Vertical Integration and Classical Economic Theory

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10/2/1997

The debates around value and price can be more fully informed by bringing empirical information to bear on the questions and framework of classical political economy. The availability of input-output accounts and related data today makes the calculation of labor values and other relevant classical economic variables possible. Empirical work that utilizes these accounts is an important dimension of research on value and price and remains largely undeveloped. The vertically integrated approach of classical political economy can easily substitute data from the input-output accounts in place of the numerical examples frequently used to make theoretical arguments. In this way, classical theoretical positions can be empirically informed.

Despite playing a fundamental role in the development of vertical integration in empirical analysis, input-output economists have not formulated their hypotheses around the issues that directly concern classical political economy. Input-output economists have mostly ignored the links of their own work to the classical economic tradition. Hence, much of the input-output literature is not theoretically motivated in the same way as classical political economy. Empirical research on technical change is often done without a clear sense of its theoretical implications. Yet, for theoretical reasons the analysis of technical change should utilize the vertically integrated framework of the classical economists in place of the direct coefficients framework frequently used in input-output analysis. The vertically integrated "sector," associated with classical economic theory, is superior to the input-output "industry" because it reduces all the complex heterogeneous inputs to a simple homogeneous input. Marx thought in terms of comparing the total labor used in the production of commodities in each industry. Vertically integrated labor coefficients are constructed through the whole intricate pattern of inter-industry connections, but capture all the labor inputs in a simple summary statistic, which Marx called value. Hence, empirical value analysis is made possible by vertical integration, and is appropriate since Smith, Ricardo, and Marx utilized vertical integration as the basis of their theories of value.

This paper addresses the concept of vertical integration in the classical economic tradition and shows how it can be given empirical content utilizing the framework developed by Sraffa (1960), Leontief (1986), Pasinetti (1980), Ochoa (1986), and Shaikh (1995). Classical "value" categories are given empirical content and pertinent theoretical issues are explored. Four major parts are presented to show the applicability of vertical integration for the analysis of technical change and to the calculation of embodied labor coefficients and prices of production. In Part One, I examine the conceptual use of vertical integration in the works of Smith, Ricardo, and Marx. In Part Two, I discuss the advantages and drawbacks of the "direct" interindustry approach and argue in favor of the vertically integrated approach. In Part Three, I show how the input-output framework can be used to calculate vertically integrated labor coefficients. In Part Four I consider the use of vertical integration in the calculation of embodied labor time, and present three numerical examples framed in the input-output framework to illustrate how vertical integration can be used to inform theory.

I. Vertical Integration in the Classical Economic Tradition

The classical economists used vertical integration in at least three important respects: to analyze the direct and indirect labor requirements needed for the production of individual commodities; to show the breakdown of profits and wages; and, to show how changes in the production conditions influence relative prices. Vertical integration can show how changes in technology, changes in distribution, and even temporary disturbances in the balance of supply and demand affect the relative prices of commodities.

Central to the classical economic tradition is the hypothesis that the relative direct and indirect labor time (i.e. vertically integrated labor time) used to produce commodities determines the relative prices of commodities. Changes in relative vertically integrated labor coefficients were identified as the principle source of changes in relative prices (Ricardo 1954: 13). Smith and Ricardo begin their analysis at the most abstract level where they argue that embodied labor time is the sole determinant of relative prices. They argue that as long as the relation of embodied labor to direct labor and the rate of turnover are the same across industries, relative vertically integrated labor coefficients can be seen as the sole determinant of relative prices. They argue that the aggregate sum of these various kinds of labor, both direct and indirect, governed the relative prices of commodities (Ricardo 1954: 25). Changes in the labor time socially necessary to produce commodities bought as capital affect the relative prices of the commodities that use them as inputs, whether directly or indirectly. The materials that act as means of production are seen to represent the indirect labor time used in the production of a given product. Ricardo illustrates this point by noting the effect of a change in the labor time necessary to produce cotton on the commodities that use cotton as an input: “If by some technical change fewer hours were required to cultivate cotton, then the price of stockings, in whose production much cotton is used, would necessarily fall relative to other commodities not affected by the technical change" (Ricardo 1954: 25).

This particular construction is representative of the “pure labor theory of value” (Pasinetti 1977: 75). Pasinetti notes that in the extreme case, where the profit rate is equal to zero, the labor theory of value holds.

where r =3D 0

$\mathbf{p}$ =3D $w\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$

$\boldsymbol{\lambda}$ =3D $\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$

p_i/p_j =3D λ_i/λ_j .

This labor theory of relative prices is the reference point for all classical political economists interested in value because it establishes the scope and method from which all further discussion of the determination of prices stem.

After thoroughly developing the thesis that the comparative worth of two commodities is entirely regulated by the relative quantity of labor embodied in the two commodities, Ricardo and Smith examine the effect of changes in distribution on the relative prices of two commodities which have different proportions of direct and indirect labor time or which have different rates of turnover. They conclude that changes in the rate of profit, given different structural properties, can affect relative prices. Both believe that the “pure” labor theory of price must be modified when the proportion of fixed to circulating capital or the rate of turnover differed across industries. Both note that there are two “great causes” of the variations in prices: One is the technical conditions of production and the other is the effect of changes in distribution on capitals with different capital compositions and turnover rates. Ricardo maintains the view that the amount of labor time embodied in a commodity is the dominant determinant of changes in relative prices. He attempts to account for the deviations of relative prices from relative embodied labor times and also to specify the direction and magnitude of these deviations by examining the role of distribution. But, Ricardo concludes that relative vertically integrated labor times rather than distribution played a dominant role in the determination of prices. According to Ricardo the greatest effect which can produced on relative price due to a change in distribution could not exceed six or seven percent (Ricardo 1954: 36).

Utilizing the technique of vertical integration the classical economists also argue that individual prices can be broken down into two constituent components: wages and profits. Adam Smith develops this notion by first looking at the direct effects (or the first “layer”) of prices which he says could be reduced to wages and profits and a remaining portion -- material (Shaikh 1984). The production conditions of the materials which act as inputs represent the indirect effects=2E The materials, which represent the second layer, can themselves be broken down into wages, profits, and the materials that represent the third layer. In turn, the materials used as means of production in this layer and all the others going back can be broken down so that the portion materials disappears and all that is left is wages and profits (Shaikh 1984). Adam Smith performs just this operation in his breakdown of the components of price which make up the price of flour:

> In the price of the flour or meal, we must add to the price of the corn, the profits of the miller, and the wages of his servants; in the price of bread, the profits of the baker, and the wages of his servants; and in the price of both, the labour of transporting the corn from the house of the farmer to that of the miller, and from that of the miller to that of the baker, together with the profits of those who advance the wages of that labour (Smith 1986: 153-54).

In theory, the component parts of price can be computed by adding up the wages and profit of each layer until the portion which represented materials approached zero (Pasinetti 1977: 66, Shaikh 1984: 66). This is equivalent to the vertically integrated approach of estimating profits and wages.

Vertical integration can be used to break down the constituent elements of any commodity into labor (both direct and indirect), or wages and profits. Shaikh (1984:65) explains how the prices of commodities are broken down into a wage component and a profit component. He notes that the price can be thought of as the sum of wage costs (wL), material costs (M), and profit (π).

p =3D $wL + \pi + M$

Shaikh notes that the price of materials (M) can be thought of as being composed of wages, profits, and the material costs of the industries which produce the means of production. He notes that the equation for prices could be rewritten to reflect this by accounting for conceptual stages of production. The materials used in the current stage of production can be thought of as M =3D $wL^{(1)} + \pi^{(1)} + M^{(1)}$. Shaikh (1984: 66) use the price equation p =3D $wL + \pi + M + wL^{(1)} + \pi^{(1)} + M^{(1)}$

to show that prices are composed of wages and profits. He notes that the residual material costs $M^{(1)}$ is smaller than the original material costs M. Following this logic Shaikh shows that the residual can be reduced until in the limit there is no material costs. In this way the price can always be conceptualized as the sum of wage costs and profits.=0D

p =3D $wL^T + \pi^T$

In this equation the term wL^T represents the vertically integrated wages and the term π^T represents the vertically integrated profits.

Following Pasinetti (1977), Shaikh (1984), and Semmler (1984) we can express the prices of commodities to equal the sum of wages (w**l**), profits (r**pA**), and intermediate materials used (**pA**) in production.

p =3D $w\mathbf{l} + r\mathbf{pA} + \mathbf{pA}$

p(**I** - **A**) =3D $w\mathbf{l} + r\mathbf{pA}$

Multiplying both side by $(\mathbf{I} - \mathbf{A})^{-1}$ we establish, through vertical integration, that the intermediate materials used in production can themselves be divided between wages and profits=2E

p =3D $w\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1} + r\mathbf{pA}(\mathbf{I} - \mathbf{A})^{-1}$

Where $\boldsymbol{\omega}$ =3D $w\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$ and $\boldsymbol{\pi}$ =3D $r\mathbf{pA}(\mathbf{I} - \mathbf{A})^{-1}$.

Hence the individual prices of commodities can be thought of as the sum of vertically integrated profits plus the sum of vertically integrated wages, p=3D $\omega + \pi$=2E Here relative prices are expressed as the sum of wages and profits.

In addition, vertical integration can be used to show the effect of an increase in the price of one commodity on all the others. Price increases in raw materials, for example oil, coal, or cotton, whether the result of changes in long-run factors like productivity, or short-run factors like discrepancies in supply and demand, affect the prices of other commodities differently. Marx, in his discussion of the effect of changes in the price of cotton on other commodities, said "if the price of raw material rises- for cotton for example - the price of cotton goods rises as well: both semi-finished goods such as yarn, and finished products such as cloth, etc. which are produced with this more expensive cotton" (Marx 1981: 207). The vertically integrated material components used in production can be a significant source of changes in relative prices and must be analyzed to account for relative price changes.=0D

In this section, I have shown that the classical economists employ the concept of vertical integration as the basis of their theories of value. They use it in three ways-- to study technical conditions of production, to break down the component part of price between wages and profits, and to assess the effects of supply and demand imbalances on the relative prices of commodities.

II. Direct Coefficients Approach

Pasinetti (1981: 109) distinguishes between two theoretical frameworks: The input-output approach, which relies principally on the analysis of direct coefficients and Sraffa's 'production of commodities' system, which relies on vertically integrated coefficients. Pasinetti claims that while both share the characteristic of being built on coefficients, the latter approach is grounded in the concept of vertically integrated sectors. Pasinetti argues that the direct coefficients approach, which he identifies with interindustry analysis, is insufficient for the study of technical change.

The interindustry approach advanced and developed by Leontief (1976) and Carter (1970) challenges the general perspective that technical change can be analyzed without explicitly analyzing changing intermediate inputs. The input-output economists conceive and empirically describe the national economy in terms of a system of mutually interrelated sectors. Leontief says:

> A national economy in operation is a process of simultaneous adjustment in which a very large number of apparently separate but actually interdependent flows of production, distribution, consumption and investment are constantly affecting each other and ultimately are determined by a set of basic characteristics of the system (Leontief 1976: v)

Leontief’s and Carter’s early studies principally focus on direct flows. In this sense their direct approach represented a huge advance over the general hypotheses of many economic theorists of technical change because they set out to examine material changes in the structure of production.

Leontief’s early work approaches the study of technical change and the evolution in the U.S. economy by examining the changing intermediate input requirements of individual industries. Viewing the economy from this perspective, Leontief shows the relations between separate sectors of the economy and how they change with time. In his early study of the U.S. economy Leontief looks at the relative percent changes in coefficients between 1919, 1929, and 1939 and describes technical change in terms of the changing input coefficients (Leontief 1976: 27). Recognizing that columns of input-output coefficients provide a great deal of detailed information, Leontief compresses this information by looking at aggregate coefficients.

Following Leontief's approach, Carter (1970) examines changes in technology by examining the movement in coefficients. She employs graphs liberally and stresses the directional and order-of magnitude aspects of technical changes. She finds direct labor saving to be the most striking feature of structural change (1970: 218). While Carter's study stresses individual coefficients, it also recognizes that changes in individual coefficients are generally only one part in a complex system of interrelated structural shifts (1970: 25). Thus, changes in individual supplying sectors mutually affected, whether directly or indirectly, other industries technical conditions of production.

Some economists contend that this approach is superior to the vertically integrated approach. Lance Taylor (1994), for example, questions whether too much information is lost due vertical integration. He says that vertical integration "=2E..leaves out precisely the details that farmers, the people from whom they buy and the to whom they sell, and the companies and researchers that provide their new technologies are likely to deem important" (1994: 8). From Taylor's perspective no information should be lost. A major drawback of this approach is that it necessitates that all the detail be analyzed before general comparative statements can be made about technical change. Moreover, it makes the work of interindustry and intertemporal comparisons of technical innovations virtually impossible. Individual processes must be compared in terms of their full complexity. For the researcher this means they have to be an expert in every field. To understand the innovation in the chemical industry the researcher would need to understand not only chemistry, but also all the processes that are used to produce the inputs used by chemists. Furthermore, the lack of a common input prevents ratio comparison and value analysis.

Vertical integration is invaluable because it facilitates the comparison of different processes through their vertically integrated labor inputs. It effectively "abstracts" from the complexity of each individual production process and permits comparison in terms of labor values. The direct approach can be used to supplement, but should not be viewed as superior from the perspective of understanding the dynamic patterns of the economy.=0D

Pasinetti (1983: 114) identifies a "gap" between the inter-industry approach, which tends to focus on direct coefficients, and the vertically integrated approach, which he associates with the classical tradition. He (Pasinetti 1983: 115) argues that while interindustry analysis is useful in understanding what happens at specific moments in time he claims that this approach "breaks down" over time as the interindustry connections change. While a particular input-output table is needed for each stage in the evolution of the economy the table cannot be consistently and "analytically linked" through time without the vertically integrated sector. Pasinetti (1983: 117) argues that the interindustry approach cannot provide "generality" in passing from one table to another. The mass of detail and changes in the structures of production make the direct approach less useful.

Vertically integrated "sectors" are superior measures of technology than direct measures based on the 'industries' used in input-output analysis. While the vertically integrated sector is “complex”, because it is constructed through the “whole intricate pattern of inter-industry connections,” it is also simple because it identifies a common element (Pasinetti 1983: 110). Marx refers to the common element of labor as value. Pasinetti notes that while the vertically integrated sector is complex in its composition its “ultimate constituent elements” have the advantage of being homo=geneous. The vertically integrated unit of productive capacity itself can be thought of in terms of vertically integrated labor. Pasinetti argues that only the vertically integrated model "allows us to follow the vicissitudes of the economic system through time" (1993: 117). Continuity through time is maintained in this approach because the vertically integrated technical coefficients acquire a meaning independent of the individual parts that compose them (Pasinetti 1993: 117).

III. The Input-Output Framework

While the conception of vertical integration plays an important role in the classical economic tradition, it was not until Wassily Leontief’s (1986) pathbreaking empirical work that vertical integration was given empirical possibilities with the Leontief inverse matrix, $(I - A)^{-1}$. From the Leontief inverse matrix the direct and indirect commodity requirements needed to produce a given level and composition of final demand can be obtained. Following Leontief’s notation we know:

n : Total number of industries.
X_i : The amount of output from industry i.
: The amount of output from industry i used as an input by industry j for the production of its own output.
Y_i : The quantity of industry i output produced and delivered for final demand.
L_j : The employment in industry j.

The technical commodity flow input coefficient is defined as:

$$a_{ij} = \frac{X_{ij}}{X_j}, \quad i, j = 1, \ldots, n$$

This represents the amount of industry i's output used per unit of j output.

Similarly, the labor coefficients are calculated by dividing the employment in each industry by its corresponding output.

$$l_j = \frac{L_j}{X_j}, \quad j = 1, \ldots, n.$$

Leontief describes the economy through a system of *n* simultaneous equations that describe the balance between total output, intermediate inputs, and final demand.

$$X_1 = X_{11} + X_{12} + \ldots + X_{1n} + Y_1$$
$$X_2 = X_{21} + X_{22} + .. + X_{2n} + Y_2$$
$$\ldots$$
$$X_n = X_{n1} + X_{n2} + \ldots + X_{nn} + Y_n$$

Alternatively, Leontief also represents the components of this equation in matrix form.

$$X = \begin{matrix} X_1 \\ X_2 \\ \ldots \\ X_n \end{matrix} \qquad Y = \begin{matrix} Y_1 \\ Y_2 \\ \ldots \\ Y_3 \end{matrix} \qquad A = \begin{matrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{matrix}$$

Here, total output is equal to intermediate input plus final demand. This simultaneous equation system can be rewritten using the technical coefficients a_{ij}.

$$X_1 = a_{11} X_1 + a_{12} X_2 + \ldots + a_{1n} X_n + Y_1$$
$$X_2 = a_{21} X_1 + a_{22} X_2 + \ldots + a_{2n} X_n + Y_2$$
$$\ldots$$
$$X_n = a_{n1} X_1 + a_{n2} X_2 + \ldots + a_{nn} X_n + Y_n$$

In matrix form this is represented by

X =3D **AX**+ **Y** , (4.3)

where **X** is the total product, **Y** is the net product, and **AX** represents intermediate inputs.

This accounting relation shows that total output minus intermediate input is equal to the net product. The conception of a net product is common to both the classical economic tradition and input-output analysis. In the national accounting total output, intermediate inputs, and the "gross national product" (final demand or the net product) are also represented in this simple accounting relation. The simultaneous system above can equally be written as:

$$
\begin{aligned}
&(1-a_{11})X_1 - a_{12}X_2 - \ldots - a_{1n}X_n = Y_1 \\
&-a_{21}X_1 + (1-a_{22})X_2 - \ldots - a_{2n}X_n = Y_2 \\
&\ldots \\
&-a_{n1}X_1 - a_{n2}X_2 - \ldots - (1-a_{nn})X_n = Y_n
\end{aligned}
$$

In matrix form we know that this can be expressed as (**I** - **A**)**X** =3D **Y**. Inverting the matrix (**I** - **A**) gives the Leontief inverse. Here, gross output is expressed in terms of the technical conditions of production and the net product.

X =3D (**I** - **A**)$^{-1}$**Y**

The Leontief inverse matrix (**I** - **A**)$^{-1}$ can be used to quantitatively calculate labor values, prices of production, and other key categories used by Marx and the classical economists.

In this framework labor values are thought of as the vertically integrated labor used in the production of a given quantity of commodities. To calculate embodied labor (or vertically integrated labor coefficients) both the commodity input requirements and the direct labor coefficients are required.

Labor values represent both the direct as well as the indirect labor used in the production of commodities. Given the direct labor requirements needed to produce a given total product and also the technical coefficients matrix, the calculation of direct labor coefficients $\mathbf{l}$ and vertically integrated labor coefficients $\boldsymbol{\lambda}$ is straightforward. The total embodied labor times (or labor values) are determined by calculating the vertically integrated labor used in the production of a given output. The unit values are given by.

$\boldsymbol{\lambda}$ =3D $\mathbf{l}(\mathbf{I} - \mathbf{A})^{-1}$.

This represents the embodied labor per unit of output. To arrive at the total value produced in each industry requires that the output be multiplied by the embodied labor coefficients.

$\boldsymbol{\Lambda}$ =3D $\boldsymbol{\lambda}<\mathbf{X}>$

If we scale this vector so that it is commensurable with total sales then direct prices (or scaled up labor values) can be compared directly with market prices and price-value deviations can be examined.

IV. Examples

To illustrate the basic principles I have outlined in this paper, I examine the question of value from both the cross-sectional and temporal perspectives. I present three examples. I assume an economy that produces two commodities, pigs and corn. In the first example I show how vertically integrated labor coefficients can be derived from an economy in which all accounting is done in terms of use-values. In the second example, I assume that there is technical change in the economy and I show how this can be measured. In the third example, I show that accounting done in terms of monetary units does not affect the calculation of vertically integrated labor.

To begin with I assume that the two commodities are expressed in physical units pigs and bushels of corn, in an economy which has the technical conditions of production exhibited in Table 4.1.

Table 4.1 Industry Use-Value Requirements in Year One

To produce 75 pigs requires:	15 pigs	125 bushels of corn	100 worker years
To produce 450 bushels of corn requires:	10 pigs	200 bushels of corn	150 worker years

Only pigs and corn are produced. The technical conditions of production in Year One can be represented in matrix form. A is the coefficients matrix, l is the vector of labor coefficients, and X is the vector of total output.

$$A = \begin{matrix} .2 & .022 \\ 1.667 & .444 \end{matrix}$$

$$l = 1.333 \quad .333$$

$$X = \begin{matrix} 75 \\ 450 \end{matrix}$$

From the equation λ =3D $l(I - A)^{-1}$ I calculate vertically integrated labor coefficients. They equal 3.182 and .727. According to the labor theory of value the relative vertically integrated labor coefficients dictate the relative prices. Hence, one pig should exchange against 4.375 bushels of corn.

Now, if I assume a second year in which the technical conditions of production change, I can estimate the change in relative vertically integrated labor and project a change in relative prices. Let us assume that the technical conditions of production change and reflect the accounts represented in Table 4.2.

Table 4.2 Industry Use-Value Requirements in Year Two

To produce 100 pigs requires:	20 pigs	100 bushels of corn	100 worker years
To produce 500 bushels of corn requires:	10 pigs	150 bushels of corn	150 worker years

Just as above, this relation can be represented in matrix form and vertically integrated labor can be calculated.

$$A = \begin{matrix} .2 & .02 \\ 1 & .3 \end{matrix}$$

$$l = \begin{matrix} 1 & .3 \end{matrix}$$

$$X = \begin{matrix} 100 \\ 500 \end{matrix}$$

Vertically integrated labor coefficients to equal 1.852 and .481. From this information I expect 1 pig to exchange for 3.85 bushels of corn.

Technical change has taken place between Year One and Two in both vertically integrated sectors. We can measure technical change by the movement in the direct labor coefficients or the vertically integrated labor coefficients. The latter is the preferable for all the reasons given in the first three sections of this paper.

To produce a pig the vertically integrated labor fell from 3.182 worker years to 1.852 years. The rate of technical innovation in the production of pigs, as measured by vertically integrated labor productivity, increased by 41.80%. To produce a bushel of corn in Year One required .727 vertically integrated worker years. In Year Two this dropped to .481. The rate of growth in vertically integrated labor productivity in the production of corn increased by 33.82%. According to the labor theory of value, the lower relative rate of technical innovation in the production of corn should cause a rise in the relative price of corn relative to pigs. The expectation is that the price of a bushel of corn relative to pigs should increase from .228 in Year One to .259 in Year Two.

The framework presented above is developed in physical terms. In the example below I illustrate how labor values can be calculated from input-output tables expressed in any unit of account. The relation between a system expressed in purely physical units and a system expressed in terms of market prices is similar (Yamada 1961, Shaikh 1984). I assume the same production structure as above, but I arbitrarily set the market price of pigs to \$50 and the market price of corn to \$25 a bushel=2E The relations expressed in terms of these market prices appear in Table 4.3.

Table 4.3 Industry Requirements in Year Two Expressed in Market Prices (Where the price of pigs=3D\$50 and the price of corn=3D\$25)

To produce \$5,000 worth of pigs requires:	\$1,000 worth of pigs	\$2,500 bushels of corn	100 worker years
To produce \$12,500 worth of corn requires:	\$250 worth of pigs	\$3,750 worth of corn	150 worker years

Despite the fact that the data is expressed in terms of dollars the value relations remain unaffected.

$$A^* = \begin{matrix} .2 & .04 \\ .5 & .3 \end{matrix}$$

$$l^* = .02 \quad .012$$

$$X^* = \begin{matrix} 5000 \\ 12500 \end{matrix}$$

Following equation 4.1, I calculate the vertically integrated labor coefficients (.03704 and .01926). Multiplying these by the respective market prices of each commodity I can recover the vertically integrated labor coefficients per unit of physical output (1.852 and

.4815). In the input-output accounting framework I do not know the units of physical output. But, I can multiply the coefficients by sales to determine the vertically integrated labor in each industry (185 and 240). This corresponds to Example Two which is done in purely physical units, and makes clear that technical coefficients expressed in terms of a monetary unit do not affect the calculation of vertically integrated labor used by each industry.

In addition, these measures can be scaled and compared directly against total industry sales and the significance of price-value deviations can be ascertained. Total sales equal $17,500 and the total number of vertically integrated worker-years equal 425. We can scale the total value (425) so that individual values are directly comparable with individual sales. The ratio κ= 41.176 is a scalar (where the sum of market prices is equal to the sum of values) which can be used to directly compare price-value deviations. Multiplying individual values by κ we calculate direct prices that are directly comparable to market prices=2E In this instance the direct price of the total outputs equals $7,617=2E56 (185*41.176) and $9,882.24 (240*41.176). Market prices for total sales were given to be $5,000 and $12,500. Absolute percent deviations between direct prices and market prices equals 52.4% for pigs and 17.6% for pigs. For the whole economy the MA%D (mean absolute percent deviation) is 35% and the MA%WD (mean absolute percent weighted deviation) equals 27.8%.

Using these very simple examples I have illustrated that categories, such as the quantity of vertically integrated labor used in each industry's production can be captured using either physical data or price denominated data. Provided price deflators accurately reflect the changing prices, the rate of vertically integrated labor productivity can also be determined. Price-value deviations can be examined by scaling the values so that they are directly comparable to market prices.

V. Conclusion

In this paper I have attempted to examine the notion of vertical integration and demonstrate how it can be used as a foundation for statistical work on the real economy. I have shown that it is a notion true to the classical theory of value and can be implemented empirically using the input-output accounts. I argued in Part One that the classical economists embraced the notion of vertical integration in their theories of value. I cited several examples in their work where they refer to the impact of a change in technology in one sector affecting other sectors. In Part Two I discussed input-output analysis and argued in favor of the vertically integrated approach to analyzing technologies comparatively and for the analysis of technical change. In Part Three I developed the framework of input-output analysis and showed how relatively simple categories such as labor values can be calculated using the input-output accounts. In the fourth part I presented a step-by-step analysis of how labor values are calculated using a two-by-two example. I showed that vertical integration could be used to analyze technical change and its relation to relative prices. I also explored the relation of input-output tables expressed in physical magnitudes with tables expressed in terms of a monetary unit. I showed by means of an empirical example that this does not affect the calculation of vertically integrated labor. In this paper I confront the task of empirically researching the relationship of classical economic theory with the empirical evidence using the vertically integrated framework.

While Smith, Ricardo, and Marx were confined to numerical examples to illustrate their theories, due to the lack of sufficient empirical data, the same cannot be said of the present generation of classical economists. Yet, only infrequently have contemporary classical economists made use of the existing data. Instead, they go on developing numerical examples to illustrate relatively minor points. Numerical examples can be helpful in illustrating basic principles, but the central objective must be to explain the world in which we live.