began, and that in which they are estimated now.

The question is: how much real value does the new £ represent? To put it another way, how much old money is the new money worth? To put it correctly, what is the ratio between the labour hours – the 'immanent measure of value' – expressed in one money and the labour hours expressed in the other? This is the true origin of Marx's famous 'first equality'. The problem is not abolished by renouncing labour values, nor is it resolved or even affected by assumptions about the structure of production, save that commodities are the products of labour. It exists for any economist who jibs at saying 'wealth appears from nowhere'. Labour values are not the problem: they are the solution.

A first and wrong answer would be to say: money has halved in value. Since all prices doubled, surely money has halved its purchasing power. This Ricardian answer neglects a vital fact: money itself was involved in the exchanges.[20] We can clarify the difficulty by denoting the old money and the new money

differently, as £old and £new, just as if there had been a currency reform, which indeed there has, though not by the intervention of any money authority.

The Ricardian theory is that £old 1 = £new 2. Let us reconstruct the table giving values in the estimated £old, to see what goes wrong.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I		£80[20]	£270[540£]	£350
Capital II	£200[25]		£30[60£]	£230

Table 11.5 Two-party exchange, all goods sold at p_1 =£16, p_2 = £8 with wrong estimates in £old

If our reconstruction had gone right, we should see an unchanged total wealth in £old. But the estimate of total social wealth comes to £old 580. This reconstruction does not work, because it suggests that exchange has destroyed £old 300.

A second answer is offered by the 'New Approach' school (see Saad-Filho in this volume). We could estimate the value of money from the value of the 'net product' it purchases. But what is the net product? We don't know where these products came from or where they are going. Which of them is net and which gross? We are analysing circulation in abstraction from all production relations, like Marx in Part 1 of Volume 1, before production has been introduced.

Value redistribution with a variable value of money

Once the problem is posed in this way there is only one answer. The fact that no wealth was created can be recognized only by converting £old to £new at a rate that ensures total social wealth, when measured in £old, does not change unless use value is destroyed or created. Any other concept of real wealth is absurd.

STOCKS	Commodity 1	Commodity 2	Money	Total wealth
Capital I		£old 121.38=£160[20]	£old 409.65[540£]	£old 531.03
Capital II	£old 303.45=£400[25]		£old 45.52[60£]	£old 348.97

Table 11.6 Two-party exchange, all goods sold at p_1 =£16, p_2 = £8, with correct value estimates in £old

Thus the correct ratio for conversion from £old to £new is one for which the total social wealth, £1160 in £new, is expressed as £880 in £old. This is given in Table 11.6 to the nearest penny. The following points should be noted:

❑ £old is a monetary measure of value. Column 3 restores the original notation in which the first number represents value, and the number in brackets represents use value. This respects the qualitative distinction between £old 409.65, the value of the money held by Capital I, and 540£ which is its use-value.
❑ We could use £new as a monetary measure of value. In this case we would have to retrospectively re-estimate the values before exchange in £new. We would find that in aggregate they amounted to £1160. Therefore, it is not enough to state that money is 'the measure of value'. It is, as Marx says, a

dynamic or variable measure which means we must also specify *at what time and what point in the circuit of capital* it applies.

❏ Labour time, on the other hand, remains a stable measure because one hour of past labour is equivalent to one our of current labour. £old 1 represents one hour of labour time, so the value magnitudes in Table 11.6 could equally well have been hours. £new 1, however, represents represents 11/14 hour of labour and if we use this conversion coefficient we arrive at the same result. Labour time, the immanent measure of value, is what the money actually measures.[21] It represents the 'real value' of goods measured in a variable denomination. It shows the real social cost of making them available for use.

❏ The sum of values in society, the same before as after, is equal to the sum of prices *provided* the two are measured in the same units.

The rise in prices *has* now redistributed value even though the two commodities exchanged in proportion to their values. Capital I was worth £400 and is now worth £531.03. It has gained £131.03, or 131.03 hours of socially necessary labour time. Capital II was worth £480 and is now worth £348.97. It has lost £131.03. Why this redistribution? Because capitals contain the money commodity too. The productive commodities originally owned by capital I have risen in value, from £200 to £303.45 Likewise those of capital II have increased from £80 to £121.38. But the *money* held by each of these capitals has lost value to the productive commodities. The value of the society's money stock has fallen from £600 to £455.17 and the balance of £144.83, transferred to its stock of productive commodities which have risen by the same amount, all in £old. As before, the extent of the redistribution does not depend on the volume of trade.
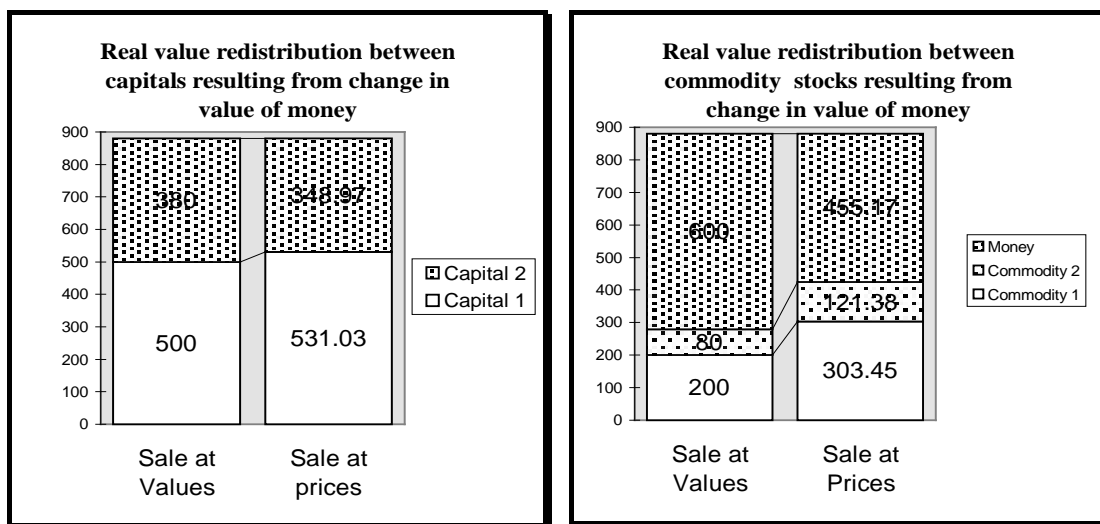

Figure 11.3 Real value redistribution brought about by a change in the value of money

The material origin of liquidity preference

Clearly, if the price of all commodities had fallen, value would have been transferred to the money commodity reflecting its increased purchasing power.

This, not some psychological disposition, lies behind liquidity preference. Capitalist competition itself drives wealth owners to hold any asset which acts as a store of value in a climate of falling prices. If money, the general equivalent, is increasing in value those who possess it can secure a rising share of social wealth.

It follows that a capitalist who merely holds money can make profit in real terms, that is, appropriate surplus value created elsewhere, as long as the value of money is rising over time, that is, has a positive derivative with respect to time as in the slump period of the business cycle. The process of competition itself operates to produce liquidity preference but only if treated dynamically. There is no static representation of this phenomenon. We will see that the price rate of profit is modified by a factor μ'/μ where μ represents the value of money.

1.6   THE ORIGIN OF PROFIT

The preceding section made no assumptions about production or the labour process at all. In and of itself, Marx's 'first equality' is not an assertion about the structure of production but the nature of exchange, and attempts to interpret it otherwise introduce serious confusion. Having said this, what then is its relation to the labour process, production, profit and surplus value?

The debate on value since Marx, and to a great extent before, is in essence about the origin of profit. Mathematically, it arises as follows: the price of any composite – and hence any capital – is a linear function of its elements, the sum of its parts. Add bread priced 20p to ham priced 20p and you have a sandwich worth 40p. And since the price of anything is a multiple of its use value, the price of any composite is a linear function of the use values in it.

The mystery of capitalist production is that from raw materials worth 40p and 20p in wages, I can get goods worth 70p. Output prices are not a linear function of input prices or quantities. 'Something for nothing' appears: value is added. This is not at all obvious or 'natural'; it is a real change in the function of money which appears when labour power is a commodity. Why does the sandwich not sink to 60p, its real cost including wages? After all, if people regard 70p as extortionate they can buy the raw materials, hire servants, and get a sandwich for 60p. Why doesn't competition level the price of all goods down to this money cost?

Proudhon, expressing the natural outrage of a dispossessed artisan class, said this discrepancy was theft caused by capitalist property. Under fair competition, if all produce sold at its 'natural' price – in proportion to the quantity of each input including labour – it would vanish. Marx simply showed that even if the price of all produce was proportionate to the quantity of consumed use values, there would *still* be a difference between the price of a commodity and the sum of the price of its inputs. This holds, incidentally, whatever the measure or source of value.

Once accepted, however, that labour like any other commodity adds value in proportion to its use value, we must ask what this use value actually is. Capitalists do not hire workers to make sandwiches for themselves but to make money from the sandwiches. The use value of the workers *is* to create value; if their sandwiches did not sell they would not be hired.

This is of course true of other commodities also; but other commodities do not walk around the market disposing of their income on an equal basis with their owners. The cost of labour power is determined *independently* of its capacity to make money for its purchaser. This, and no other reason, is why profit exists. If labourers were hired directly as slaves, robots, beasts of burden or servants, then whether or not labour time were the measure of value, surplus labour would not be extracted in the form of money profits but directly, like domestic labour.

Both Marx and Ricardo therefore said no more than this: that all inputs add value in proportion to the quantity consumed. Since one particular commodity is directly involved in the production of every other commodity, the value added by all other commodities can be reduced to the value added by this particular commodity, namely labour power.

The origin and nature of price-value deviations

Ricardo stopped at this point. The difficulty, however, is that the actual money price of any commodity still differs from a linear sum of inputs: from value. His school foundered because it could not explain how goods whose value, based on a sum of the prices contributed by their inputs, is 70p, can sell for 80p or 65p.[22]

Like Smith he treated such deviations as accidental, so that an average over time yields a 'natural price' equal to value. But in at least two cases prices diverge *systematically* from values: rent, and capitalist competition, when the supply of any product adjusts only until capital cannot obtain a higher return by migrating. In this case even the average price is only exceptionally equal to value.

Vulgar economics approaches this problem as things appear to the capitalist; it treats capital as an extra commodity. The natural price of a product becomes its raw material cost, plus wage costs, plus the cost of 'capital'. The main objection is that this cost is already accounted for. Capital comprises commodities whose costs enter the product directly. If I already charge 60p for eating the sandwich, how can I charge an extra 10p for having it? Still more awkward, the solution implies wealth can be created without use value. If money creates value, why bother putting it in a factory? Why not just leave it in the fridge and watch it make free lunches?

But in any case the idea does not solve the problem posed. What if the market price deviates from the new 'natural price'? The dichotomy of Ricardian value theory has not been abolished; equilibrium or long-run price has simply supplanted value. Deviations of real, market prices, from this ideal are no better explained and worse still, they render the price of 'capital' non-uniform.

Equilibrium theories, as Carchedi discusses in this volume, escape by acting as if market prices did not exist. The equalisation of profit rates is taken as achieved fact. Even so, what determines the rate? Why 5 or 10 per cent, not 100 per cent? The two main answers to this reflect the two faces of the commodity. Neoclassical theory derives profit from exchange value, as the 'price' of capital; the surplus approach school derives it from use values – a putative 'physical surplus'. The result is a man fighting his shadow. The moves are impressive[23] but no-one can win.

Over this debate looms the suspicion that because of the errors in Marx's own price theory there is no rigorous alternative in his framework. There is.

Price as the outcome of value transfers

Marx pointed out that deviations of market prices from values could not be considered in isolation from each other. Consider first a single use value X. If £P is its price then we can work out the value transferred between X and the rest of society as the difference between its value and its price. Call this £E: then

$$£E = £P – £X$$

Clearly, for every unit of X, a certain amount of value is transferred to or from owners of X. If the value of iron was £10 per ton and its price rises to £15, then for every ton of iron I own, I will gain £5. Value transferred is thus +£5 per ton, just as the price was £15 per ton. Call this e and note that $e = £E/X$. The unit price p is then the value plus this modification:

$$p = \lambda + e$$

This is altogether different from the relation proposed by Bortkiewicz, for whom price is a multiple of value.[24] Moreover, if we know e for every commodity, we can compute the value lost or gained by any given capital. If ham is undervalued by 5p per slice and bread is overvalued by 2p per slice, sandwich-owners will lose a penny for every sandwich for which they hold the title deeds.

We can calculate the value lost or gained by every capital in society as a consequence of any change in prices at any time. In matrix terms, though this is not needed to follow the argument, the value transferred between capitals is

$$£E = e\mathrm{X}$$

The sum of these, $\Sigma_j £E_j$, is the difference between the price of all the goods in circulation and their value, including (as always) money. This sum is the value gained or lost by society as a whole arising from any change in prices. The 'equality of total prices and values' means this is zero. There is no net gain or loss of value when commodities exchange without a change of form, that is, without destruction or creation of use values. If we give a different answer, then we say that a rise in money prices is the same thing as an increase in wealth when no extra consumption or enjoyment results. By *reductio ad absurdam,* hyperinflation is the wealthiest state a nation can attain.

Thus the outcome of any given set of market prices is summarized by the *transfer* vector *e*. Just as value added £L^t summarizes the value-creating effects of production, so *e*^{t} summarize the redistribution of this value effected by circulation. It contains all the information there is about the effect of price changes on capital; for every set of prices there is a unique *e* and vice versa.

But there is more: *e* defines the relation between profit and surplus value.

Profit as surplus value plus value transfers.

The relations above were all derived without reference to production, as a prelude to explaining the origin of profit. Now suppose use values X are produced with inputs C and a value-product L of workers working for V(=L) hours. The value contribution of C is given, as explained throughout this book, by their current price *p*C or just C. The workers therefore create the following value:

$$£X = \Sigma £C + £L \qquad (18)$$

The cost to the capitalist, however, is

$$£C + £V$$

the sum of constant and variable capital. The difference between the two is thus

$$£S = £X - (£C + £V) = £L - £V$$

the surplus value added by the workers. Now consider what happens if the product sells for £P, different from £X. This is given by

$$£P = £X + £E \qquad (19)$$

The capitalist makes a profit, the difference between sales and costs:

$$£\Pi = £P - £C - £V$$

At first sight, this bears no relation to £S, because it contains no term directly related to £L, the value-product. But equations (18) and (19) show that

$$£\Pi = £X + £E - £C - £V \qquad \text{from (19)}$$
$$= £C + £L + £E - £C - £V \qquad \text{from (18)}$$
$$= £L - £V + £E$$

that is

$$£\Pi = £S + £E$$

that is, profit equals surplus value plus the transfer vector £E. Summing now gives

$$\Sigma £\Pi = \Sigma £S + \Sigma £E = \Sigma £S$$

the famous 'second equality'; the sum of profits equals the sum of surplus values.

> We thus have a mathematically exact demonstration of why the capitalists, no matter how little little love is lost among them in their mutual competition, are nevertheless united by a veritable veritable freemasonry vis-à-vis the whole working class as a whole. (1981:198)

Why has this simple relation eluded Marxologists since Bortkiewicz? Because Bortkiewicz's model, by eliminating time, conflates and identifies two transfers of value which take place at different points in time, forcing them to be identical.

We noted above that the value contribution £C is equal to pC, the current price of consumed constant capital. This of course differs from the value with which £C emerges from production, and the difference between pC and λC is in turn a transfer vector eC. *But this vector arises at a different point in time*. It expresses transfers from the last cycle. It is not equal to £E^t except under Bortkiewicz's restrictive assumption that inputs are purchased at the *same* price as outputs. Of course, everything just said applies also to this special case; but the conceptual framework imposed by it utterly obscures these simple basic identities.

1.7   THE MATHEMATICS OF PRICES AND VALUES

In the next more mathematical section we derive the above results generally and rigorously and match them to Marx's writings on the subject.

Circulation has two distinct and independent results. First, it transfers use values from one owner to another. After circulation C, V, X and all other stock magnitudes have changed because outputs have been transferred from X to their purchasers. These movements are governed by social and historical laws specific to any given economy. Second, however, these movements are effected at definite prices by exchange with money. Price changes transfer value from one capital to another *independent of the movement of use values*. Value-price analysis has to define the laws governing these transfers.

Recall that £K=$\lambda\hat{K}$ gives the total value of each commodity in the economy. Note that this is different from λK, the value of each capital in the economy. Now let

$$\pounds E = p\hat{K} - \lambda\hat{K}$$

defining

$$e = \pounds E\hat{K}^{-1}$$

gives

$$p = \lambda + e$$

Therefore to any set of prices p corresponds a unique transfer vector e.

The transfer vector and constant capital

Consider the simple case where all capital is turned over uniformly in a single period, which means we can continue to blur the distinction between turnover and stock. In section 5 we showed that at a given time t unit values are given by λ_t

$$\lambda^{t+1}X = \lambda^t C + \pounds L. \tag{15}$$

Tradition has it that Marx forgot to transform inputs. But this transformation is already implicit, and in several places explicit, in his analysis of exchange. It consists in assessing the contribution to value of consumed constant capital

$$£C^t = \lambda^t C$$

when C are purchased at prices different from values. As explained by Marx and at many points in this book, the cost price of X includes value transferred to C in the previous phase of exchange. The value of consumed constant capital is

$$\lambda^t C + e^t C$$

Thus in place of $\lambda^t C$ in equation (15) we write $\lambda^t C + e^t C$ to get

$$\lambda^{t+1} X = \lambda^t C + e^t C + £L^t \tag{20}$$

or

$$\lambda^{t+1} X = p^t C + £L^t$$

As Marx (1972:167) puts it in a previously-cited quotation:

> the cost price of constant capital [$p^t C$] – or of the commodities which enter into the value of the newly-produced commodity as raw materials and machinery [or] labour conditions – may likewise be either above or below its value. Thus the commodity comprises a portion of the price [$e^t C$] which differs from value [$\lambda^t C$], and this portion is independent of the quantity of labour newly added. [$£L^t$]

This correction makes no reference to the price at which the output is sold. It is independent, therefore, of p^{t+1}, and hence $£E^{t+1}$, in fact of any magnitudes from time $t + 1$, since $t + 1$ had not happened when the inputs were purchased. It is also clear that 'this portion [$e^t C$] is independent of the quantity of labour newly added' since the value contribution of labour power is, as before, $£L^t$.

The transfer vector and variable capital

We can divide both the consumption and the stocks of society into two categories: goods acquired by workers, and everything else.[25] The latter includes V, the commodity labour power, which has a value and a price. As with any other commodity a certain quantity of value $e_L{}^t V$ is transferred to or from the commodity labour power when its price differs from its value. Thus

$$p_L{}^t V = \lambda_L{}^t V + e_L{}^t V$$

But equally, since the wage W is a set of commodities with a value and a price,

$$p^t W = \lambda^t W + e^t W$$

Since here we assume the money wage is completely spent in a period (ignoring consumer durables); the value of V is the same as the value of W and the price of V is the price of W.[26] It follows that

$$e^t W = e_L{}^t V$$

that is, the value lost or gained by workers is equal to the difference between the price and the value of the commodity labour power:

> The workers must work for a greater or lesser amount of time [$e^t W$] in order to buy back these commodities (to replace them) and must therefore perform more or less necessary

labour [$e_L{}^tV$] than would be needed [λ^tW] if the prices of production [*in general market prices*, p^tW] of their necessary means of subsistence did coincide with their values. (Marx 1981:309)

Value, price and the value product

The output of period [*t*, *t* + 1], contained in X, is sold at time *t*+1 at prices in general different from values. The (vector of) output values, given by (20), is £X. Their price $£P^{t+1}$ differs from $£X^{t+1}$ by the transfer vector $£E^{t+1}$ for the *current* period, that is

$$£P^{t+1} = £X^{t+1} + £E^{t+1}$$

Substituting for $£X^t$ from equation (20) gives

$$£P^{t+1} = \lambda^tC + e^tC + £L^t + £E^{t+1} \qquad (21)$$

Value of elements of constant capital	λ^tC
Value transferred in last period of circulation	e^tC
Value product	$£L^t$
Value transferred in this period of circulation	$£E^{t+1}$
Market Price $£P^{t+1} = \lambda^tC + e^tC + £L^t + £E^{t+1} = £X^{t+1} + £E^{t+1}$	

This exhibits the 'two ways' in which the conversion of £X into £P takes place. Values λ^tC emerge from production between *t*–1 and *t*. At time *t*, prices $£P^t$ effect transfers of value e^tC. A new cycle of production adds new value £L, *and then* prices $£P^{t+1}$ effect further transfers $£E^{t+1}$ at time *t+1*. There is no redundancy, no circularity, and no error. The price is a linear sum of value contributions from dead labour, live labour and value transfers effected by the price system. The value of the output is perfectly distinct from its price, the difference being £E; moreover it is independent of variable capital or the wage.

> Variable capital, whatever difference between value and cost-price it may contain, is replaced by a certain quantity of labour [$£L^t$] which forms a constituent part of the value of the new commodity, irrespective of whether its price [$£P^{t+1}$] expresses its value [$£X^{t+1}$] correctly or stands above or below the value. (Marx 1972:167)

The next three sections illustrate the main magnitudes of Marx's value theory using the formulae derived. We omit the time subscript when it is equal to *t* (start of the current period), giving it only when it is *t* + 1 (end of the current period)

Surplus value

Variable capital is in general less than £L, the value product. The difference is a vector we denote by £S, surplus value.

Value product	£L
Variable capital	$£V = p_LV$
Surplus value	$£S = £L – £V = £L – p_LV$

This can be broken down to separate out value transfers from the previous period:

$$£S = £L - p_L V = £L - (\lambda_L V + e_L V)$$

The term $e_L V$ represents transfers of value in the *previous* period of circulation; it is the difference between the price and the value of labour power.

Cost price

The cost price of the period is the sum of the constant and variable capital turned over in this period, namely

Constant capital	$\Sigma^i £C = pC$
Variable capital	$£V = p_L V$
Cost price	$\Sigma^i £C + £V = pC + p_L V$

Profit, surplus value, and Marx's second equality

Output is in general sold at a price different from its value. The difference between market price and cost price is capitalist profit, a row vector we call Π.

Market price	$£P^{t+1} = £C + £L + £E^{t+1}$
Cost price	$£C + £V = p(C + V)$
Profit	$£\Pi^{t+1} = £P^{t+1} - £C - £V = £L - £V + £E^{t+1} = £S + £E^{t+1}$ (22)

Equal profit rates are not assumed, though these results are equally valid for the special case where they do equalise. No necessary law governs the actual profits realised in different sectors. Most important of all, when we look beneath sectoral averages we find *individual* profit rates realised by different producers of the same commodity. Whatever the sectoral averages, these differ vastly and are the motor of the investment mechanism. As Marx repeatedly argued, the pursuit of an above average profit rate, brought on by an exceptionally productive new technique, is the real motive for capital movements.

> In actual fact, the particular interest that one capitalist, or capital in a particular sphere of production has in exploiting the workers he directly employs is confined to the possibility of taking an extra cut, making an excess profit over and above the average [E^{t+1}] (Marx 1981:299)

Whatever the time average of Π, each actual sale will deviate from it.[27] Nevertheless, just as a general law regulating exchange (the first equality) applies to all market prices, a second general law regulates profits. Summing (22) gives

$$\Sigma £\Pi = \Sigma £L - \Sigma £V + \Sigma £E^{t+1}$$

But $\Sigma £E^{t+1}$ is 0; therefore

$$\Sigma £\Pi = \Sigma £L - \Sigma £V = \Sigma £S.$$

Marx's 'second equality'. Being established for the general case where profits are not equal, it is certainly true for the special case where they do, that is where market prices equal prices of production.

1.8   CAPITAL

We now turn to the study of capital as such. To this end we must correct the most basic flaw of General Equilibrium and the hidden basis of the simultaneous equation construction: the assumption that the market clears. This brings us to a threshhold. Everything said until now can be stated in a more limited way in an equilibrium framework; the results that follow cannot. They have no parallel in equilibrium and directly contradict it. There are therefore two distinct approaches to the study of a market economy, which can and should be tested by the normal method of science: which best explains the observed facts.

What is capital?

Marx succinctly defined capitalism as 'generalized commodity production', a society in which the production, circulation and distribution of the material means of existence takes the form of use values produced for sale. Generalized does not mean 'everything' – domestic labour is still not paid. Capitalism means:

❏ The *means of production* are commodities, and
❏ specifically *labour* enters production as a commodity, labour power.

   Every element of production except labour-power is itself a product of value. Value requires nothing for its own production except labour and itself. Though labour remains the source of value, capital – past labour – dominates living labour and organizes society around its own reproduction, securing all the conditions of its existence. Dead labour becomes a self-reproducing, self-expanding and self-evolving social relation; in modern jargon, artificial life.[28]

> If we pin down the specific forms of appearance assumed in turn by self-valorising value in the course of its life, we reach the following elucidation: capital is money, capital is commodities. In truth, however, value is here the subject of a process in which, while constantly assuming the form in turn of money and commodities, it changes its own magnitude, throws off surplus-value from itself considered as original value, and thus valorises itself independently. (Marx 1976a:255)

   Neoclassical theory duly accords capital the power of procreation. But its forms of existence, the commodity and money, each unite in themselves two aspects, use and exchange value. The theory divides: one personality assigns creation to machines and the other to exchange. Macroeconomics demands things from money, and microeconomics supplies money from things. For Marx, in contrast, production is a unity:

> Like the commodity, which is an immediate unit of use-value and exchange-value, the process of production, which is the process of production of commodities, is the immediate unity of the processes of labour and valorisation. (Marx 1976a: 978-9)

This does not just mean the output is a commodity. The elements of production – machines, work in progress, labourers and money – not only produce but *exist* as commodities, unities of use and exchange value. They transmit value to their products not because they once had value but because they still do. Their ability to mobilize living labour is not derived from their individual characteristics or history but their relation to all other commodities of the same type.

If these lose or gain value for any reason, this is transmitted not only to the products of capital but to capital itself. Its creative power cannot therefore be reduced to a purely technical nor a purely monetary function. If matter could make value, money would grow on trees. But if value could make matter, then trees should grow on money. The task is to unite in a single dynamic relation the independent determination and mutual interaction of all aspects of capital.

Capital as a stock of commodities and the dynamics of the stock-flow relation

Capital accumulates as stocks and acts as such for the capitalist. My wealth is measured not by what I handle but what I have, or bank clerks would be rich beyond the dreams of avarice. This is an enormous problem for equilibrium theories of all types, whose approaches fall into three categories:

❑ that of Walras, who separates all commodities into two species: fixed capital which lasts for ever, and circulating capital which is consumed instantly.[29]
❑ that of Bortkiewicz, who treats all capital as completely turned over in one 'period' so that stocks are always equal to flows.[30]
❑ that of von Neumann, Sraffa and the surplus approach school, who treat fixed capital as a series of flows from machines of different ages or vintages.[31]

Nothing indicates the effect of equilibrium theory on mental health more than the contortions induced by a meeting with simple facts. *All* commodities act on the same basis as components of capital. If I buy a sausage machine, that is an investment. If I stock up meat, that is an investment. If I pay a week's wages, that too is an investment, and if I buy an old sausage machine I may pay less and make worse sausages, but it is an investment just like the others. If I stockpile sausages even *they* are an investment until they putrify. My capital consists of everything I need to sell sausages, its size is their current monetary worth, and my profit is the rate it grows. That is how my banker sees it and that, under capitalism, is how it is.

Everything which exists as a flow, a quantity in motion, forms itself into a stock, a quantity at rest. A river does not merely pass through the land but takes up space within it. If water flows into a space at one point and out at the other, the space holds a variable but definite quantity of liquid. The rate at which this rises

or falls is the net flow, the difference between inflows and outflows. The problem is not to make a scholastic distinction between one type of flow and another but to understand how *all* flows of value are dragooned into service as capital.

This does not require a metaphor of substance; it applies wherever one cause augments a thing and another diminishes it. Production and circulation are *par excellence* activities of this type. At each stage of the circuit commodity stocks are increased because of what went before and decreased because of what comes after. This is not metaphysics but bookkeeping.

Why supply does not match demand, and where the difference goes

It is precisely such bookkeeping which simultaneous equations exclude. If the economy reproduced perfectly and identically, stocks could not differ from flows because they would neither rise nor fall. In reality reproduction is incessantly interrupted or capitalism would not exist. The gap between supply and demand appears as changes in stock levels, providing the signals that drive price changes and tell producers what is socially necessary. This is the pulse of capitalism.

Simultaneous equations impose an immediate identity of supply and demand; if these do not match there is nowhere for the excess to go or the shortage to come from. Mismatches are relegated to an impenetrable subjective domain which by definition has no visible expression, which is why neoclassical theory is constantly driven to seek psychological explanations of material phenomena. There is no means of forming prices, no movement of capital, no technical change, and no capitalism. Equilibrium posits a living corpse, blood with no heart.

To illustrate this, consider the stocks which would result from the flow activities described by Tables 11.1 and 11.2. The technology of period 1 did not actually use up the output of period 0. We have an unsold surplus: five units of unsold C_{II}, fifty of C_I and two hundred unemployed people as shown in Table 11.7. One table no longer represents the economy. We need an independent record of these stocks.

STOCKS		C_I	C_{II}	L		C_I	C_{II}	Labour Power
Producer P_I	owns	35		300	and	5		
Producer P_{II}	owns	10		200	and		50	
Labourers	own		50		and			200

Table 11.7 Stocks after one period of production with supply-demand mismatches

Stocks, value transfers and accumulation

Stocks exist whether or not flows proceed smoothly. Productive capital collects as machinery or work-in-progress; output as inventory, money as hoards, new purchases as goods in transit and even private consumption as weekly shopping or consumer durables. Capital, the money value of these stocks, is what the capitalist advances and expects a return on. This cannot be reduced to the annual

turnover of capital except on Bortkiewicz's preposterous assumption that workers are paid annually, machines replaced annually, and raw materials purchased annually, an assumption that has become the bedrock of Walrasian Marxism.[32] We illustrate this with a simple extension: suppose fixed capital turns over once every two periods.

STOCKS		C_I	C_{II}	L		C_I	C_{II}	Labour Power
Producer P_I	owns	70		300	sales stocks	0		
Producer P_{II}	owns	20		200	sales stocks		0	
Labourers	owns		50		sales stocks			0

Table 11.8 Simple reproduction with fixed capital, stocks at the beginning of period 1

Suppose production begins with the stocks given in Table 11.8 and proceeds with the turnover given in Table 11.1 for one period. At the end of this period, stocks are as in Table 11.9. Half of C_I, all of C_{II} and all labour power has been used up, but they have been reproduced as sales stock.

STOCKS		C_I	C_{II}	L		C_I	C_{II}	Labour Power
Producer P_I	owns	35		0	sales stocks	50		
Producer P_{II}	owns	10		0	sales stocks		100	
Labourers	owns		0		sales stocks			250

Table 11.9 Simple reproduction with fixed capital, stocks at the end of period 1

The value advanced to run this cannot be reduced to the capitalists' annual purchases. They must buy everything in Table 11.8 before they can even start. But far more important, while this goes on *all prices and values change.* The remaining 35 units of C_I are no longer worth $35 \times \lambda^0$ but $35 \times \lambda^1$. They have appreciated or depreciated, and the differences confront the capitalists as gains or losses.

The problem which Bortkiewicz wishes away is now clear. Everyone holds stocks inherited from previous times, not by accident but because these are necessary to the act of consumption. We buy meat by the pound, not by the hour. Existing as commodities, these stocks are a component of supply and take part in price formation as long as they are available for circulation. Capital, made up of commodities, is therefore constantly re-estimated by the pricing system. It follows that, in addition to the value transferred between current goods – flows – value is incessantly moved between accumulated goods – stocks.

If I bought a computer for £3000 last year, then even if functioning perfectly it will lose value, not because it has decayed but because cheaper and better machines have driven down its price and drained it of value. If they did not appear, it would not lose value.[33] But this has a converse. I advanced the original, not the new value of the computer. My debts have *not* fallen. £3000 is what I must find from my sales, and is the basis on which my rate of profit is calculated. The fundamental error in the equilibrium vision is that it loses sight of this fact. It idealizes the process whereby capital settles accounts with its own past, above all its brutality and blindness. This is why the rate of profit really does fall, whatever Marx's inquisitors have to say; this is why the constructive power of technical

progress unleashes the destructive power of bankruptcy, mass unemployment, social devastation, periodic crisis, and all its attendant ills.

This is not secondary. It is *the* decisive phenomenon of accumulation because capital depends for its existence on endless revolution in production. With the stage that Marx terms the 'real subsumption of labour by capital', the production of relative surplus value, it harnesses every resource of labour and nature to accumulation, which enslaves it. Prometheus begets Faust. No matter what damnation awaits or what devastation trails, it exists to expand and expands to exist. It consumes its past to create its future. Even as its latest creations start to live out their days, newer and cheaper rivals have numbered them.

If capitalism could continously revolutionize the productivity of all human labour, so that every capital on the globe individually realized the benefits of each technical advance and no human labour were devalued by it, we would live in a world something like the idealization of equilibrium analysis. This would be the world of the Okishio theorem, the factor-price theorem, 'balanced growth' and all other idealizations of capitalist progress. This world might be unacceptable for other reasons – it would still contain rich capitalists and poor workers – but it would not be ravaged by war, disease, famine and death. The opening to the world market would not have projected Eastern Europe into the third world and much of the third world into hell. 'Modernization' would not be a synonym for doom, children would not be born to feed Chronos, and the four horsemen would not ride out on steel-clad steeds with hearts of crystal.

It would not be the world we live in. Capitalist progress is simultaneous destruction and construction irrevocably intertwined. In raising the *average* productivity of human labour it directly lowers the productivity of *most* human labour because it concentrates the value of each commodity in the hands of a minority, those who deploy the most advanced technology. Otherwise there would be no incentive to deploy the new technology. The more technology becomes a universal component of all means of production, the more pronounced this phenomenon and the less protection the benefits of nature afford to those denied the fruits of technology. This, one of the absolute limits on the capitalist mode of production, has been surgically excised by the mainstream theories, both non-Marxist and supposedly Marxist, which seek to understand it.

Age doesn't matter: money does

Insofar as equilibrium theories of all kinds have grappled with the impact of price movements on existing capital, they have turned from the changing *money costs* of capital and dealt with the passage of time by distinguishing between the *physical properties* of commodities of different ages. This misses the point. The restless movement of value and price applies to all goods of all ages. When house prices rise, they all rise including old houses, because they take part in a common market with a common use value. The age of a stock is of secondary importance.

There *is* no general way to distinguish between new and old goods from their intrinsic properties. What is old copper? Copper is a pinkish conductive substance. Its date of production is not stamped on its atoms. As for machines, the market cares only how and whether they work. Old machines differ from new ones only if they undergo bodily change or if the new machines perform differently. Physical difference, not age, alters use value. What constitutes used software – do its bits fall off?[34] The vast and resourceful literature on scrapping, vintages, and joint production is beside the point; when prices change *for whatever reason*, goods and capitals alike lose or gain value. It makes no difference to profits if some accounting date passes and a machine has a birthday. Theories of aging belong in the theory of production; attempts to explain price by age originate with the misguided belief that value is a component of physical being.

It is equally mistaken to think changes of use are the motor force of price movements. In a certain sense every factory is a distinct use value, a unique combination of parts which belong together.[35] These may decay, survive, or change their function. *It doesn't matter*. What counts is that either in parts or together it can be bought and therefore has a price.[36] Internal changes of use modify its technical composition; if a machine is reduced to scrap we have one less machine and one more ton of metal. But the value transferred between me and society remains the difference, after adjusting for changes in the value of money, between what I paid for the factory and what I will get if I sell it, in whatever form.[37]

Indeed, price movements determine use. If the price of scrap rises sixfold, dead machines wake to money's kiss.[38] If the price of steel collapses, the finest furnace in the world may be sold for scrap. And if a segment of production is isolated from the world market, either being forcibly removed – as in Russia, China and Eastern Europe, or in less extreme forms by protection and import substitution – or because it is in a backwater of innovation, it can survive and indeed advance for decades and in some cases millenia.[39]

1.9 VALUE IN THE PRESENCE OF STOCKS

The calculation of all value magnitudes has to be modified to take into account, in a rigorous manner, the modification of previously-existing values by both price and value changes after they have been produced. This is a natural extension of Marx's method for calculating social or market values from individual values:

> The individual commodity does not only appear materially as a part of the total produce of capital, but as an aliquot part of the total produced by it. We are now no longer concerned with the individual autonomous commodity, the single product. The result of the process is not individual goods, but a *mass of commodities* in which the value of the capital invested together with the surplus-value – i.e. the surplus-labour appropriated – has

reproduced itself, and each one of which is the incarnation of both the value of the *capital* and the surplus-value it has produced. The labour expended on each commodity can no longer be calculated – except as an average, i.e. an ideal estimate … This labour, then, is reckoned *ideally* as an aliquot part of the total labour expended on it. When *determining the price* of an individual article it appears as a merely ideal fraction of the total product in which the capital reproduces itself. (Marx 1976a :954)

Once a unified market is established, value and price emerge as an average over all the output of society. Marx concentrated his attention on the relation between individual producers and this market value. But everything he wrote logically applies to the entire stock of society; it would not make sense to exclude any portion of this on the basis of an arbitrary accounting separation which adjudges it an output of the 'last period' and therefore ineligible to take part in the formation of a uniform market price.

The value calculation

Production begins with a definite quantity of each commodity possessing a definite value. During production some of it metamorphoses and transfers part of this value to whatever it becomes. It loses both the use value and the exchange value of this consumed part. But it also gains new use values from production, and with them individual value transferred from inputs and added by labour power.

But these two contributions are independent. Total use value is the initial stock less what was consumed plus what was produced; while its exchange value is the initial stock less what was consumed, plus value transferred in production, plus the value product. Dividing the second by the first gives the new market value of the commodity, arising from the two sources of existing stocks and new product.

To illustrate this, we again present Tables 11.8 and 11.9 but in value terms, on the assumption that as before initial unit values are $\lambda_1 = 40$, $\lambda_2 = 7$, and hence $\lambda_L = {}^7/_{10}$.

STOCKS		C_I	C_{II}	V		C_I	C_{II}	Total
Producer P_I	owns	2800[70]		210[300]	sales stocks	0		3010
Producer P_{II}	owns	800[20]		140[200]	sales stocks		0	940
Total Value		3600	0	350		0	0	3950

Table 11.8a Simple reproduction with fixed capital, values at the beginning of period 1

The new assumption that constant capital turns over at half the speed of living labour means, of course, that the proportions of living and dead labour in the product are not the same as before. We calculate the individual value of outputs as before, by adding together the consumed dead labour and the added living labour:

STOCKS		C_I	C_{II}	V		C_I	C_{II}	Total
Producer P_I	owns	1400[35]		0	sales stocks	1700[50]		3100
Producer P_{II}	owns	400[10]		0	sales stocks		600[100]	1000
Total Value		1800	0	0		1700	600	4100

Table 11.9a Simple reproduction with fixed capital, individual values at the end of period 2

As before, there is a contradiction between the output and input values of C_I. The 50 units of output have an individual value given, as usual, by the sum of metamorphosed inputs (1400) and value product (300). Their unit individual value is therefore $1700 \div 50 = 34$. If it were not for the 35 units of preserved stocks of C_I, this would be the market value. But these preserved stocks also contain the value with which they started, namely 1400, corresponding to the old unit value of 40.

There is only one coherent way to resolve this contradiction, which is to estimate the new market (social) value of C_I as the average of the whole value contained in the whole stock of C_I:

> if an increase in the price of raw materials takes place with a significant amount of finished goods already present on the market, at whatever stage of completion, then the value of these commodities rises and there is a corresponding increase in the value of the capital involved. The same applies to stocks of raw materials, etc., in the hands of the producers. This revaluation can compensate the individual capitalist, or a whole particular sphere of capitalist production – even more than compensate, perhaps – for the fall in the rate of profit that follows from the raw material's rise in price. Without going into the detailed effects of competition here, we may remark for the sake of completeness that (1) if there are substantial stocks of raw material in the warehouse, they counteract the price increases arising from the conditions of their production; (2) if the semi-finished or finished goods on the market press heavily on the supply, they may prevent the price of these goods from rising in proportion to the price of their raw material … The smaller the amount of stock to be found in the production sphere and on the market at the end of the business year, at the time when raw materials are supplied afresh on a massive scale (or, in the case of agricultural production, after the harvest), the more visible the effect of a change in raw material prices.(Marx 1981:207-208)

The market will insist on this whether we like it or not, because it will assign a uniform price, and thereby a uniform value, to this stock. Table 11.10 illustrates the result. There are 95 units of C_I in existence, consisting of 45 units which were preserved intact and 50 units just produced. The exchange value contained in them is likewise the value of the new stock, 1700, plus the preserved value, 1800, totalling 3500. Dividing by the total use value gives the new unit market value, namely $^{700}/_{19}$. This is less than the old 40, but greater than the new individual value of 34 emerging from production. As for C_{II}, its value is the same as in simple reproduction because there are no preserved stocks. The 'standard' calculation is thus a special case of the general technique.[40]

As an equation, the calculation looks like this:

$$(45+50)\lambda_1^{\ 1} = 35\lambda_1^{\ 0} + 300 + 45\lambda_1^{\ 0} \tag{23}$$
$$100\lambda_2^{\ 1} = 10\lambda_1^{\ 0} + 200 \tag{24}$$

This alters our previous conclusions in only one way: through the transfer of value brought about by the revaluation of stocks. Total new value is still equal to the value product 500, which replaces the value of variable capital, 350, to increase the total value in society from 3950 to 4100. Values arise from production and will now circulate at prices different from these values in

accordance with Marx's first equality. Stocks therefore have an impact on value *prior* to the formation of market prices, a point to which we shall return.

Values do not immediately sink to the level of the cheapest available technology. This occurs only when the product has been manufactured in sufficient quantities to replace all existing stocks and become the actual, and not just the potential new technique used by society. This has profound implications.

Commodity C_I, period 1	Use Value	Exchange Value	Average
Conserved in unconsumed stocks	**45**	**1800**	**40**
Metamorphosed/Transferred in production	50	1400	–
Added by Labour power	–	300	–
Subtotal; new stock	**50**	**1700**	**1700 ÷ 50 = 34**
Total	**95**	**3500**	**3500 ÷ 95 = $^{70}/_{19}$**

Table 11.10 Value calculation with fixed capital stocks

First, it contradicts the prime assumption of the Okishio theorem and all comparative statics – that new technology can be immediately, universally and costlessly deployed. The introduction of new technology is a process over time – usually years and often decades – and during this time values change continuously.

Second, it contradicts the view that 'socially necessary labour time' is the time which *would* be needed if the latest technology were universal. The latest technology is *never* universal: as fast as it is introduced, it is superseded.[41]

Finally, it means that the calculation of profit and surplus value themselves have to be modified to take account of the transfers of value effected by revaluation.

Surplus value and profit

The impact of price changes on commodities is relayed through capital, which the market reduces to a money sum. If the elements of production lose or gain value through the operation of the price system, this communicates itself to my profits. If any part of my capital depreciates through technical change, this is registered as a loss of profits. The concept of profit does not therefore make sense unless variations in the price of stock are taken into account.

Suppose I own 1000 tons of iron worth £2000, of which 500 (worth £1000) are consumed in production to make steel that sells for £3000. Suppose the wage was £1000. If the price of iron has not changed meanwhile, my profit is the difference between costs and revenues, £1000. But suppose in the meantime the price of iron halves. My remaining stock of 500 tons is now worth only £500. I have lost £500 through price changes. *This is a deduction from profits.* It will be balanced by rises in prices elsewhere so that others make windfall profits. Over the whole of society, total profit is unaltered. But this cannot help the individual capitalist whose books show, according to normal accounting practice, a *cost* of £500 in stock depreciation to be found from revenues. Profit is therefore not £1000 but £500.

Gross worth at start of production		Gross worth at end of production	
Iron: 1000 tons	£2000	Iron: 500 tons	£500
Labour contracts (variable capital)	£1000	Labour contracts	None
Steel	None	Steel worth its sale price, that is	£3000
Gross worth	**£3000**	**Gross worth**	**£3500**

Table 11.11 Profit taking into account depreciation

This amounts to the following; profit is no longer simply the difference between revenues and costs. It is the change in gross worth of the business, just as the capitalists calculate it. My advances are given on the left hand side of Table 11.11. My results are given on the right hand side.

Profit is the difference between the two: £3500 – £3000 = £500. This is not an accounting foible: it is enforced by the market. If I claimed profits of £1000 I would soon be forced to recognize the error. Other capitalists would purchase iron at its new market price. If they managed to sell steel for £3000 they would secure an excess profit of £500. If not, I would be forced to take a further loss. Whether or not the price of steel reflects this general devaluation, we confront each other as capitals using the *same* production process, in the technical sense, but with *different* productivities in value terms and hence different individual profits.

Without depreciation, this method yields the normal result: the 500 tons of unused steel would have the same price. Depreciation registers exactly as if the loss in value had transferred to the product. The accounts will read:

Sales:		£3000
Costs:	Materials £1000	
	Depreciation £500	
	Labour £1000	Total £2500
Profits:		**£500**

We can thus separate out depreciation into two components: actual usage (£1000) and moral depreciation (£500).[42] We could treat the £500 as a transfer of *surplus* value from the previous cycle of production. Marx, however, considered it a component of the *value* of the product.[43] In this case labour in steel production becomes less productive, as if it had been deskilled, since it now creates only £1500 in the same time it previously created £2000. Elsewhere, constant capital *appreciates* and consequently transfers less to its product; labour in these branches becomes more productive and adds more value. Both approaches are consistent but the first follows capitalist practice.

In either case the surplus value created in any given period remains equal to the value product, less the value of consumed capital; and total profits equal total surplus value in accordance with Marx's second equality.

Note once again that the profitability of a production process cannot be derived from technical conditions alone. Producers of the same product with the same inputs and the same methods will secure different profits depending on when they buy their steel.[44] Note finally that the resultant profit rate is *different* from that predicted by equilibrium theory. Productivity-enhancing technical

change produces a falling rate of profit in conditions where equilibrium theory predicts a rising rate, as Andrew Kliman's chapter in this volume shows.

Figure 11.4 illustrates this point, giving the rate of profit and the value of total capital stock for the case we have been discussing under the following unexceptionable conditions: C_I and C_{II} turn over once every ten periods and the technique of producer P_{II} remains fixed; however P_I is able to invest its physical surplus, 2.5 times its output, so that its invested constant capital rises continually without expanding the labour force; that is, labour productivity in both sectors steadily improves due to the cheapening of C_I. The real wage remains fixed at half the output of P_{II} The equilibrium calculation shows a rising profit rate and a falling capital stock; the correct calculation shows a rising capital stock and a falling profit rate.
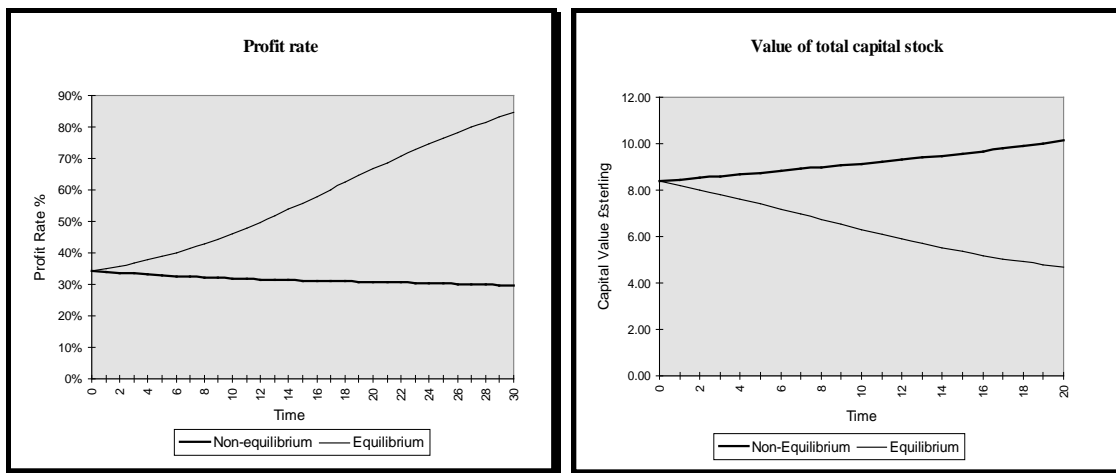

Figure 11.4 Equilibrium and non-equilibrium profit rates and capital stock

What is fixed capital? the period of reproduction and continuous time

Equilibrium theories, as we have seen, are forced to locate the value-creating potential of capital in either its exchange-value or its use-value aspect, neglecting the unity of the two.

In the absence of technical change, there would be no difference between the two. If use value and exchange value formed an indissoluble unity in the commodity, there would be no need to consider them separately. They would behave like the weight and the volume of a liquid or a powder, which behave as interchangeable measures There are conversion scales from fluid ounces to pints, or ounces to tablespoons, which cooks use happily every day of their lives, so that recipes work equally well in either.

If prices never changed, values and quantities would be linked in the same way. Price and quantity would be invertible expressions of the same thing, and could be used interchangeably, just as the French Revolution *defined* its unit of weight, the gramme, to be the weight of a volume of water – one cubic centimetre. The £ sterling could be used as a universal measure of quantity and cake could be made with £2 of flour and £1 of butter, just as the neoclassical

production function says. The only debate in economics would be which numéraire from the infinite number available was the most aesthetically or politically pleasing, be it gold, paper, labour, standard commodities, socks or fish.

It is a different matter when two quantities vary independently. If one is used as the standard of the other, a change of form appears as a change of magnitude. If France had defined weight as a volume of gas, its balloonists would have made matter out of hot air and Marie Antoinette would have kept her head. And if there were no incompressible substances, the Académie could without doubt have sustained a long and heated debate on whether weight or size mattered most.[45]

The neoclassical macroeconomic production function therefore makes it appear as though money creates money, because it uses a changing standard of price as a measure of its capacity to mobilize or create value.

Those variants of equilibrium theory which eschew this choice attempt to do so by identifying a special, value-creating type of capital – fixed capital. In fact, however, this elevates an arbitrary accounting unit to the level of an economic constant. This unit is the period of reproduction.

Many neoclassical models secretly depend on this constant. For example, the formula giving the rate of interest on currencies that are expected to devalue is usually given as $i + h$ where i is the normal rate of interest on the currency under threat, and h is a hedge factor or risk premium given by the expected rate of fall in value of the currency under the impact of a devaluation. Unfortunately h has time in its denominator. Therefore the expected rate of fall should be adjusted for the *time* at which it is expected to happen. As the hour of doom approaches, h becomes infinite. This happened on Black Wednesday when at one point the Bank of Sweden raised the interest rate on the Krone to 500 per cent, in vain.

Why does C_I appear as fixed, and C_{II} as circulating capital, in Tables 11.5 and 11.6? Because we took as our period of reproduction a unit of time in which C_{II} is completely used up. But there is no basis for this choice. If we had taken the period of reproduction to be a week instead of a month, or a day instead of a week, C_{II} and indeed variable capital would have turned over only partially in this time.

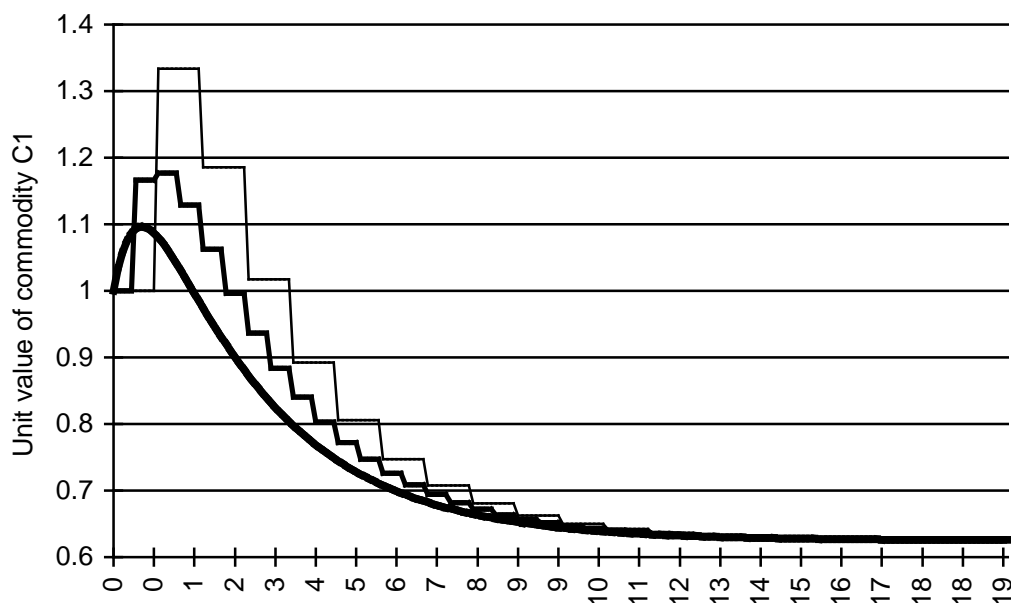**Successive approximations to time-varying values**

Figure11.5 Uniform convergence of sequentially calculated values with fixed capital

As we have seen, in reality all components of capital exist as stocks and money-capital must be advanced for their use. Marx's extensive discussion of turnover time is not destined, as all subsequent authors have taken it, to establish the *difference* between fixed and circulating capital but to establish their inherent *identity* insofar as they function as capital.

Our final correction, therefore, is to remove the arbitrary assumption of a fixed period of reproduction and treat all elements of capital equally. This finally deals with an abstraction we have so far made: in effect, that circulation ceases until production is complete. In fact they proceed in parallel. The assumption that a period of circulation alternates with a period of production therefore introduces a distortion.[46] How do we know that this distortion is not fatal?

The isolation is *analytically* correct, because there are distinct acts in the life of each commodity, which are separated in time for each individual commodity: the production of the commodity, and its sale. But while it is true that a definite period of time elapses while each commodity is produced, this period is not the same for each commodity, so that we cannot act as if all commodities were sold at once.

General Equilibrium theory deals with this problem by eliminating the passage of time altogether, assuming an immediate identity between production and consumption, a fact that Marx made the centre of his criticism of Ricardo. Producers are made to pay for their inputs at prices which do not yet exist, on the assumption that 'in the long run' this assumption will justify itself, which it does by the extreme measure of removing every source of change in prices. This

solves the paradox of Achilles and the Tortoise by killing the tortoise before the race.

The method we propose, classical in the natural sciences and in mathematics in general, is to make the period of reproduction progressively shorter and shorter This gives a more and more accurate view, until in the limit – the continuous case – the distortion is eliminated. When the period used for accounting is reduced successively, the magnitudes calculated by the formalism reduce to a well-defined trajectory; in mathematical parlance, the sequence of values and prices should converge uniformly. What they converge uniformly to is the continuous case, in which the period of reproduction is treated as infinitely small.

The condition for this is the absence of singularities or 'sudden steps' in the stock or price vectors.[47] This is clearly true for changes in stock and in the value product, although at moments of financial crisis, vast volumes of commodities can now change hands in a very short space of time, approaching a singularity. It will definitely be violated for the transfer vector £E at moments of crisis and of sudden shifts in such quantities as exchange rates, when all commodities exchanging for a given money are instantly revalued in terms of another money. It is precisely at such points that the value-price distinction asserts itself most emphatically. Outside of such moments, the distinction is blurred by the smooth operation of the market and it becomes impossible to distinguish changes effected by the price system from changes resulting in underlying real value movements. What remains true, however, are three fundamental laws:

❑ The new value entering the economy – the value product – is proportional to the time worked by employed labourers;

❑ Surplus value and profit in total add up the difference between this value product and the wage;

❑ As will be demonstrated, the total value of stock (equal to its total price) increases for as long as the capitalists invest any portion of this surplus value, and falls only when – as in a crisis – a forcible disinvestment takes place.

Figure 11.5 shows how the successive reduction of the period of reproduction yields sequences of values which converge on a single trajectory.

1.10   THE MATHEMATICS OF ACCUMULATION

Reproduction is an alternation of production and circulation, the unity of these two moments. Any mathematical formalism will stand or fall by the manner in which it represents this unity. If it obliterates distinctions which actually exist in the real world, it will remain nothing more than idealization.

The errors in the simultaneist approach to reproduction and accumulation exactly parallel the errors in its approach to exchange. In exchange it assumes constant prices, in reproduction constant proportions. In exchange this gives rise to a separation into the two spheres of 'value' and 'price'. The image of this in its

treatment of reproduction is a separation into the two spheres of 'use value' and 'value'. The founding catechism of the Surplus Approach school is the idea that a *single technology* corresponds to a *single price system*. In simultaneous equations this is true, but in the real world and in Marx it is false.

Our first task, therefore, is to re-integrate production and circulation in such a way that the distinct effects of each process on both use value and value are properly represented; and our second task is to re-unite these two separate aspects of reproduction in such a way that their concrete unity does not destroy their abstract difference.

Relative surplus value

Virtually the whole of the simultaneist interpretation of Marx assumes simple reproduction. Marx, on the contrary, conducted his analysis in the framework of relative surplus value. The two are incompatible. For Marx there is no *necessary* reproduction of the material elements of production; as we explain in chapter 1 this is an ideological construction which has paralysed economic thinking from 1897 onwards. To take but one example: what is the meaning of a 'physical surplus'? For the whole linear production school of which the Surplus Approach school is but a part, it means this; first we subtract from the physical product all those commodities necessary to restart production with *exactly* the same inputs in *exactly* the same proportions.[48]

This never happens, and Marx makes this abstraction at only one point, during the construction of his simple reproduction schemes in Volume II. This assumption is immediately dropped and does not apply in Volume III (or Volume I), all of which are in the framework of relative surplus value. In practice society reproduces a mixture of old with new and different commodities which permit the capitalists to reproduce their *capital*, that is, their money or more generally, its capacity for growth. They therefore resume production of different goods, using different techniques, in different proportions, in every way a modification of what went before. A physical surplus in the pure sense does not even exist, because many use values are consumed and never even produced again, having been superseded technically. Space does not allow us to develop this in full but we draw the reader's attention to Volume 34 of his collected works, which contains the final part of the 'second draft of capital', and which he wrote immediately after the *Theories of Surplus Value*, itself a break in his study of relative surplus value. This volume also contains the first worked-out version of his reproduction schemas.

We draw attention to just three points which run completely counter to the simple reproduction formalism. First, for Marx reproduction is the *conversion of surplus value into capital*. But in simple reproduction, surplus value cannot be converted into capital or it would not be simple. Moreover surplus value is for Marx always converted into *more efficient* means of production, because it seeks

a higher individual rate of profit. The cheapening of the means of production was not for him an afterthought but the starting point. Thus we find, for example:

> it is the tendency and the result of the capitalist mode of production continuously to raise the productivity of labour, hence continuously to increase the amount of the means of production converted into products with the same additional labour, continuously to distribute the newly added labour over a greater quantity of products, so to speak, and therefore to reduce the *price* of the individual commodity, or to *cheapen* commodity prices in general. (Marx 1994:369)

Secondly, if the framework of the transformation is that of simple reproduction, what is it doing sitting just before the chapter on the falling rate of profit? How can the rate of profit fall in simple reproduction when it is due to the *accumulation* of commodities, that is, the conversion of surplus value into capital?

Thirdly, we actually find that Marx has a completely different concept of physical surplus:

> One should not imagine for that reason that *surplus produce* arises merely because in reproduction the amount of products increases as compared with the original amount. All *surplus value* is expressed in *surplus produce*, and it is only this that we call *surplus product* (the surplus of use value in which the surplus value is expressed). On the other hand, not all of the *surplus product* represents surplus value; this is a confusion found in Torrens and others. Assume, for example, that the year's harvest is twice as large this year as the previous year, although the *same* amount of objectified and living labour was employed to produce it. The *value* of the harvest (disregarding here all deviations of price from value brought about by supply and demand) is the same. If the same acre produces 8 qrs of wheat instead of 4 qrs, 1 qr of wheat will now have half as much value as before, and the 8 qrs will have no more value than the 4 had. In order to exclude all outside influences, assume that the seed was cultivated on specific fields, which yielded the same product as the previous year. Thus a qr of seed would have to be paid for with 2 qrs of wheat, and all the elements of capital as also surplus value would remain the same (similarly the ratio of the surplus value to the total capital). If the situation is different in this example, this is only because a part of the constant capital is replaced *in natura* from the product; hence a smaller part of the product is needed to replace the seed; hence a part of the constant capital is set free and *appears* as surplus produce.(Marx 1994: 220)

The legacy of Sraffa is the idea that with a given technology, prices are given solely by the division of the product between capital and labour: that 'technology determines prices'. From this point of view the statement that 'a qr of seed would have to be paid for with 2 qrs of wheat' is inconceivable. The seed *is* wheat and in material terms the wheat replaces itself directly. How, then, can one qr of wheat exchange on a basis of equality with 2qrs of wheat? Does this mean that the price of wheat is twice itself?

Moreover it is widely assumed that one may divide the gross product independently of prices into three portions representing replacement means of production, wage goods and luxury goods, and that the value distribution in society is simply equal to the value of these three portions of the product. But this is simply not so. If constant capital has cheapened and indeed changed its material nature during a period of production, and the value of portion of the

gross product which pays for its replacement is smaller than the value of the constant capital of the last period, then as Marx puts it in many places in all three volumes, constant and variable capital is 'freed up'; accumulation is fuelled *not* just by surplus value but by this additional surplus. Marx's definition of surplus produce is therefore this: it is the portion of the gross product left over after the *value* of consumed capital has been replaced, *not* after its use value has been replaced.

However, it is necessary to analyse the total movement of use values distinctly from the movement of values. The most important thing, therefore, is to make no prior assumptions, such as simple reproduction, which impose an a priori constraint on the reproduction of values. This in turn is impossible unless we recognize, as Marx did in his treatment of expanded reproduction, that accumulation does not consist of the immediate redeployment of produced goods in production, but in a *prior accumulation of unused use values* and a *prior accumulation of idle money*. That is, we have to account systematically for the conversion of flows into stocks.

The representation of stocks and flows

We must thus represent properly the relation between turnover and stock. The first hurdle, at which the simultaneous method falls, is to recognize that turnover alters stock, to realize there is any dynamic relation at all between the two. But this is not enough. There are two causes of the rise or diminution of any stock, and Marx's analytical construction is constructed through and through to distinguish between them. On the one hand, production decreases C, W, and B and raises X; and on the other, circulation diminishes X and increases C, W and B. We must distinguish systematically between these sources of variation; the inability to do so is one of the most important conceptual failings of neoclassical economics.

In the deepest sense, a stock *is* an aspect of a flow, just as any existing thing is an empty abstraction if divorced from what it was and what it will be. The movement is the primary entity.[49] The notation of the differential calculus is clumsy in this regard since it takes the entity itself, X, as given, and the variation of X, ΔX, as the variation of it. But we are not in a position to escape this here.

When we need to make the distinction clear we use the symbol Δ_C, the variation in circulation, to represent the change in any magnitude due to circulation, and the symbol Δ_P, the variation due to production, to represent the change of the same magnitude due to production. The symbol Δ is thus the sum of the two, the total variation in a quantity, giving an operator identity

$$\Delta = \Delta_C + \Delta_P$$

Our analysis moves from the discrete to the continuous case, which means we need to be able to represent the *rate of flow* of a given magnitude. By analogy we shall write

C_C for the rate at which C is changed in circulation;

C_P for the rate at which C is changed in production.

C' for the total rate at which C is changed in reproduction, so that $C' = C_P + C_C$

Note that C' has its normal significance in calculus as the rate of change of C. This also permits us to move away from the confusing use of a different symbol for price and for value, which are actually the same thing at different points on the circuit of capital. As reproduction progresses, the price of any commodity changes for two reasons; because of changes in the productivity of labour and because of the redistribution of surplus value in the sphere of circulation. From one period to the next, *p* changes, as it were, twice; once because the outputs of production contain a different amount of socially necessary abstract labour time

$$p^t \rightarrow p^t + \Delta_P p^t$$

a quantity we have hitherto called λ, and again because circulation redistributes surplus value

$$\lambda^t \rightarrow \lambda^t + \Delta_C p^t = p^{t+1}$$

where $\Delta_C p$ is simply what we have so far called *e*. The overall movement is

$$p^t \rightarrow p^t + \Delta_C p^t + \Delta_P p^t = p^{t+1}$$

Analogously we have

p_C for the rate at which *p* changes in circulation;

p_P for the rate at which *p* changes in production.

p' for the total rate at which *p* changes in reproduction, so that $p' = p_P + p_C$

The important and difficult thing in a truly general dynamic analysis, as we have said, is to separate and analyse these two sources of variation, production and circulation, in relation to the two moments of the commodity, use value and value, and then unite them in such a manner that their concrete unity is expressed without obliterating their abstract differences. We proceed to do this by separately analysing, first the general laws governing the reproduction (production and circulation) of use values; and then those governing the reproduction (production and circulation) of value. This is conducted for the discrete case. We then bring the two together and reduce the period of reproduction to zero to produce a general, continuous differential equation governing reproduction as a whole.

The reproduction of use values

Production, in which we include reproduction and hence personal consumption, destroys and create use values. We cannot predict how much. Workers may consume all, part, or none of W and the capitalists of B. Investments may, or may not, be used, and output may or may not be sold. But we can quantify the outcome: each stock is either augmented or diminished by its turnover.

Circulation on the other hand alters the distribution of stocks, so strictly speaking C^t, for example, actually stands for two different magnitudes: before circulation and afterwards.[50]

In circulation commodities are redistributed in four main ways:

❑ One part is purchased by labourers and either consumed by them or laid up as a consumption fund. This is thus added to the wage fund W.

❑ A second part is purchased by the capitalists and employed – or lies idle – in production in the next period. This is thus added to constant capital C.

❑ A third part is purchased or set aside by capitalists for private use in B.

❑ A fourth part remains as unsold inventory X in the hands of the capitalists.

Again there is no automatic way to predict the proportions of these exchanges. Thus the only relation we can rely on is the definition:

$$K = X - C - W - B \qquad\qquad (25)$$

Equation 25 is the most general statement we can make. If any of the magnitudes in it are specified in more detail – for example by a production function or a theory of consumer demand – then we have a particular model of the economy, not a general theory. We can say, however, that the same law applies to any changes of stock levels, so that

$$\Delta K = \Delta(X - C - W - B) \qquad\qquad (26)$$

However, the same is true for any isolated source of change, so that

$$\Delta_C K = \Delta_C(X - C - W - B) \qquad\qquad (27)$$

But this means we can say something specific about circulation since it can neither create nor destroy use values. The quantity $\Delta_C K$ may change in circulation through a redistribution of commodities but the total commodities in it cannot. It follows that the row sum of $\Delta_C K$ is zero.

Therefore summing (27) across rows – capitals – produces a fundamental statement, a sort of Kirchoff's Law of circulation, which any commodity economy must obey:

$$\Delta_C \Sigma_j (X - C - W - B) = 0 \qquad\qquad (28)$$

In consequence the quantity ΔK, changes in K over the whole of reproduction, can *only* be due to production (in which, recall, we include private consumption).

Therefore

$$\Delta K = \Delta_P \Sigma_j (X - C - W - B) \qquad\qquad (29)$$

We term these last two equations the *fundamental stock accounting identities*. They are the most general statements which can be made about the reproduction of use values in a market economy and therefore, firstly, must be true in any particular case and secondly, impose no hidden a priori assumptions.

The calculation of value in the presence of fixed capital

At the beginning of a period of production at time *t*, following circulation, the total goods in circulation K, that is all goods in society available for sale, comprise the following use values:

C productive stocks;
W consumption goods owned by labourers;
B consumption goods owned by capitalists;
X sales inventory owned by capitalists;
K total stocks excepting labour-power, the sum of the above;
V the total labour power in the economy.

Assume for simplicity that workers consume all wage goods in the current period. Consumed variable capital V is therefore always equal in price and hence value to the price of consumed wage goods pW consumed during the same period.[51]

After production each stock has diminished except X, which has grown because production has created new use values Δ_PX. A portion of K^t survives intact to subsequent periods and preserves the value it has inherited. This portion, plus Δ_PX, makes up K^{t+1}, the total goods now in circulation. It follows that this intact portion has magnitude

$$K^{t+1} - \Delta_P X^t$$

or

$$K^t + \Delta_P K^t - \Delta_P X^t$$

(Another way of deriving the same result is to say that this intact portion is equal to K^t less consumption of C, V, W and B.) This preserves the value it possessed when production began, and contributes this to the total supply of value in society as if it had just been produced. This component of new value is equal to

$$p^t(\hat{K}^t + \Delta_P \hat{K}^t - \Delta_P X^t)$$

Production creates new goods whose value comprises two components, namely the value transmitted by the consumed constant capital $\Delta_P C^t$ and the value added by labour power $\Delta_{\pounds} L^t$. The total value in the economy following production is therefore the sum of preserved and new values,

$$p^t(\hat{K}^t + \Delta_P \hat{K}^t - \Delta_P X^t) + p^t \Delta_P C^t + \Delta_P \pounds L^t$$

On this basis, new unit values are formed. These are a social average, equal to the total value of each commodity divided by the total use value of the same commodity. Representing new unit values as $p + \Delta_P p$, the total value of all stocks in circulation is also given by

$$(p + \Delta_P p)\hat{K}^{t+1}$$

that is

$$(p + \Delta_P p)(\hat{K} + \Delta_P \hat{K})$$

where we drop the time subscript since only subscripts at time t are now involved.

hence $\qquad (p + \Delta_P p)(\hat{K} + \Delta_P \hat{K}) = p(\hat{K} + \Delta_P \hat{K} - \Delta_P X) + p\Delta_P C + \Delta_P \pounds L$

Expanding and simplifying yields

$$\Delta_P p\hat{K} + \Delta_P p\Delta_P \hat{K} = -p\Delta_P X + p\Delta_P C + \Delta_P \pounds L$$

that is

$$\Delta_P p \hat{K} + p\Delta_P X = p\Delta_P C + \Delta_P \pounds L + o(2)$$

We now divide through by Δt and pass to the limit as $\Delta t \rightarrow 0$ This gives the *value accounting identity*

$$p_P \hat{K} + pX_P = pC_P + \pounds L_P \tag{30}$$

or, in slightly more familiar form

$$p(X_P - C_P) = \pounds L_P - p_P \hat{K} \tag{31}$$

This should be compared with the value equation when all stocks are considered to turn over during the period of production:

$$p(X_P - C_P) = \pounds L_P$$

The difference is $p_P \hat{K}^t$, the *revaluation term*. This expresses the redistribution of value brought about by depreciation of commodities due to technical change.

Suppose now that in circulation goods sell, not at prices equal to values $p + \Delta_P p$ but at new prices $p + \Delta p$ where in general $\Delta p = \Delta_P p + \Delta_C p$, the value change brought about by production plus the value change brought about by circulation. The same reasoning yields the *price accounting identity*

$$p'\hat{K} + pX = pC + \pounds L + p_C K \tag{32}$$

or

$$p'\hat{K} + pX = pC + \pounds L + \pounds E \tag{33}$$

Equations (32) and (30) are the basic dynamic relations of price and value taking into account fixed capital. Given only the observed data of the economy they are determinate and distinct vectors of values and prices.

They can be rearranged to show how new value is created and redistributed in the economy thus:

$$p'\hat{K} + p(X - C) = \pounds L \tag{34}$$

that is, *new value enters the economy at the rate* £L, and

$$p'\hat{K} + p(X - C) = \pounds L + \pounds E \tag{35}$$

showing how this new value is redistributed through by transfer vector £E.

Surplus value and profit with fixed capital

The capitalists begin production with stocks K – W, that is, everything except wage goods, and variable capital V whose value is pW. Their gross value is therefore

$$p(K - W) + pW = pK$$

At the end of production they have used up $\Delta_P C$, $\Delta_P V$ and $\Delta_P B$ and created new use values $\Delta_P X$. They therefore own stocks equal to

$$\Delta_P(K + X - C - B)$$

and have also used up $\Delta_P V$ of their variable capital. Their new worth is equal to the new price of their stocks

$$(p + \Delta_P p)(K + \Delta_P X - \Delta_P C - \Delta_P B) - \Delta_P \pounds V$$

and assuming that the value of variable capital is equal to the current price of wage goods, this is equal to

$$(p + \Delta_P p)(K + \Delta_P X - \Delta_P C - \Delta_P B - \Delta_P W)$$

However, luxury consumption B is a deduction from their wealth; it is part of what they appropriated. Gross wealth including current consumption is therefore

$$(p + \Delta_P p)(K + \Delta_P X - \Delta_P C - \Delta_P W)$$

Subtracting current gross wealth from initial gross wealth gives net surplus value:

$$(p + \Delta_P p)(K + \Delta_P X - \Delta_P C - \Delta_P W) - pK$$
$$\Delta_P p K + p\Delta_P X - p\Delta_P(C - W) + o(2)$$

But the value equation (30) established that

$$\Delta_P p \hat{K} + p\Delta_P X = p\Delta_P C + \Delta_P £L + o(2)$$

from which

$$p\Delta_P X = p\Delta_P C + \Delta_P £L - \Delta_P p \hat{K} + o(2)$$

Substituting for pX yields the rate at which surplus value is produced or the *rate of surplus value generation*

$$\Delta_P £S = \Delta_P p(K - \hat{K}) + p\Delta_P C + \Delta_P £L - p\Delta_P(C + W) + o(2)$$

The two terms in $p\Delta_P C$ drop out leaving

$$\Delta_P £S = \Delta_P p(K - \hat{K}) + \Delta_P £L - p\Delta_P W + o(2)$$

But we have assumed the value of consumed variable capital $\Delta_P £V$ is equal to the price of consumed wage goods, disregarding consumer durables, giving

$$\Delta_P £S = \Delta_P p(K - \hat{K}) + \Delta_P(£L - £V) + o(2)$$

Dividing by Δt and passing to the limit yields

$$£S_P = £L_P - V_P + p_P(K - \hat{K}) \tag{36}$$

This is the value-product of labour power £L, less variable capital £V, plus a *redistribution* term $p_P(K - \hat{K})$. This reflects the result of the competitive struggle between capitals through depreciation. All capitals whose value has risen have appropriated surplus value from all capitals whose value has fallen through depreciation. The rate of *profit generation* is given similarly by

$$\Pi' = £L - £V_P + p(K - \hat{K}) + £E \tag{37}$$

that is, the rate of surplus value generation plus the transfer vector £E. Marx's second equality follows from two facts: the sum of the components of £E is zero, and

$$\Sigma K = \Sigma \hat{K}$$

Lastly the equations of price and profit yield a simple relation connecting price and profit on a sectoral basis

$$\Pi' = p'K + p(X_P - C_P) - V' \tag{38}$$

Capitalist accumulation

The wealth of society falls into two main portions: the wage fund W, which is owned by workers, and everything else, which is owned by capitalists. This latter is capital; it consists of those commodities which, broadly speaking, enter into the equalisation of profit rates. In this we include the wealth of collectors, speculators, hoarders and rentiers; in short every form of wealth which acts as a receptacle for surplus value and which, as a component in a portfolio of wealth, may be exchanged for other commodities in pursuit of a higher rate of growth of real value, that is, profit. Neglecting variable capital this is given by $K - W$.

However capital also seeks a return on variable capital along with all other advances of money. The value of the capital seeking a share of surplus value is therefore simply the scalar quantity

$$\Sigma_j p K$$

The total rate of accumulation of society is the rate at which this magnitude grows. (For the rest of this section we are concerned only with total social magnitudes and we will therefore drop the summation sign.) Differences between p and λ which cancel out over all of society. This total rate of accumulation whether goods sell at prices or values, is therefore

$$£K' = p'K + pK'$$

the sum of two quantities, one the result of the accumulation and capitalist consumption of use-values and the other the result of price and value changes. But the second of these terms is given by the equation of value production:

$$p'\hat{K} + pX_P = pC_P + £L + £E$$

When we sum over the whole of society $\hat{K}$ and K are the same and $£E$ vanishes:

$$£K' = £L + p(K_P - X_P + C_P)$$

However, the stock accounting identity tells us

$$K_P = X_P - C_P - B_P - W_P$$

Thus the rate of growth of capital, summed over society, is therefore

$$£K' = £L - £B_P - £W_P$$
$$£K' = £S_P - £B_P \tag{39}$$

The only way this can be negative is if the bourgeoisie disinvest in value terms.

The general law governing the rate of profit

We are now in a position to state the general law governing the variation of the rate of profit. Since we have made no special assumptions concerning wage rates, supply and demand, capitalist behaviour or the structure of production, this law is absolutely general and must therefore apply in all special cases.

The general or average rate of profit is given by the ratio between $£S_P$, the rate at which profit is generated, and K, the volume in value terms of capital seeking a

return on investment. Between one period and the next, this changes by an amount

$$r' = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{£S_P}{£K}\right) = \left(\frac{£K£S - £S_P£K'}{£K^2}\right) = \left(\frac{£S_P - r£K'}{£K}\right)$$

But we can substitute from the numerator using equation (39), to give

$$r' = \left(\frac{£S_P - r(£S_P - £B_P)}{£K}\right) = \left(\frac{£L - £V - r£I}{£K}\right)$$

where £I is the rate of investment, that is, surplus value less capitalist consumption. We can now formulate precisely the conditions for this to be a positive magnitude (rising profit rate) or a negative magnitude (falling profit rate). First, if £L and £V are zero (constant rate of value creation and constant wage in value terms), then *the rate of profit must fall unless the capitalists disinvest in value terms*, that is, unless I, the rate of investment, is negative. Thus (the law as such) investment produces a continuously falling profit rate.

Second, this can be offset (countervailing tendencies) by raising £L – making the workers work harder or employing more of them – or by decreasing £V, the share of national product which they consume in value terms. However there are *absolute limits to either*. £L here is the social average. Over all of society, differences between less or more skilled labour average out, and therefore it is in a fixed ratio to hours worked. And £V cannot be decreased below zero or the workers die.

We thus find – an astonishing and salutory result – that after a hundred years of nit-picking at Marx's original statement of the general law of the falling rate of profit, that this law is not merely valid, but scientifically and rigorously exact.

1.11 CHANGES IN THE VALUE OF MONEY

Under general price inflation, anyone who holds commodities other than money will make profits in money terms, whether or not these profits correspond to a real increase in their command over either people or things. This is not a special feature of Marxist analysis but applies in any conceivable economic framework.

Suppose I purchase 100 tons of steel for £100 on 1 January and do nothing with them; and all prices rise by 10 per cent during the year. My steel is worth £110 on 31 December and my profit is therefore £10, in money terms.

No-one, no matter how ideologically blinded or prejudiced against value theory, could possibly claim that this represents an increase in real wealth. There is thus a real problem in economics which has to be dealt with in any analytical framework, although most microeconomics evades or ignores it: how can we distinguish between profits which are the result of purely monetary phenomena, and profits which in some sense represent an increase in real wealth?

In an equilibrium framework this is incomprehensible. Monetary inflation can be simulated in comparative statics by changing the numéraire from one period to the next. But this does not exhibit the false profits induced by inflation. The

numéraire appears in both the numerator and the denominator of the profit expression, which appears therefore as if it were unaffected by the value of money. The real basis on which monetary variations affect profits – the variation of asset prices from one time to the next – simply cannot be represented in such systems.

This is a deeply practical question. Accountants, who understand many of these issues better than economists, have devised inflation-accounting systems for eliminating false profits of this type. Working economists distinguish between real and nominal value. Macroeconomic theory attempts to separate out the effects of changes in the price level from movements in the 'real economy'.

Working economists lead strangely schizophrenic existences. From 9 until 5, for however many days a week they are paid to produce useful, or at least marketable results for governments, accountants, market researchers or perhaps investment banks, they sit in offices and carefully adjust figures with scrupulous professional attention using price indexes calculated with minute care to disentangle the real values of the assets under discussion from their monetarily-inflated prices. Then, during the hours left for reading, writing, or attending learned conferences, they sit and read, or perhaps write, theoretical tracts which have been crafted with equal care for a hundred years around the single proposition that all prices are relative, the value-price distinction is meaningless, and that accounting for social effort in labour hours is a theoretically-discredited and fruitless activity.

This schizophrenia is self-induced and uncalled for. In the framework we propose, money – which was present from the start – enters in a natural and obvious manner into the calculation. As shown in section 5 the commodity serving as money at all times has a known and calculable value, as does every other commodity. This may be considered in one of two ways which are formally equivalent. First, we may take the value of money at some given initial starting point as the standard of value (and hence price). Thus, if in 1980 the total assets of the economy were priced at £1000 billion and in 1981 the same goods would have been priced at £1250 billion, then a 1981 pound is worth 1.5 times a 1980 pound; the value of £1 has thus fallen to $£^4/_5$ measured in 1980 pounds.

Alternatively, we may wish to express these magnitudes directly in hours. As Carchedi and de Haan show in this volume, this calculation is perfectly practical in principle. The only difficulty is that the accuracy of measurement is affected by the time period chosen. By calculating the price of the new goods created over some definite time, correcting as we will show for the (known) change in the value of money and dividing by the total hours worked in society that created these new goods, we can calculate the value product £L of an average hour of socially necessary labour time in 1980 pounds. Since we are converting to a constant value measure (1980 pounds), the wealth of society in 1980 may now be estimated in hours, as may all magnitudes previously calculated in pounds.

Either calculation yields a coefficient μ^t, the quantity of value expressed in one unit of current money. How does this affect the calculation of profit? Begin in the current period; gross money wealth is

$$\mu p K$$

After production and circulation gross wealth including current consumption is

$$(\mu + \Delta\mu)\{(p + \Delta p)(K + \Delta X - \Delta C - \Delta W) + \Delta\pounds E\}$$

Subtracting current gross wealth from initial gross wealth gives net profits in money terms:

$$(\mu + \Delta\mu)(p + \Delta p)(K + X - C - W) - \mu p K$$

Clearly, the part of this equation that is multiplied by μ will yield the same expression for the rate of profit generation as before but multiplied by μ, namely

$$\mu\{p'(K - \hat{K}) + \pounds L - \pounds V + \pounds E\}$$

All elements of the second part, multiplied by $\Delta\mu$, will vanish in the limit except

$$\Delta\mu p K$$

and this must be added to the expression above to yield the *money rate of profit generation*

$$\Pi_m = \mu\{p'(K - \hat{K}) + \pounds L - \pounds V + \pounds E\} + \mu' p K \tag{40}$$

where the extra term $\mu' p K$ shows that profit must be adjusted for the *rate of change of the value of money*, multiplied by the price of capital stock.

This can be summed to yield the rate of profit generation in the whole economy, the general rate of profit generation – remembering that when summing over society many of the terms drop out or can be simplified:

$$\Sigma_j\Pi_m = \mu\Sigma_j\pounds S + \mu'\Sigma_j K \tag{41}$$

Finally, dividing through by the money price of the total capital stock μK yields the *money rate of profit*

$$r_m = \frac{\mu\Sigma_j S + \mu'\Sigma_j K}{\mu K}$$

$$= r + \frac{\mu'}{\mu}$$

the normal rate of profit plus a term representing variations in the value of money

$$\frac{\mu'}{\mu} \quad \text{or} \quad \frac{d(\log \mu)}{dt}$$

The importance of this is as follows:[52] during a period, such as the boom phase, when all prices are generally rising, the money rate of profit is raised artificially by this general rise. The effect, however, is limited to periods in which prices are *rising*, not when they are simply high; it is a dynamic effect with no static equivalent. High money profits act as an attractor for investment so that investment-led growth creates and re-enforces the demand for all goods, feeding the rise in prices. Money itself becomes a source of losses, since its purchasing

power is falling. Value is thus transferred out of society's stock of money and into its stock of productively active goods.

However, the resultant accumulation begins to raise the value of invested capital stock, reducing the actual underlying profit rate. Initially this is not perceived because it is offset by general price inflation, but eventually comes to dominate. At a certain point, the reduction in demand provoked by this fall in the profit rate, or perhaps some external or specific endogenous event – it is irrelevant what the immediate cause is – will bring to an end the period of generally rising prices. Now, however, the term μ'/μ becomes *negative* and instead of offsetting the fall in the underlying rate of profit, it re-enforces it. The feedback mechanism goes into reverse; now investment cuts off, existing productive stock becomes idle or bankrupt, and demand falls, re-enforcing the fall in the value of money. Money becomes a source of gain, since its purchasing power is rising and additional stores of value are sought such as precious metals, jewellery and collectors items. Value thus flows out of the stock of productively active goods and into society's stock of money and other stores of value.

A point is reached where the value of the capital in society – including money and the like – has actually devalued because society has physically drained them of value. This can happen in a number of ways. Value may be transferred into spheres which do not participate in the equalisation of the profit rate, such as armaments or other state expenditures. If society continues producing goods with new technology, even at a reduced rate, then the physical stock of goods gradually declines in value towards its theoretical equilibrium rate (old assets are written off, depreciated or liquidated) so that the mass of value entering the equalisation of the profit rate falls generally towards its theoretical equilibrium magnitude. The underlying profit rate begins to recover; the stage is set for a new cycle of accumulation on an expanded scale – until the next time.

NOTES

1   The word 'simplification' is abused in the literature. The axiomatic method abstracts from particular factors which may be re-introduced at a later stage. The power of Euclidean geometry, the most beautiful classical example of this method, lies in the formulation of axioms concerning lines and points which state *only* the relations between them. The thickness of a Euclidean line or size of a Euclidian point is not zero: it is undefined. I can build a projective geometry out of Meccano or out of my head, as I choose. The 'simplification' that profit rates are equal, or that supply matches demand, is of a different order. It simplifies by constraining, not by removing constraints.

2   Magic numbers are the raw material of sorcery and religion alike: think of the pentangle, the trinity, the seven-branched candelabrum, the number of the beast. Cabbalism, of which neo-Ricardianism at times seems a reincarnation, was dedicated to discovering the secret forms of God in the numbers and symbols He bequeathed. The famous Tower of Babel, built in Babylon, was a a magical monument with seven rising stages, each dedicated to a planet. Its angles symbolized the four corners of the world. 'The old tradition of a fourfold world was reconciled with the seven heavens of later times.' says Seligman (1975:38) 'For the first time in history numbers expressed the world order'. Not for the last.

3   The notion, originating with Plato, that the geometry expresses divine relations, was the conscious basis of a political system. So long did it take to break free that Kepler, who established the modern laws of planetary motion, experimented for years with circular orbits believing the Creator could not possibly have taken the ellipse as His model for the universe. See Farringdon (1939), Lerner (1992)

4   Nearly everything said here applies also to the systems of linear inequalities pioneered by von Neumann (1937) and further developed by Morishima (1973) and other writers.

5   This order of presentation is logically incorrect for pedagogical reasons as exchange should have been introduced before production. Otherwise the order of presentation follows the development found in *Capital*.

6   Note that these were calculated independent of workers' consumption, which affects only the value of labour power; 6 hours of labour power are worth $24 \times v_2 = 4$ hours so $v_L = {}^4\!/_5$

7   Moreover the output of period 0 will not all be used in period 1, that is, the market will not clear. This is dealt with in section 8

8   Suppose I build an infernal device, the Laplace Integral Engine, deploying the latest Sraffa-Heisenberg Inference Technology to digest all information about the planet including the state of Schrödinger's cat and predict infallibly all prices on 1st April 1999. Suppose I sell the results for £1 a prediction. The information being cheap and worth having, I sell a few billion and retire. The customers, however, did not buy the information for pure interest but to make a few bob; they behave differently. But this falsifies the predictions, whose premises have been changed by the information deduced from them. The machine contradicts its own existence. The economic future can be predicted only if it is consciously controlled; that is, if humans reach prior agreement as to courses of action they wish to pursue in knowledge of the consequences, and stick to them because knowing these consequences does not divert them from it. But such a situation has nothing to do with a market economy.

9   The expression of L in hours would be hrsL, slightly clumsily. But this is simply equal to V. However £V, the value of labour power, is not equal to £L: labour power adds value in proportion to its magnitude (number of hours worked), not in proportion to its price. This proportion is assumed the same for all labour but a general treatment of skilled and complex labour would make it a vector of coefficients. See for example Giussani (1987). See also Carchedi and de Haan in this volume.

10   The reader should not think in terms of the linear production convention that columns represent quantities and rows prices or values. Our variables represent *commodities*, unities of use and exchange value. Columns represent capitals and rows represent commodities, in *each* space representing values/use values, stocks/flows. Each table has $3n^2$ degrees of freedom where n is the number of sectors.

11   This is borrowed from tensor analysis. There is no implication that values are contravariant and capitals covariant vectors, although it is an interesting idea.

12   I am in debt to Bruce Roberts for drawing my attention to this problem in a very patient reading of a first draft of a section of this paper.

13   An 'augmented form' can be constructed; labour is a distinct row of X and C and v is partitioned into its labour and non-labour components: $v^+ = [v_{\text{non-labour}}; 1]$. Then $v^{t+1} = v^{+\,t} c$ where $c = CX^{-1}$

14   Every money, even paper, has a cost of production and therefore an intrinsic value. It requires a certain number of socially necessary labour time to bring it into existence. But the cost of production (value) of every money including gold diverges from its rate of exchange for other commodities, which Marx sometimes terms the exchange value of money, and sometimes simply the value of money. The term 'value of money' covers, we think, what Rodríguez calls 'exchange value of money'. When we wish to distinguish the intrinsic value of money we call it 'the value of the commodity which serves as money'.

15   This issue is dealt with exhaustively in the section on continuous dynamics.

16   The apparent 'technical' requirement to replace inputs arises only because money tied up in machines is lost unless it panders to their appetite. But raw material purchases rise and fall, and stop if the machine becomes unprofitable. And when the machine itself is due for replacement only an insane capitalist buys the *same* machine instead of the latest.

17   See Walras' (1965:89) theorem: 'The effective demand for or offer of one commodity in exchange for another is equal respectively to the effective offer of or demand for the second commodity multiplied by its price in terms of the first.'

18   'But a further series of factors have also to be taken into account in our analysis, factors which affect the sizes of C, V and S in a decisive way, which must therefore be briefly mentioned. Firstly, the *value*

of money. This we can take as constant throughout' (Marx 1981:142, emphasis in original). This is rather important. If the value of money affects the magnitude of C, V and S in a 'decisive way', how does this square with the universally-accepted view that constant capital transfers to the product only the value with which it emerges from production, instead of the value it realizes in exchange?

[19] I place the 'supply and demand' for money in quotes because it is in my view of a different order from the supply and demand for commodities in the normal sense. These express the rate of consumption or production, not the absolute amount in existence. Hume's concept is that metal money, which possesses a substantial intrinsic value that may exceed its extrinsic value, appears to be affected by laws of supply and demand when coin is melted down – though the reverse (conversion of bullion into coin) is rare because of the laws against forgery. But regardless of the empirical validity of this 'law', a different conception is involved from supply in the sense of the *rate at which it is produced*. The so-called 'supply of money' is the supply of a stock, not a flow. To confuse the two is to confuse a quantity with its differential. Equilibrium theory can make this elision because in it, there are no differentials.

[20] 'It [surplus value] is the sum total of the realized unpaid labour, and this grand total is represented, just like the paid labour dead and living, in the total mass of commodities *and money* that accrues to the capitalists'. (Marx 1981:274, my emphasis); 'The *sum of values* remains the same, even if the expression of that total *sum of values* were to grow in money, hence the sum of 'exchange-values' rises, according to Herr Wagner. This is the case, if we assume that the *fall in price* in the sum of the other commodities does not cover the *over-valued price* (excess price) of the corn. But in that case the exchange-value of money has, *pro tanto*, fallen below its value; the sum of values of all commodities not only remains *the same*, it even remains the same in *monetary expression*, if money is reckoned among the commodities'. (Marx 1975:188, emphasis in original)

[21] 'It is not money which renders the commodities commensurable. Quite the contrary. Because all commodities, as values, are objectified human labour, and therefore in themselves commensurable, their values can be communally measured in one and the same specific commodity, and this commodity can be converted into the common measure of their values, that is into money. Money as a measure of value is the necessary form of appearance of the measure of value which is immanent in these commodities, namely labour-time' (Marx 1976a:188)

[22] Ricardo and Marx both accept that the value added by labour power is a variable function of the time worked. Some workers add more value than others because they are more skilled or work harder. It is reasonable to assume that the same type of worker in the same conditions creates the same amount of value in the same time. Multiplying by a coefficient for each type of labour under average conditions gives the value it creates in one hour. From now on, with Marx, we assume this reduction as given. This necessary correction does not remove the problem: When price deviates from value, it is still not the sum of value created and transferred.

[23] See for example Harcourt (1972), Eichner (1979)

[24] Seton (1957) does pose the price-value relation as an additive rather than multiplicative difference, although he also introduces price-value multipliers.

[25] Space does not permit a full treatment of expenditures left out of this account, which can be assigned to one or other of variable capital or profit. Thus the unproductive costs of circulation come from profits; the services of the state to labourers are part of variable capital while taxes on labourers are a deduction from it; the services of the state to capital are part of profits while taxes on the capitalist class, either privately or as capitalists, are a deduction from profits. See Moseley (1990), Freeman (1992c).

[26] V is neither W nor the money wage. It is the labour-power contracted to the capitalists, for which they pay the money wage V, which is then spent separately on *W* at a time of the workers' choosing. Recall that for clarity V is not included in K; its price and value are the scalars p_L, λ_L.

[27] 'Market-value (and everything that was said about this applies with the necessary limitations also to the price of production) involves a surplus profit for those producing under the best conditions in any particular sphere of production … this holds good for all market-prices, no matter how much they might diverge from market values or market prices of production. The concept of market price signifies that the same price is paid for all commodities of the same kind, even if these are produced under very different individual conditions and may therefore have considerably different cost prices' (Marx 1981:300-301).

28   Moreover when the stage of 'specifically capitalist production' or the 'real subsumption of labour' is reached, capital organizes not merely its reproduction but the continuous revolutions in technology that drive it forward. 'This entire development of the productive forces of *socialized labour* (in contrast to the more or less isolated labour of individuals) and together with it the *use of science* (the *general* product of social development), in the *immediate process of production*, takes the form of the *productive power of capital*. It does not appear as the productive power of labour, or even of that part of it that is identical with capital … The mystification implicit in the relations of capital as a whole is greatly intensified here, far beyond the point it had reached or could have reached in the merely formal subsumption of labour under capital' (Marx 1976a:1024).

29   'I define *fixed capital*, i.e. *capital in general*, just as my father did in his *Théorie de la richesse sociale* (1849) as all durable goods, all forms of social wealth which are not used up at all or are used up only after a lapse of time, i.e. every utility limited in quantity which outlasts its first use, or which, in a word, can be used more than once, like a house or furniture. And I mean by *circulating capital* or *income* all non-durable goods, all forms of social wealth which are used up immediately, i.e. every scarce thing which does not outlast its first use, or which, in short, can be used only once, like bread or meat' (Walras 1984:212).

30   '[I]t will be convenient, in order not to complicate the presentation, to introduce the same limiting assumptions which Tugan-Baranowsky made use of, namely, that the entire advanced capital (including the constant capital) turns over once a year and reappears again in the value or the price of the annual product' (Bortkiewicz 1984:199-200).

31   'The *j*th column of B [the output matrix–AF] represents the quantities of commodities produced by production method *j*, where that list of commodities is taken to include all partially used items of fixed capital. (Thus machines, etc., of different ages are treated as *distinct* commodities and are represented as such in the columns of both A and B.)' (Steedman 1977:164).

32   Marx makes no such assumption. Nearly two-thirds of Volume II, which deals with reproduction, is dedicated to the turnover time of capital. Marx's 'correctors' all base their account on his statement in Volume III that *solely* in order to study the formation of profit he will abstract from differences in turnover time. It is illegitimate and absurd to apply this to reproduction and in the event Marx does not even use it in Volume III; in all his tables capital advanced differs from capital consumed.

33   'If, as a result of a new invention, machinery of a particular kind can be produced with a lessened expenditure of labour, the old machinery undergoes a certain amount of depreciation, and therefore transfers proportionately less value to the product. But here too the change in value originates outside the process in which the machine is acting as a means of production' (Marx 1976a:318).

34   Worn-out economic theories still fetch the same as two hundred years ago, even allowing for inflation. This is because nothing new has hit the market.

35   Accountants normally allow for depreciation on a 'going concern' basis; that is, they assume the investment is functioning as part of a totality that is selling its product.

36   'Thus if an increase in the price of raw material takes place with a significant amount of finished goods already present on the market, at whatever stage of completion, then the value of these commodities rises and there is a corresponding increase in the value of the capital involved. The same applies to stocks of raw material, etc. in the hands of the producers. This revaluation can compensate the individual capitalist, or even a whole particular sphere of capitalist production – even more than compensate, perhaps – for the fall in the rate of profit that follows from the raw material's rise in price' (Marx 1981:207-208). Note once again that a change in *price* modifies the *value* of the existing capital, as Marx then explicitly notes: 'Our whole investigation has proceeded from the assumption that any rise or fall in prices is an expression of real fluctuations in value. But since we are dealing here with the effect that these price fluctuations have on the profit rate, it is actually a matter of indifference what their basis might be. The present argument is just as valid if prices rise or fall not as a result of fluctuations in value, but rather as a result of the intervention of the credit system, competition, etc.'

37   'The *destruction of capital* through crises means the depreciation of *values* which prevents them from later renewing their reproduction process as capital on the same scale. This is the ruinous effect of the fall in the prices of commodities. It does not cause the destruction of any use values. What one loses, the other gains. Values used as capital are prevented from acting again as *capital* in the hands of the same person.The old capitalists go bankrupt. If the value of the commodiites from whose sale the capitalist reproduces his capital = £12,000, of which say £2,000 were profit, and their price falls to

£6,000, then the capitalist can neither meet his contracted obligations, nor, even if he had none, could he, with the £6,000, restart his business on the former scale, for the commodity prices have risen once more to the level of their cost prices. In this way, £6,000 has been destroyed, although the buyer of these commodities, because he has acquired them at half their cost price, can go ahead very well once business livens up and may even have made a profit' (Marx 1969b:496).

[38] In the early 90s in London because of a shortage of London Stock bricks the labour time of stealing the contents of a house sank lower than stealing its fabric. A new crime developed; instead of breaking into houses, entrepreneurs broke the houses themselves and sold the bricks.

[39] The Roman builders of a number of still-functioning Italian aqueducts would be gratified but astonished to find they had started a self-sustaining joint production process which was to last two thousand years.

[40] For simplicity we have omitted the commodity stocks of labour power, and those held by labour power. Because the value of C_I does not change in this example, this does not affect the calculation.

[41] There is not scope to go into the authoritative and profound Japanese debate on market value discussed in Professor Itoh's (1980) book. However, we believe that the insight provided by the fact that previously-existing stocks enter the formation of market values does substantially change the terms of the debate; it means, for example, that society does not immediately determine what is socially-necessary and what is not, but only after a lapse of time; and that the movement of stocks is one of the main indicators that allow producers to judge whether their selling price corresponds to what is socially necessary labour.

[42] In the extreme case of software, which is in principle indestructible, *all* depreciation is moral. How can a vintage theory possibly explain its contribution to value?

[43] 'If the short working life of the machines (their short life-expectancy *vis-à-vis* prospective improvements) were not counter-balanced [by extension of the working hours] they would transfer too great a portion of their value to the product in the way of moral depreciation' (Marx 1981:209).

[44] At one stroke, incidentally, this eliminates the neoclassical production function: there *is* no fixed relation between the value of outputs and the value of inputs derived from the production condition of a single process. The marginal product of either capital or labour simply ceases to exist in the normal sense of General Equilibrium theory.

[45] The sizeists would have won.

[46] We are indepted to Paolo Giussani for pointing this out.

[47] Stock matrices X and the like are functions of both production and circulation. Turnovers $\Delta X / \Delta t$ are the first partial derivatives of *stock* matrices, maintaining circulation zero. We could have completed the analysis more rigorously by introducing a turnover due to circulation, the first derivative maintaining production zero. The continuity condition states that both these partial derivatives should be bounded.

[48] The concession of a change in scale is permitted in the concept of a von Neumann ray, or balanced growth, in which all production expands proportionately. This does not alter the argument that follows, since balanced growth requires that *at least* the previous inputs to production should be reproduced.

[49] The recognition of this fact is, I think rightly, considered by Hegel as almost the first act of philosophy. 'Becoming is the first concrete thought, and therefore the first notion; whereas Being and Nought are empty abstractions … As the first concrete thought-term, Becoming is the first adequate vehicle of truth. In the history of philosophy, this stage of the logical Idea finds its analogue in the system of Heraclitus. When Heraclitus says "All is flowing", he enunciates Becoming as the fundamental feature of all existence' (Hegel 1975:1323).

[50] We could have made this distinction formally, but this would have overloaded the notation. The context always makes clear whether stock variables are being considered before or after circulation.

[51] This can be corrected to allow for *secondary exploitation,* transfers of value to and from consumer durables, but we shall omit this correction here.

[52] I am grateful to Professor Itoh for an extremely useful discussion on the mechanism of crisis, though I am responsible for the interpretation which follows and particularly any errors it contains.