ON THE RELATION BETWEEN LABOR TIME AND MONEY IN A TWO SECTOR SYSTEM OF COMMODITY PRODUCTION AND EXCHANGE

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Abstract

The issue of the relation between labor time and money in a two sector system of commodity production and exchange is investigated. An interrelation may be derived from consideration of the physical data alone - the commodity and labor time inputs, the commodity outputs, and the real wage - in which the prevailing two prices and money wage separately multiplied by a certain relation between labor time and money are determinate. This relation is the ratio of the total man-hours of direct labor time to the price of the net product. This same result may alternatively be derived in an argument that begins instead with the elements of the labor value system.

I

Introduction

Marx defined the value of a commodity as the amount of socially necessary labor time needed to produce it, but there are however well-known difficulties in applying this concept of value to the theory of competitive profit rate-equalizing prices. In certain special circumstances in a two sector system - equal compositions of capital or an equalized zero money rate of profit - the relative labor value equals the relative price or, the same thing, the individual commodity {labor value/price} ratios are equivalent. Furthermore, this common relation equals the ratios of these corresponding aggregates: {total value/total price}, {total input value/total cost price}, {total surplus value/total profit}, and {total direct labor time/price of net product}.

The difficulty here is that under the usual conditions of differing compositions of capital and an equalized nonzero rate of profit the individual {labor value/price} ratios must always differ, and there was no reason to prefer one to the other.

Now Marx was aware of this but he asserted that despite these differences total price could be equated with total value while, simultaneously, total profit could be equated with total surplus value:

> ...the sum of the profits in all spheres of production must equal the sum of the surplus-values, and the sum of the prices of production of the total social product equal the sum of its value. [Marx, 1894, Chap. X, p. 173].

[Marx throughout this argument assumes that the total capital turns over once in one period.] This assertion may be expressed as follows: under all two sector conditions the aggregate ratio {total value/total price} equals the aggregate ratio {total surplus value/total profit}. This also equals the aggregate ratio {total input value/total cost price}. [Equivalence between any two of these implies equivalence with the third.] This argument holds if and only if the rate of profit in terms of prices equals the rate in terms of values.

The difficulty here is that under the usual two sector conditions of differing compositions of capital and an equalized nonzero money rate of profit these two rates differ and the above three ratios must then all differ. These furthermore usually differ from the fourth aggregate ratio mentioned above, {total direct labor time/price of net product}.*1*

To summarize: except under certain special circumstances the individual commodity {labor time/money} ratios differ and the aggregate ratios all differ. It consequently seemed that in general no definite relation between any amount of labor time and any price or sum of prices could be concluded or preferred, again, under the usual conditions obtaining in a two sector system [Seton, 1957, p. 153].

The entire issue appeared to be rendered irrelevant in a further development. In an approach usually associated with the work of Piero Sraffa in his Production of Commodities by Means of Commodities, subsequently termed the quantity system argument in this paper, it was shown that given just the "physical" data - the commodity and direct labor time inputs, the commodity outputs, and the real wage - as premises, the money rate of profit and the relative price were determinate [Sraffa, 1960]. It was quite interesting that certain monetary relations were calculable without knowing or assuming anything in regards to the nature of money. To be sure, it could be shown that profit [a nominal money magnitude] was positive if and only if surplus value [a certain amount of labor time] was positive, and this conclusion was accorded the status of the "Fundamental Marxian Theorem" [Morishima, 1973]. But this conclusion was merely qualitative and none of the quantitative relations between labor time and money discussed above existed in this argument.

The calculation of the relative price and the money rate of profit in the quantity system argument does not require knowledge of the absolute levels of the physical data. Rather, it is only necessary to know the physical data represented in terms of certain relative quantities, and the argument of this paper originates in a critique of this particular representation. Specifically, this representation leaves out of account a certain relative datum defined by the absolute physical data, and this information is that which fixes the relative size of the sectors. When this information is taken into account then "relative" prices and money wages in a certain different sense may be calculated. That is, given the physical data alone the separate commodity prices and the money wage each multiplied by a certain factor, three combined magnitudes, are determinate, and this factor is the aggregate ratio {total direct labor time/price of net product}. In this "expanded" quantity system argument this quantitative relation between labor time and money stands in relation to nominal unit prices and money wages, and the combined magnitudes are invariant with respect to any purely nominal variances. The argument here thus affirms, by this route, the preferred relation between labor time and money as put forward originally by Gérard Duménil and Duncan K. Foley [Duménil, 1980. Foley, 1982]. [Section III]

It will also be shown that while the two sector labor value system - defined here as the two individual labor values, the value of labor power, and the rate of profit in terms of values - is determinate from the real premises, it happens that except in one extraordinary situation one of these elements, the value rate of profit, is indeterminate absent consideration of the relative size of the sectors. When this calculation is shown then the above expanded relative price and money wage determination may be completed in an argument beginning with the elements of the labor value system and applicable in all circumstances. The quantitative analysis of profit and surplus value may also then be undertaken. [Sections IV & V]

The representation of the two sector system in the usual quantity system argument is also indeterminate in important other respects, and these deficiencies are also corrected when this same neglected information is taken into account. [Section VI]

[It is the two sector system alone that is analyzed in this paper. The major difficulties in understanding the role of the relation between labor time and money are describable within the confines of this abstracted system, and it might be expected that a satisfactory solution, if any, to these difficulties should also be describable at this same level of abstraction. The application of this analysis to the n-sector system is an obvious project beyond the purpose here.]

II

Notation and the Quantity System Argument

The method of exposition will be to develop a notation for a two sector system and describe the quantity system argument. This will introduce the main arguments in the succeeding sections.

The two sector model considered here consists of a Sector I producing capital goods exclusively and a Sector II producing consumption goods; workers and capitalists consume from Sector II alone. This system is simplified in that it is assumed that there is no fixed capital; the circulating capital turns over once per production period; production takes one period; and the wage is advanced at the beginning of the period. In the following N1 and N2 are the units of nonlabor inputs in the capital goods and consumer goods sectors respectively; L1 and L2 are the respective labor inputs, the man-hours of direct labor time; Q1 and Q2 are the respective units of output; P1 and P2 are the respective nominal unit prices of the outputs; w is the nominal hourly money wage; and r is the money rate of profit. The real output of Sector I Q1 is assumed to be at least equal to the sum of the nonlabor inputs of the two sectors {N1 + N2} [this issue is revisited in Section VI]. The hourly money wage here represents the purchase of one hour of labor power which yields one hour of social labor. If competition among workers and capitalists equalizes w and r across the sectors, the equations determining prices in the two sectors may be written:

I Capital Goods $[N1P1 + L1w][1 + r] = Q1P1$ (1)

II Consumption Goods $$[N2P1 + L2w][1 + r] = Q2P2 \qquad (2)$$

These equations are now divided through by the respective L1 or L2. The relations N1/L1 and N2/L2 are designated k1 and k2 respectively; the relations Q1/L1 and Q2/L2 are designated q1 and q2 respectively; and the equations become:

$$[k1P1 + w][1 + r] = q1P1 \qquad (3)$$
$$[k2P1 + w][1 + r] = q2P2 \qquad (4)$$

The quantity system argument may be demonstrated after dividing equations (3) and (4) through by the price of the consumer commodity P2. w/P2 is the hourly real wage and is designated RW:

$$[k1P1/P2 + RW][1 + r] = q1P1/P2 \qquad (5)$$
$$[k2P1/P2 + RW][1 + r] = q2 \qquad (6)$$

The argument may now be stated. Given the absolute physical data - the total real wage and the real inputs and outputs - as premises, the relative magnitudes {k1 k2 q1 q2 RW} are defined. The rate of profit and the single relative price P1/P2 are then determined from equations (5) and (6).*2* This quantity system determination is outlined as follows:

physical data   -> {k1 k2 q1 q2 RW} -> {P1/P2 r}

The premises of this quantity system argument, the absolute physical data represented in terms of the relative quantities {k1 k2 q1 q2 RW}, do not reflect changes in the relative size of the sectors [for example multiplication of {N1 L1 Q1} by a factor of two]. But there is no logical reason for considering only some, and not all, of the relative information defined by the absolute levels of the physical data. When this additional information is after all taken into account then one may go beyond the calculation of the relative price P1/P2, i.e., the nominal prices relative to one another. As shown in the next section one may in fact calculate the two prices and the money wage each multiplied by a certain relation between labor time and money, the aggregate ratio {total direct labor time/price of net product}. The labor value system may also then be completed, and certain other indeterminacies in the usual quantity system representation of the two sector system are corrected.

III

Essential Prices and Money Wages

In the two sector system as represented in equations (1) and (2) the real information appears in the form of absolute levels, e.g. N2 and Q1, and the prices and the money wage appear as current dollar magnitudes. It is evident that if all of the real data were multiplied by some common factor and/or if the two prices and the money wage were multiplied by some common factor, an essentially identical reality would be generated. It would be desirable to represent the two sector system in the form of information, or variables, that abstract from such purely proportionate differences, and only such differences. Such a representation may be characterized as the "essential definition" of the two sector system, and it may be derived from an "expansion" of the usual quantity system argument.

In the following L, or the total direct labor time, is the sum of L1 and L2 and N is the sum of N1 and N2.

The first step is to return to equations (1) and (2) and simply sum them. The right side of this sum equation, the total price of the output, is written as the sum of the total price of the means of production inputs NP1 and the price of the net product, here designated Y:

$$\text{sum:} \quad [NP1 + Lw][1 + r] = NP1 + Y \qquad (7)$$

The next step is to divide this equation through by L, and the ratio N/L, the average composition of capital, is twice generated.*3* This quantity, designated k, is a weighted average of k1 and k2 depending on the relative distribution of social labor time. This datum is most conveniently represented by the ratio L2/L and is designated d. Therefore by definition:

$$k = [1 - d]k1 + dk2 \qquad (8)$$

The six absolute real data in equations (1) and (2) - {N1 N2 L1 L2 Q1 Q2} - may be essentially represented in terms of five relative quantities, and in the notation here these are {k1 k2 q1 q2 d}. d fixes the relative size of the sectors, and the other four variables are invariant with respect to d.

The ratio Y/L is the inverse of the relation between the total direct labor time and the price of the net product, and this relation is designated m. The sum equation may now be rewritten:

$$[kP1 + w][1 + r] = kP1 + 1/m \qquad (9)$$

The final step is to multiply (3), (4) and (9) through by m:

$$[k1mP1 + mw][1 + r] = q1mP1 \quad (10)$$
$$[k2mP1 + mw][1 + r] = q2mP2 \quad (11)$$
$$[kmP1 + mw][1 + r] = kmP1 + 1 \quad (12)$$
And:
$$RW = mw/mP2 \quad (13)$$

The quantity system argument may now be restated in an expanded manner as follows. Given the physical data as premises {k1 k2 q1 q2 d RW} are defined and hence k(8) is determined. The rate of profit and the two prices and the money wage each multiplied by m are then determined from equations (10)-(13). This argument is outlined as follows:

physical data -> {k1 k2 q1 q2 d RW} -> {mP1 mP2 mw r}

Multiplying equations (3), (4) and (9) by any of the alternative relations between labor time and money [designated m', for example v1/P1 or the ratio {total value/total price} discussed in Section I] would yield the combined magnitudes {m'P1 m'P2 m'w} along with a remainder {m'/m} on the right side of the converted equation (9). The only thing to do with this remainder is to proceed to divide the equations through by it, yielding (10)-(12) as written.

The proposition that the ratio {total direct labor time/price of net product} or m was the preferred relation between labor time and money was first put forward independently by Gérard Duménil and Duncan K. Foley [Duménil, 1980. Foley, 1982]. As Foley expressed it:

> ...the money value of the whole mass of net production of commodities expresses the expenditure of the total social labor in a commodity-producing economy...[Foley, 1982, p. 37]

Foley discusses the "transformation problem" in his Understanding Capital: Marx's Economic Theory. In the course of presenting a numerical example illustrating his argument this particular relation between labor time and money, which he termed the "value of money," is introduced in equation (6.7) [Foley, 1986, p. 100]. John Roemer, one of the proponents of the usual quantity system approach, criticized Foley's theory in a review of this work. To him this preferred relation between labor time and money was just a "...given..." and he characterized this equation defining it as a "...deus ex machina..." whose "...economic motivation..." he "...[could not] understand..." [Roemer, 1990, p. 1728]. Foley's equation, designed to fix the value of money at unity in his example, equated the price of the net product - Y - with the total man-hours of direct labor time - L. [In this equation Y takes into account the relative composition of net output and the relative size of the sectors is hence determinate.] If, from (7), {[NP1 + Lw][1 + r] -

NP1} is substituted for Y and if this is equated to L/m in a generalization of the equation, then equation (12) above is easily derived. With this equation this theory may be situated in relation to both the usual quantity system argument and "labor value analysis" [Section IV].

To proceed, the ten relative quantities in the above schema represent an infinity of essentially identical two sector economic realities as discussed at the beginning of this section, and the analysis of the two sector system may be conducted in terms of these quantities. Given these data any one of the essentially identical realities are defined given one of the absolute physical data, e.g. Q1, along with the magnitude of m. Equations (1) and (2) may then be written in full.

The premises on the left, the physical data essentially represented as {k1 k2 q1 q2 d RW}, abstract from proportionate quantity change, and only such change, and given these premises the economic reality is essentially determined. By contrast, the premises of the usual quantity system argument also abstract from a particular disproportionate change, i.e., a change in the relative size of the sectors.

The combined variables {mP1 mP2 mw} on the right, determinate from the physical data alone, are here termed the "essential" prices and money wage. With no change in the essential economic reality any variations in the nominal magnitudes {w P1 P2} are exactly counterbalanced by inversely proportionate variations in the magnitude of m. {mP1 mP2 mw} are unchanged and such nominal variances thus amount to, in essence, no price or wage change at all.

By contrast any change in the essential economic reality, i.e., some change in {k1 k2 q1 q2 d RW}, must always be reflected in some change in the essential prices and money wage. In the case of an isolated change in the relative size of the sectors [an isolated change in d] {mP1 mP2 mw} are affected because the magnitude of m changes in this circumstance.

In the following section it is shown how the above determination given the essential physical data may be arrived at in an argument that begins instead with the elements of the labor value system.

IV

Labor Value Analysis

The difficulty with "labor value analysis," defined here as somehow showing the determination of the "price system" beginning with the elements of the "labor value system" alone, is obvious. One may construct numerical examples of two sector realities in the form of equations (1) and (2), and proceed to continue constructing such examples ad infinitum, wherein: 1) the labor value system in each example is identical; and 2) the relative prices and the money rates of profit in these examples are all different.

For Steedman, who in his Marx After Sraffa presented what he characterized as "the Sraffa-based critique of Marx," the reason for this difficulty was plain: labor value analysis was just irrelevant. Two of his arguments warrant quotation:

> i) the conditions of production and the real wage paid to workers, both specified in terms of physical quantities of commodities, suffice to determine the rate of profit [and less importantly, all prices of production];
> ii) the quantities of labour embodied in the various commodities, which can themselves only be determined once the conditions of production are known, thus play no essential role in the determination of the rate of profit [or the prices of production]; [Steedman, 1981, p. 14]

Later in the work [p. 48] these points are schematized as follows:

```
                      (a)
              ------------------->  All value quantities
              ^                              |
              |                              |
              |                             ===  (c)
              |
              |
     Physical production  ------->  Profits and prices
        and wage data       (b)
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The purpose of this section is to present, based of the arguments of the preceding sections, a critique of this "Sraffa-based critique of Marx" in the form of a reexamination of paths (a), (b) and (c).

(b)

Path (b) was discussed in the preceding sections. Given the limited representation of the "physical production and wage data" in the premises of the usual quantity system argument, it is just not true that "prices" - plural in any sense in a two sector system - are calculable. The relative price and the real wage alone are determinate within this interrelation. By contrast when all of the relative information defined by the "physical production and wage data" is taken into account then the essential prices and money wage { mP1 mP2 mw } are separately determinate, and these results abstract from any purely nominal variances.

(a)

It is certainly a fact that the elements of the "labor value system" - the two individual labor values, the value of labor power, and the value rate of profit - are calculable from the physical data. What has not been appreciated is that one of these elements, the value rate of profit, is incalculable given only the limited premises of the usual quantity system argument.

This determination of the labor value system is here described utilizing the same notation as in the previous sections. The defining equations for the individual labor values v1 and v2 are:

$$N1v1 + L1 = Q1v1$$
$$N2v1 + L2 = Q2v2$$

Or:

$$k1v1 + 1 = q1v1 \qquad (14)$$
$$k2v1 + 1 = q2v2 \qquad (15)$$

The hourly real wage multiplied by v2 defines the value of labor power vl, or the labor time needed to produce the real wage:

$$vl = v2RW \qquad (16)$$

{v1 v2 vl} are therefore determined from (14), (15) and (16) given the premises of the usual quantity system argument {k1 k2 q1 q2 RW} [Section II].

The rate of profit in terms of values rv is first written in the CVS notation:

$$rv = S/[C + V]$$

S or surplus value is the difference between the total direct labor time and the total value of labor power V, and V in turn equals Lvl. C is the total value of the means of production inputs, or Nv1. Therefore:

$$rv = [L - Lvl]/[Nv1 + Lvl]$$

After dividing the numerator and the denominator by L the ratio N/L appears in the denominator. This is the average composition of capital as defined in Section III and it is again designated k(8).

The equation for the value rate of profit now becomes:

$$rv = [1 - vl]/[kv1 + vl]$$

This may be rewritten as follows:

$$[kv1 + vl][1 + rv] = kv1 + 1 \qquad (17)$$

It is evident that rv is indeterminate absent knowledge of d, hence k(8), and it is thus shown that the complete determination of the labor value system requires consideration of all of the information essentially defined by the physical data. This determination is outlined as follows:

physical data -> {k1 k2 q1 q2 d RW} -> {v1 v2 vl rv}

It will be noticed that equation (17) is similar to equation (12) in the essential price system [Section III]. Neither equation can be written if the average composition of capital k(8) is indeterminate, and this is the case if the variable d and hence the relative size of the sectors is indeterminate. This information, determinate within equations (1) and (2), is simply lost in the course of the derivation of equations (5) and (6), the foundation of the usual limited quantity system argument, as already discussed.

(c)

When this complete determination of the labor value system from the physical data is understood along with the revision of path (b) described previously, the reconstruction of path (c) may be undertaken. This may be briefly outlined, with details below, as follows: the elements of the essential price system {mP1 mP2 mw r} may in all cases be calculated from the elements of the labor value system {v1 v2 vl rv} given the additional knowledge of the individual compositions of capital {k1 k2}, and this argument itself may be generalized. Path (b) may in fact be replaced by this reconstruction of path (c).

The first step is to define exactly what is known given the elements of the labor value system alone. To begin with, from {v1 v2} what is known, from equations (14) and (15), are certain

interrelations among {k1 k2 q1 q2}; then given {vl} the real wage RW is calculated from (16); and then given {rv} the average composition of capital
{k} (8) may be calculated from (17). {k1 k2 q1 q2 d} remain specifically incalculable. This limited determination is outlined as follows:

labor value system
{v1 v2 vl rv} -> {v1 v2 k RW} with (14) & (15)

The determination may now be completed with the additional knowledge of {k1 k2}. {v1 v2 k1 k2} are now known and {q1 q2} are calculated from (14) and (15). Then with {k k1 k2 q1 q2 RW} all now known the four elements of the essential price system {mP1 mP2 mw r} are determined from the four equations (10)-(13). This general determination is outlined as follows:

labor value system				essential price system
{v1 v2 vl rv}	+	{k1 k2}	->	{mP1 mP2 mw r}

[Of course if the embodied labor times {v1 v2 vl} alone are given then the determination is completed given the additional knowledge of all of {k k1 k2}, and the calculation of rv is a by-product.]

The essential price system is thus calculated beginning with the labor value system in a common argument that applies in all circumstances. Within this single argument it may now be understood how the elements of the labor value system may coincide with the elements of the essential price system and why they usually differ.

With equal compositions of capital the average composition of capital {k} determinate within the labor value system ipso facto defines {k1 k2} since they are all equivalent. {mP1 mP2 mw r} are then calculated according to the above argument, and it may then be discovered in a separate and subsequent investigation that these elements are identical to the corresponding elements of the labor value system.

[The precise logic is as follows. If k1 equals k2 then {k k1 k2} are all equivalent; the left sides of equations 10)-(12), and hence their right sides, are therefore equivalent. The left sides of (14) and (15), and hence their right sides, are also therefore equivalent.
From the equivalence between the right sides of (10) and (11) the relative price P1/P2 equals q2/q1; and from the equivalence between the right sides of (14) and (15) v1/v2 also equals q2/q1. P1/P2 therefore equals v1/v2. From the equivalence between the right sides of (10) and (12) mP1 equals the inverse of [q1 - k]; and from (14) v1 equals the inverse of [q1 - k1]. Then since k equals k1, v1 equals

mP1; and then since P1/P2 equals v1/v2, v2 equals mP2. Then from (13) and (16) vl equals mw, and then from (12) and (17) rv equals r.]

With differing compositions of capital the determination may be completed if the variables {k1 k2} are given as additional information [the only constraint being that one of {k1 k2} must be greater than {k} and the other must be less]. {mP1 mP2 mw r} are again calculated and it is apparent that these results [and hence the relative price] are dependent upon the exact magnitudes of {k1 k2}. The elements {mP1 mP2 mw r} must differ from the corresponding {v1 v2 vl rv} except in two particular circumstances [Section VI].

[The special circumstance of a zero money rate of profit r may be mentioned. At this limit it is only necessary to know {v1 v2} along with r = 0. Here mw is calculated at unity from (12), and (10) and (11) then become identical to (14) and (15) respectively. mP1 then equals v1 from (10) and (14), and mP2 then equals v2 from (11) and (15). It is not necessary to know any of {k1 k2 q1 q2 d RW}.]

It is possible, in an entirely analogous argument, to travel in reverse and calculate the labor value system beginning with the essential price system. From {mP1 mP2 mw r} the average composition of capital k and the real wage RW are again determinate, here from (12) and (13). With {k1 k2} again additionally given {q1 q2} are calculated from (10) and (11), and {v1 v2 vl rv} are then calculated from (14)-(17). This argument may be outlined similarly:

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essential price system                labor value system
    {mP1 mP2 mw r}   +   {k1 k2}   ->   {v1 v2 vl rv}
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More generally, either of these schematized arguments may be completed given the additional knowledge of two of {k1 k2 q1 q2 d} with the exceptions of {q1 k1} and {q2 k2}.

It may be said that the labor value system alone under usual conditions amounts to a partial definition of the essential economic reality, but a similar criticism is here made in regards to the usual limited quantity system construction. The difference is that labor value analysis alone is short by two data while the quantity system argument is short by only one, the variable d. The relative price P1/P2 is determinate in the latter, hence the seductive appeal of this approach, but it remains incomplete as described.

V

Profit and Surplus Value

In this section the concepts of profit and surplus value are investigated in light of the above arguments. Mention was made earlier [Section I] of the "Fundamental Marxian Theorem," the conclusion of the quantity system argument that profit is positive if and only if surplus value is positive. It will be shown presently that quantitative judgments regarding profit and surplus value may be concluded. It will be demonstrated that: the totals of both of these quantities are definite numbers of man-hours of labor time; they differ under usual two sector conditions; this difference merely reflects the usual difference between the essential price and labor value of the consumer commodity; and there is a commonsense interpretation of this difference.

Surplus value was already defined in Section IV as the difference between the total direct labor time L and the total value of labor power, or Lvl, or Lv2RW:

Surplus value = L - Lv2RW

Lv2RW is the labor time needed to produce the total real wage; it is the necessary labor time.

The relation between the total current dollar money wage and the net income may be said to represent the extent to which the collective working day resolves into its paid and unpaid portions. If total wages were one-third of the net income then it could be said that the workers had spent one-third of the total labor time working for themselves, and this one-third portion of the total is the paid labor time. But this ratio of total wages to the net income equals the essential hourly money wage mw:

total wages/net income = paid time/total time = mw

And therefore: paid time = Lmw

The unpaid labor time then is the difference between the total direct labor time and this paid time:

Unpaid labor time = L - Lmw = L - LmP2RW

If the nominal profit [Y - Lw] is multiplied by m the same expression is generated. This "essential" profit is the unpaid labor time. The formulae for surplus value and essential profit are similar and it is evident that both quantities are definite numbers of man-hours of direct labor time. These coincide if and only if the essential price of the consumer commodity equals its labor value, and since these usually differ the essential profit usually differs from surplus value. The "meaning" of this difference amounts to just this: the time it takes to produce the real wage differs from the time it takes to earn the money to buy it. There is no other meaning. The collective working day simply fractionates in two different ways, unpaid/paid and surplus/necessary, depending on this usual difference between essential price and labor value.

VI

Other Indeterminacies in the Usual Quantity System Construction

The criticism, generally stated, of the usual quantity system argument advanced in this paper is that the two sector system is underdetermined within the confines of this approach. The two sector system is indeterminate in other important respects apart from those previously discussed, and for the same reason, and these are described in this section.

*   *   *

Marx contended that the capitalists, if they were to survive as such, had no choice but to attempt to accumulate, to extend their capital [Marx, 1867, Chap. XXIV, Section 3]. That is, they could not be represented as perfectly free to spend their profits on personal consumption. On the other hand it could not be imagined that they would allow themselves to starve. These assertions may be more succinctly expressed as follows: no definite trend in the fraction of profit consumed by the capitalists can be postulated, and therefore for analytic purposes this quantity should be considered a constant. But before it can be fixed it must be defined. In the notation here capitalists' consumption equals [N2P2 - wL] and total profit equals [Y - wL]. The fraction consumed is the ratio of the two, here designated F, and the following equation is easily derived [wherein m equals L/Y as before]:

$$1 - [1 - F][1 - mw] = dq2mP2 \qquad (18)$$

The fraction of profit consumed by the capitalists F is indeterminate absent consideration of the variable d. [This conclusion is independent of any judgment regarding any tendency of F; it follows from its definition alone]. Furthermore, it just so happens that upon this manipulation of the ratio defining F the coefficient m, the relation between the total direct labor time and the price of the net product, appears again in relation to w and P2.

*　　　　　*　　　　　*

If the relative size of the sectors is indeterminate the two sector system may or may not even be viable, in two respects. To begin with, the variables {k1 k2 q1 q2} define an upper limit for the real wage, and this limit is the inverse of the labor value of the consumer commodity v2. The money rate of profit r is positive if and only if RW is below this limit:*4*

$$RW < 1/v2 \qquad (19)$$

In the following this inequality is assumed and for further purposes here is rewritten as follows:

$$RW/q2 < 1/q2v2$$

Given only that there are two sectors, i.e., L1 and L2 are both positive, d or L2/L must in the first place be a positive fraction less than unity, or $0 < d < 1$. But the two sector system is viable if and only if d is further constrained, and these constraints are in fact defined in the above inequality. That is, d must be such that the following holds:

$$RW/q2 < d \ \text{<or=} \ 1/q2v2 \qquad (20)$$

The inequality on the right reduces to the inequality Q1 >or= N1 + N2, and the real output of the capital goods sector is at least equal to the sum of the real nonlabor inputs of both sectors.*5* If d exceeds 1/q2v2,
Q1 < N1 + N2 and the system is not viable; if d equals 1/q2v2, Q1 = N1 + N2 and simple reproduction exists.

If on the other side RW/q2 > d, then F < 0 and capitalists' consumption is extinguished.*6*

To summarize: the money rate of profit r is positive, capitalists' consumption exists, and simple reproduction at least exists if and only if the double inequality (20) holds. The latter two conditions are indeterminate absent knowledge of d, hence absent knowledge of the relative size of the sectors.

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It was noted in Section I that under the usual two sector conditions of differing compositions of capital and a nonzero equalized money rate of profit the aggregate {labor time/money} ratios would all differ. When the relative size of the sectors is taken into account, and only then, three exceptions to this rule may be discovered. [Proofs of the first and third of these exceptions are omitted from this paper.]

Firstly, and again assuming the above usual conditions, in one particular circumstance the aggregate ratios {total value/total price}, {total input value/total cost price}, and {total surplus value/total profit} are equalized; the money rate of profit equals the value rate of profit; and Marx's postulated double equality [Section I] holds after all. This occurs when the essential physical data satisfy the following equation:

$$RW = q2dk/[q1[1 - d]], \qquad (21)$$

where k = [1 - d]k1 + dk2 (8). This equation holds if and only if the aggregate [L1w] in equation (1) equals the aggregate [N2P1] in equation (2). It should be noted that with differing compositions of capital and a nonzero equalized money rate of profit the above common ratio cannot equal the aggregate ratio {total direct labor time/price of the net product}, or m.

Secondly, in the extraordinary circumstance of simple reproduction - where Q1 = N1 + N2 - the ratio {total surplus value/total profit} equals m. Whether or not simple reproduction exists depends on knowledge of the relative size of the sectors. It was noted above that d equals 1/q2v2 under simple reproduction and therefore v2 equals 1/dq2. F in (18), the fraction of profit consumed by the capitalists, is unity in this circumstance and the same value for mP2 is calculated. v2 equals mP2 and therefore from (13) and (16) vl equals mw. Total surplus value therefore equals the essential profit, or the nominal profit multiplied by m [Section V]. The ratio {total surplus value/total profit} therefore equals m under simple reproduction.

Thirdly, the ratio {total input value/total cost price} equals m if and only if the following equation holds:

$$RW = q2dk/[q1[1 - d] - k], \qquad (22)$$

again where k = [1 - d]k1 + dk2 (8).

VII

Concluding Remarks

The genesis of this paper was a dissatisfaction with the usual quantity system argument wherein the rate of profit and relative price alone are determinate given the physical data. Any relation between labor time and money evaporates within the confines of this argument, but the objection does not originate with this fact. The difficulty rather is that this argument mishandles its own premises: it omits from consideration, and quite without any justification this writer is aware of, a certain datum defined by the physical data, the relative distribution of social labor time that fixes the relative size of the sectors.

Furthermore, this omission is selective since it is "known" that the rate of profit in terms of values rv does not, under usual two sector conditions, equal the rate in terms of prices. But this knowledge cannot be arrived at if rv is indeterminate, and this is the case if the relative size of the sectors is not taken into account. Further still, when this information is after all considered it may be discovered that in one accidental circumstance the two rates after all do equalize.

It is only when the relative size of the sectors is taken into account that the average composition of capital is determinate, and it is only then that the essential price and labor value systems may be completed. Specifically, it is only then that equations (12) and (17) may be written. At the same time it becomes plain that given either system alone the essential economic reality is only partially determined, and it becomes evident exactly why any one labor value system may coexist with an infinity of disproportionately disparate essential price systems and why any one essential price system may coexist with an infinity of disproportionately disparate labor value systems.

FOOTNOTES

1 Certain of the aggregate {labor time/money} ratios may equalize in certain extraordinary or accidental circumstances despite differing compositions of capital and a nonzero equalized rate of profit. These exceptions are discussed in Section VI.

2 The solution is to combine (5) and (6), eliminating [1 + r]. The following quadratic equation is then derived:

$$X^2 + X[A - BC] - AB = 0,$$

where $X = P1/P2$; $A = RW/k2$; $B = q2/q1$; and $C = k1/k2$. This is solved for X or P1/P2 in the usual way and r is then calculated from (5) or (6).

3 This concept of the "average composition of capital" appears at the beginning of Chapter X in Volume III of Capital [Marx, 1894, p. 173].

4 From (12) mw = 1 at r = 0, and then from (13) RW = 1/mP2. mP2 = v2 at r = 0 [Section IV] and therefore RW = 1/v2 at this upper limit.

5 This follows from (14) and (15) along with the definitions of d, k1, k2, etc.

6 Equation (18) may be rewritten as follows after substituting mw/RW for mP2 (13):

$$d/[RW/q2] = 1 + F[1/mw - 1]$$

[1/mw - 1] on the right is always positive since mw < 1. d/[RW/q2] is therefore greater than unity, or RW/q2 < d , if and only if F is positive, i.e., capitalists' consumption exists.

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